PROBABILITY AND STATISTICS, H21, FINAL EXAM

Name: ________________________ Student number______________________

(1) (3.5 marks) In Terrebonne one in 120 car owners drives an electric car. If two hundred Terrebonne households are sampled at random.
   (a) Argue informally that the conditions to use a Poisson approximation to the Binomial are satisfied. Use the Poisson approximation to answer the questions which follow.
   (b) Calculate the probability that in 200 Terrebonne car owners, at least three people drive an electric car.
   (c) How many Terrebonne car owners must be sampled so that the probability of including at least one driver with an electric car is 98% or more?

(2) (3 marks) A sample of 6 calls to the office of a family physician gives the following wait times in minutes

$$5.3 \quad 14.8 \quad 4.4 \quad 6.7 \quad 3.8 \quad 10.9$$

The secretary claims that the response is within 5 minutes give or take 4 minutes. Assuming that the distribution of waiting times is normal test $H_0 : \sigma = 4$ versus $H_1 : \sigma > 4$.

(3) (3.5 marks) FraiseBec is the largest producer of strawberries in Canada. The length of time X (in hours), needed for me to complete a round trip to their fields in Sainte-Anne-des-Plaines (time to pick the strawberries included) is distributed according to

$$p(x) = \begin{cases} k(x^3 + x) & 1 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k that makes $p(x)$ a probability density function
(b) Find the cumulative distribution function.
(c) Compute the expected value and the standard deviation.

(4) (3.5 marks) The amount of trash generated by eight Candiac households, in kgs per week is given below

$$3.8 \quad 4.6 \quad 12.0 \quad 10.5 \quad 8.1 \quad 9.2 \quad 5.0 \quad 6.4$$

Assume that the population is normally distributed.

a) Compute the sample mean and the sample variance.

b) Construct a 95% confidence interval for the population mean.

c) Construct a 95% confidence interval for the population standard deviation.

(5) (3 marks) Every year Montreal college welcomes 6400 students. 800 of them are foreign students. Use normal approximation to estimate the probability that in a random sample of 1000 students from this college between 100 and 120 (inclusive) are foreign students.

(6) (3 marks) Let μ denote the true average length of sleep per night of college students during the final exam period. Consider testing $H_0 : \mu = 7.4$ versus $H_1 : \mu < 7.4$ based on a sample of size $n = 28$ at $\alpha = 0.01$ level of significance. Assume that the population is normal with standard deviation of 2.4 hours. What is the probability of making a type II error when $\mu = 6.9$ hours? What is the power of the test?

(7) (3.5 marks) At the Forex desk of a major bank orders for Swiss Francs come according to a Poisson process with rate of 3 orders per hour.

a) Determine the pdf for the waiting time for fifth order.

b) Determine the mean and the variance for the waiting time for the fifth order.

c) An hour has passed with no orders for Swiss Francs. Determine the probability that more than additional 2 hours will ellapse before five orders for Swiss Francs are placed.

(8) (3.5 marks) The average number of peperoni slices on two competing brands of frozen pizza were as follows: Brand A: sample size 10; sample mean 16.4; sample standard deviation 2.3; Brand B: sample size 12; sample mean 14.6; sample standard deviation 3.4. You can assume that the populations are normal.

a) Compute a 98% confidence interval for the ratio of population variances. Based on this interval can you claim that the variances are different with 98% confidence?

b) Compute a 98% confidence interval for the difference of population means. Based on this interval can you claim that the mean number of peporoni slices differ for the two brands of frozen pizza with 98% confidence?

(9) (3.5 marks) A RV X has moment generating function $M_X(t) = exp(-4t+6t^2)$.

a) Determine $p(-7 \leq X \leq 0)$.

b) Determine the probability density for the RV $Z = \exp Y$.

(10) (3 marks) A teacher claims that the average number of extension requests in a Calculus class is about thirty per semester. A sample of 14 semesters has a mean number of extension requests equal to 22.3 with sample standard deviation of 5.8. Test $H_0 : \mu = 30$ versus $H_1 : \mu < 30$ at the 1% level of significance. Report a range for the p-value and make sure to draw a conclusion in the context of the problem.

(11) (3.5 marks) A Prob/Stats course has two exams. For a randomly selected student, let X be the number of points earned on the first exam and Y be the number of points earned on the second exam (rounded to nearest multiple of 5). Suppose that the joint pmf of X and Y is given by the accompanying table:

		Y		
	$p(x,y)$	30	35	40
	0	0.07	0.02	0.01
X	5	0.20	0.20	0.10
	10	0.16	0.14	0.10

a) Compute the marginal probability distributions of X and Y.

b) Compute the conditional probability mass function of Y given that $X = 10$.

c) Compute the conditional mean of Y given $X = 10$. Write a sentence in English interpreting your findings.

d) Compute the correlation between the RV's X and Y.

(12) (3.5 marks) An experiment consists of tossing two (unfair) Belgian Euro coins and recording the number of heads. Assume $p(H) = 0.47$ and $p(T) = 0.53$, Approximate (using CLT) the probability that the average number of heads obtained by repeating this experiment 100 times will be in the range from 0.9 to 1.1.