1.1 Four Ways to Represent a Function (Cont'd)

The Domain of A Function

Being able to determine the domain of a function is a crucial skill in calculus, because it gives you the capability to assess where a function is properly defined. Since each function is different, there is no "one size fits all" strategy which can be deployed across all functions to compute their domains. However, for the vast majority of functions, this can be resolved by ensuring that values which lead to:

- division by zero
- negative values under an even root

are excluded from the domain.

Example 1: 1.1.39

Compute the domain for $f(x) = \dfrac{x+4}{x^2-9}$

Solution

Example 2: 1.1.40

Compute the domain for $f(x) = \dfrac{x^2+1}{x^2+4x-21}$

Solution

Example 3

Compute the domain for $f(x) = \dfrac{2x^2 + 5x + 3}{10 - 3x - x^2}$

Solution

Example 4: 1.1.42

Compute the domain for $g(t) = \sqrt{3 - t} - \sqrt{2 + t}$

Solution

Example 5: 1.1.44

Compute the domain for $f(x) = \dfrac{x + 1}{1 - \frac{1}{x+1}}$

Solution

Example 6

Compute the domain for $g(x) = \sqrt[4]{2x+5} + \sqrt[3]{x-9}$

Solution

Example 7

Compute the domain for $g(x) = \sqrt{x-10} - \sqrt{x-5} + x^2$

Solution

Example 8

Compute the domain for $h(x) = \dfrac{1}{x-9} - \sqrt{x}$

Solution

Example 9

Find the domain for $k(x) = \dfrac{\sqrt{4-x}}{x^2-4x-12}$.

Solution

Techniques For Finding Domains of Harder Functions

In the next few examples, we will show how to find the domain of functions for which the expression in the radicand are nonlinear. Working out the domains of these functions require a two-pronged approach:

1. Determining the zeros of the expression, and if required, values which result in division by zero.
2. Ascertaining the sign $(+/-)$ of the argument by directly plugging in the values which occur before and after the zeros into the expression.

Example 10

Find the domain for $f(x) = \sqrt{x^2-16}$.

Solution

Example 11

Find the domain for $f(x) = \sqrt[4]{\dfrac{x-1}{(x-3)(x+5)}}$

Solution

Graphs

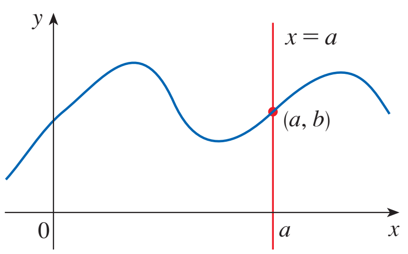
The graph of a function often reveals properties of the function which are not always apparent from it algebraic form. For example, on which intervals the function is increasing/decreasing on; if it is continuous on its entire domain or not; if it possesses symmetry about the y−axis or about the origin; and so on.

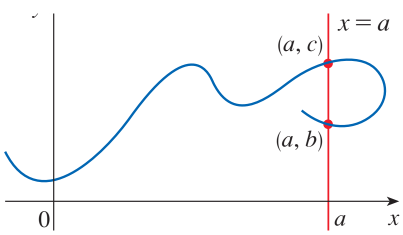
The Vertical Line Test

In mathematics, the vertical line test is a visual way to determine if a curve is a graph of a function or not.

Theorem: The Vertical Line Test

A curve in the xy−plane is the graph of a function if and only if no vertical line intersects the curve more than once.

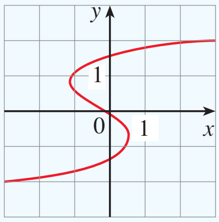




The curve on the left **is a** function; it passes the vertical line test. The one on the right **is not** a function; when $x = a$ a vertical line cuts the curve more than onece.

Example 12

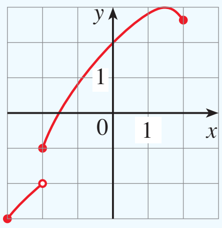
Consider the following curve. Determine if it is a function or not using the vertical line test. If it is, state the domain and range of the function.



