Name: ______________________________
Date:

Business Data Analysis
201-316-VA

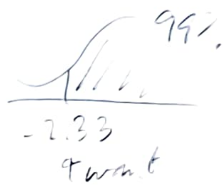
In Class Exercise #11: Normal, Sampling, Approx

1. **Satellite Insurance**

   A relay microchip in a telecommunications satellite has a life expectancy that follows a normal distribution with a mean of 90 months and a standard deviation of 3.7 months. When the microchip malfunctions, the entire satellite is useless. An insurance company will insure the satellite, making a $50 million payout in the case of a satellite failure. Assume that all the other components of the satellite will work indefinitely.

   (a) For how many months should the satellite be insured to be 99% confident that it will last beyond the insurance date?

   $P(Z < 2.33) = 0.99 \rightarrow P(Z < -2.33) = 0.01$

   $X = Z\sigma + \mu$

   $= -2.33(3.7) + 90 = 81.379$

   around 81 months

   (b) If the satellite is insured for 84 months, what is the probability that it will malfunction before the insurance coverage ends?

   $Z = \dfrac{X - \mu}{\sigma} = \dfrac{84 - 90}{3.7} \approx -1.62$

   $P(Z < 1.62) = 0.9474$

   $P(Z < -1.62) = 1 - 0.9474 = 0.0526$

   (c) If the insurance company charges $3 million for 84 months of insurance, how much profit does the company expect to make? Show your computations.

   They make $3 million

   They lose on average $50 mil × 0.0526
   = $2.63 mil

   Profit = $3 mil − $2.63 mil = $370,000

2. **Impulse Buying**

Let x represent the dollar amount spent on supermarket impulse buying in a 10-minute unplanned shopping interval. The mean of the x distribution is about \$20 and the estimated standard deviation is about \$7.

(a) Consider a random sample of 100 customers who have a 10-minute unplanned shopping time.

i. In words, what does the random variable $\bar{x}$ represent?

The average spending of the 100 customers

ii. What can we say about about the probability distribution of $\bar{x}$?

It's approx. normal with mean $\bar{X}_\mu = \$20$

& std dev $\bar{X}_\sigma = \$7/\sqrt{100} = \$0.7$

$\sigma/\sqrt{n}$

iii. What theorem did you use to answer ii)? What assumptions did you make, if any?

Used the Central limit theorem

Since there are 100 customers, $n = 100 \geq 30$.

No assumptions

(b) What is the probability that $\bar{x}$ is between \$18 and \$22?

$$z_1 = \frac{\bar{x}_1 - \mu}{\sigma/\sqrt{n}} = \frac{18-20}{0.7} \approx -2.86$$

$$z_2 = 2.86$$

$1 - 0.9979 = 0.0021$

0.0021

$P(18 < \bar{x} < 22) = P(-2.86 < z < 2.86)$

$P(z < 2.86) = 0.9979$

$P(-2.86 < z < 2.86) = 0.9979 - 0.0021 = 0.9958$

(c) If we assume that x has a distribution that is approximately normal, what is the probability that x is between \$18 and \$22?

$$z_1 = \frac{x - \mu}{\sigma} = \frac{18-20}{7} \approx -0.286 \qquad z_2 = 0.286$$

$P(18 < x < 22) = P(-0.29 < z < 0.29)$

$P(z < 0.29) = 0.6141$

0.3859

$1 - 0.6141 = 0.3859$

$= 0.6141 - 0.3859 = 0.2282$

3. **New Products**

A study shows that 80% of all new products introduced in grocery stores fail and are taken off the market within 2 years. A grocery store chain introduces 66 new products. Approximate the probability that within 2 years...

(a) 15 or more succeed

Bin: $\mu = np = 66(0.2) = 13.2$

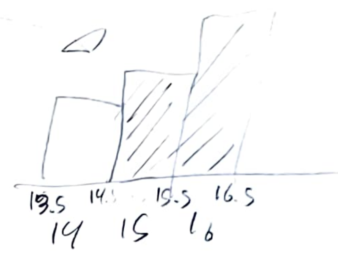
$\sigma = \sqrt{npq} = \sqrt{66(0.2)(0.8)} = \sqrt{10.56} \approx 3.2496$

$P(r \geq 15) = P(x \geq 14.5)$

$z = \dfrac{x-\mu}{\sigma} = \dfrac{14.5-13.2}{\sqrt{10.56}} \approx 0.40$

$= P(z \geq 0.40) = 1 - P(z < 0.4) = 1 - 0.6554$

$= 0.3446$



(b) fewer than 10 succeed

$P(r < 10) = P(x < 9.5)$

$z = \dfrac{x-\mu}{\sigma} = \dfrac{9.5-13.2}{\sqrt{10.56}} \approx -1.14$

$P(z < -1.14) = P(z > 1.14) = 1 - P(z \leq 1.14)$

$= 1 - 0.8729$

$= 0.1271$