

# SN1 Practice Test 2

## General Information and Recommendations

- It covers lectures:
  - L4. Probability Theory (Introduction to Probability)
  - L5. Probability Theory (Counting Techniques)
  - L6. Probability Theory (Rules of Probability)
  - L7. Probability Mass Functions
  - L8. Expected Value and Variance of a Discrete Random Variable
- It is strongly advised that you go **over all of the problems covered in class, the class exercises, examples on Probabilia, and the problems on this test.**

**Practice Test 2 - A**

Winter 2025

Name: 





This test consists of 7 questions.

You will have **1.5 hours** to complete this test.

Instructions:

- Write your answers directly on the questionnaire.
- Show all work. Your solutions will be scored on the correctness and completeness of your methods and use of proper notation as well as your answers. A final answer with no work, calculations, and/or explanations will result in a grade of zero for that questions - even if it is correct.
- Notation counts. Poor notation = Loss of marks.
- All cell phones and listening devices must be turned off. All unauthorized materials must be put away.
- Only non-graphing, non-programmable calculators are permitted.
- Give exact answers and reduce all fractions.  $\sqrt{2}$  is exact, 1.41 is an approximation of  $\sqrt{2}$ . If using decimals, please give answers to four significant decimal places.

Note:

- Some questions will take more time, some less. Manage your time.
- Start by reading over the entire test.
- Start with a question you find easy.

Good Luck!

Marks	
1	/
2	/
3	/
4	/
5	/
6	/
7	/
Total:	
	/
	(      % )

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Cheating and plagiarism are serious academic offences. Anyone caught cheating, or aiding in the act of cheating, will immediately be given a mark of zero for this test, and a note will be placed in his or her file.

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## 1. Multiple Choice

Sheldon, Howard, and Raj are three students taking a multiple choice exam in Quantum Mechanics. Each question has five choices, but only one correct answer among them. For Question #1, Sheldon has no idea of the correct answer, Howard correctly identifies one answer that is wrong, and Raj correctly identifies two wrong answers. All three decide to guess at random from the answers they think stand a chance of being correct. Calculate the probability that

- (a) none of them chooses the correct answer.
- (b) Raj is the only one to choose the correct answer.
- (c) exactly one of the three students chooses the correct answer.

## 2. Rock, Paper, Scissors

In 2005 a Japanese company wanted to auction off their priceless art collection. To choose an auction house, they invited Christie's and Sotheby's to take part in a Rock Paper Scissors battle. Christie's won.

A researcher asked 10 people to predict the winning move in a one-round Rock, Paper, Scissors match. Each participant selected one of the three options. It was found that 40% of the participants chose Rock, 30% chose Paper, and the remaining chose Scissors.

However, not all predictions were equally rational. Some participants were subconsciously influenced in their choices. The likelihood of a prediction being biased depended on the move selected. Among those who chose Rock, 5% were considered biased. Of those who chose Paper, 10% were biased. For Scissors, the rate of bias was 20%.

A person is selected at random.

- (a) What proportion of all predictions were biased?
- (b) If a prediction was biased, what's the chance it was Scissors?
- (c) If a prediction was not biased, what's the chance it was Rock?
- (d) What's the probability that a person made an unbiased Paper prediction?
- (e) What's the chance a person either made a biased prediction or chose Scissors?

### 3. Urn

An urn contains  $R$  red balls and  $W$  white balls, where  $R$  and  $W$  are strictly positive integers. Balls are drawn at random one after another. Every time a ball is drawn, it is replaced back in the urn together with  $N$  new balls of the same color.

- (a) Compute the probability that a red ball is drawn on the first draw.
- (b) Compute the probability that a red ball is drawn on the second draw.

#### 4. Who?

In a 2008 survey, nearly a quarter of Britons polled thought Winston Churchill was a fictional character, and 58% thought Sherlock Holmes was real. Suppose that we randomly select four British citizens.

- (a) Let  $X$  denote the number of British citizens in the sample who believe that Sherlock Holmes is a real person. Determine the probability mass function of  $X$ , and present your solution in a clearly organized table.
- (b) What is the probability that at least 1 person in the group believe he is real?
- (c) What is the probability that at most 2 people in the group does not believe that Sherlock Holmes is real?

## 5. Bulgarian Lottery Mystery

In 2009, the same six numbers were drawn twice in a row in Bulgaria's national lottery. The odds were 1 in 4 million. No one won the first round, but 18 people picked those same numbers in the second and each won 10,164 leva ( $\approx 10,129$  CAD).

Five thousand lottery tickets are sold for \$10 each. One ticket will win \$2,500, two tickets will win \$800 each, and five tickets will win \$50 each. Let  $X$  denote the net gain from the purchase of a randomly selected ticket.

- (a) Create a probability distribution for  $X$ .
- (b) Find the expected value  $E(X)$  and interpret the meaning of this value in the context of the problem.
- (c) Calculate the variance and standard deviation of  $X$ . Include units in your answers.

## 6. Yahtzee

In the game of Yahtzee, five balanced dice are rolled simultaneously. Find the probability of getting

- (a) four of a kind (ie. exactly four of the five die are show the same number of dots).
- (b) a full house (three of a kind and a pair).
- (c) three of a kind (exactly three of the five die show the same number of dots, and the other two show exhibit different numbers from each other and the triple).

## 7. PIN Number

You are creating a 4-digit pin code. How many choices are there in the following cases?

- (a) With no restriction.
- (b) No digit is repeated.
- (c) No digit is repeated, digit number 3 is a 0 .
- (d) No digit is repeated, and they must appear in increasing order.
- (e) No digit is repeated, 2 and 5 must be present.