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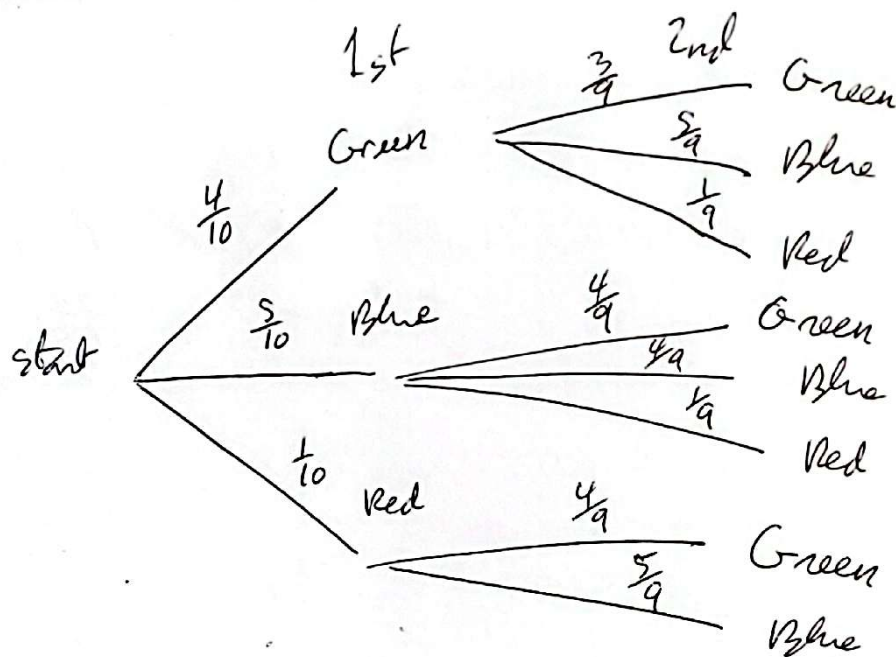
Business Data Analysis
201-316-VA

In Class Exercise #9: Counting and Probability Distributions

1. Balls in urn

An urn contains 10 balls that are identical except for color: 4 green balls, 5 blue balls and 1 red ball. Two balls are drawn from the urn, one after another and without replacement.

- (a) Draw a tree diagram that includes probabilities to map out the possible results for the two draws and their associated probabilities. Use exact fractions. Check that the probabilities sum to 1.



$$\begin{aligned}
 P(GG) &= \frac{12}{90} = \frac{2}{15} \\
 P(GB) &= \frac{20}{90} = \frac{2}{9} \\
 P(GR) &= \frac{4}{90} = \frac{2}{45} \\
 P(BG) &= \frac{20}{90} = \frac{2}{9} \\
 P(BB) &= \frac{20}{90} = \frac{2}{9} \\
 P(BR) &= \frac{5}{90} = \frac{1}{18} \\
 P(RG) &= \frac{4}{90} = \frac{2}{45} \\
 P(RB) &= \frac{5}{90} = \frac{1}{18}
 \end{aligned}$$

$$\Sigma = 90/90 = 1$$

- (b) Let x be the number of green balls drawn. Use your answer in part a) to create a probability distribution for x in a small table. Use exact fractions. Check that the probabilities sum to 1.

x	$P(x)$
0	$30/90 = \frac{1}{3}$
1	$48/90 = \frac{8}{15}$
2	$12/90 = \frac{2}{15}$

$$P(BB) + P(BR) + P(RB)$$

$$P(GB) + P(GR) + P(BG) + P(RG)$$

$$P(GG)$$

- (c) Find the mean and standard deviation for x from part b)

x	$P(x)$	$xP(x)$	$x^2P(x)$
0	$\frac{1}{3}$	0	0
1	$\frac{8}{15}$	$\frac{8}{15}$	$\frac{8}{15}$
2	$\frac{2}{15}$	$\frac{4}{15}$	$\frac{8}{15}$
Σ	1	$\frac{12}{15} = \frac{4}{5}$	$\frac{16}{15}$

$$\mu = \Sigma xP(x) = \frac{4}{5}$$

$$\sigma^2 = \Sigma x^2P(x) - \mu^2 = \frac{16}{15} - \left(\frac{4}{5}\right)^2 = \frac{32}{75}$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{32}{75}} \approx 0.6532$$

2. Deck of Cards

A standard deck of 52 cards contains 4 suits (heart, diamonds, clubs, spades), and each suit has 1 card of each of the 13 ranks (10 number cards, and 3 face cards: J, K, Q)

Answer in fractions, and show your steps. When drawing cards without replacement from the deck...

- (a) What is the probability that when drawing 3 cards, all 3 are hearts?

$$P(HHH) = P(1st H) P(2nd H | 1st H) P(3rd H | 1st H \& 2nd H) \\ = \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} = \frac{1716}{132,600} = \frac{11}{850}$$

- (b) What is the probability that when drawing 2 cards, one is a heart and one is a spade?

$$P(H \& S) = P(1st H \& 2nd S) + P(1st S \& 2nd H) \\ = P(1st H) P(2nd S | 1st H) + P(1st S) P(2nd H | 1st S) \\ = \frac{13}{52} \cdot \frac{13}{51} + \frac{13}{52} \cdot \frac{13}{51} = \frac{338}{2652} = \frac{13}{102}$$

- (c) In a certain game, you draw 2 cards, and you will win if both are hearts or both are faces. What is the probability of winning?

$$P(\text{win}) = P(2H) + P(2F) - P(2H \text{ and } 2F) \\ = \frac{13}{52} \cdot \frac{12}{51} + \frac{12}{52} \cdot \frac{11}{51} - \frac{3}{52} \cdot \frac{2}{51} \\ = \frac{282}{2652} = \frac{47}{442}$$

3. Counting

An activities club has 12 members.

- (a) How many different committees can they make consisting of 4 members?

$${}_{12}C_4 = 495$$

- (b) How many different committees can they make consisting of 1 president, 1 vice-president, 1 treasurer and 1 secretary?

$${}_{12}P_4 = 11,880$$