Solution

Example 13

Consider the following curve. Determine if it is a function or not using the vertical line test. If it is, state the domain and range of the function.



Solution

Piecewise Defined Functions

Piecewise-defined functions are not defined by a single equation, but by two or more; and equation is valid over some interval.

Definition 1: Piecewise Defined Functions

A **piecewise-defined function** is function that is defined over a sequence of intervals. They have general form

$$f(x) = \begin{cases} \text{rule 1} & ; x \in \text{domain 1} \\ \text{rule 2} & ; x \in \text{domain 2} \\ \text{rule 3} & ; x \in \text{domain 3} \\ \vdots & \end{cases}$$

Example 14

Let

$$f(x) = \begin{cases} 2x^2 + 5 & ; \quad x < 1 \\ x^2 & ; \quad 1 \le x < 3 \\ x + 1 & ; \quad x \ge 3 \end{cases}$$

Determine the following values

a. $f(0)$

b. $f(3)$

c. $f(1)$

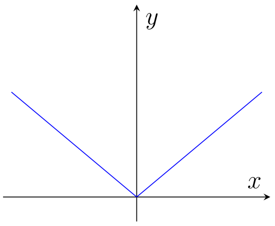
d. $f(-5)$

e. $f(10)$

Solution

Piecewise-defined functions occur frequently in engineering and commerce applications. The Heaviside, step, and floor-functions are examples of piecewise defined functions. Perhaps the most famous piecewise-defined function is the absolute value function. It has two pieces, and both the algebraic and graphical form of the absolute value function are presented below.

$$|x| = \begin{cases} -x & if\ \ x < 0 \\ +x & if\ \ x \geq 0 \end{cases}$$



Example 15

Find the domain and sketch the function of $f(x) = 4 - 2x$

Solution

Example 16

Find the domain and sketch the function of $f(x) = x^2 - x - 2$

Solution

Example 17

Find the domain and sketch the function of $f(x) = |2x - 1|$

Solution

Example 18

Find the domain and sketch the function of

$$f(x) = \begin{cases} x + 2 & ; x < 0 \\ 1 - x & ; x \geq 0 \end{cases}$$

Solution

Example 19

Find the domain and sketch the function.

$$f(x) = \begin{cases} x+2 & ; x \leq -1 \\ x^2 & ; x > -1 \end{cases}$$

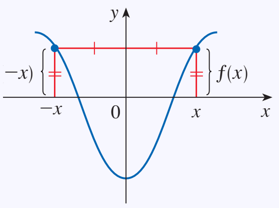
Solution

Symmetry

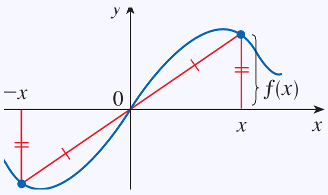
Calculus has a number of applications, and one that you will learn about in this course is curve sketching. Before sketching the graph of a function several tests are carried out to streamline the process. One of these tests, is a test for symmetry.

Definition 2

If a function satisfies $f(x) = f(-x)$ for all values of x in its domain, then f is symmetric about the y–axis, and is classified as an **even function**.



If the function satisfies $-f(x) = f(-x)$ for all values of x in its domain, then f is symmetric about the origin, and is classified as an **odd function**.



Example 20

For each of the following function, determine if it is even, odd, or neither.

a. $f(x) = x^2 - x^4$

b. $g(x) = \dfrac{1}{x\sqrt{x^2 - 1}}$

c. $k(x) = x^3 + x^5 - 2x$

Solution

Example 21: 1.1.36

Evaluate the difference quotient for the given function and simplify your solution.

$$f(x) = x^3 \qquad ; \qquad \frac{f(a+h) - f(a)}{h}$$

Solution

Example 22: 1.1.38

Evaluate the difference quotient for the given function and simplify your solution.

$$f(x) = \sqrt{x+2} \qquad ; \qquad \frac{f(x) - f(1)}{x - 1}$$

Solution