SN1 Practice Test 2

General Information and Recommendations

- Test 1 is scheduled to take place on **Thursday, April 9, 2026**.
- It covers lectures:
  - L4. Probability Theory (Introduction to Probability)
  - L5. Probability Theory (Counting Techniques)
  - L6. Probability Theory (Rules of Probability)
  - L7. Probability Mass Functions
  - L8. Expected Value and Variance of a Discrete Random Variable
- It is strongly advised that you go **over all of the problems covered in class, the class exercises, examples on Probabilia, and the problems on this test**.

201-SN1-05 Probability and Statistics

Practice Test 2 - A

Winter 2025

Name:	Solutions

This test consists of 7 questions.
You will have **1.5 hours** to complete this test.

Instructions:

- Write your answers directly on the questionnaire.
- Show all work. Your solutions will be scored on the correctness and completeness of your methods and use of proper notation as well as your answers. A final answer with no work, calculations, and/or explanations will result in a grade of zero for that questions - even if it is correct.
- Notation counts. Poor notation = Loss of marks.
- All cell phones and listening devices must be turned off. All unauthorized materials must be put away.
- Only non-graphing, non-programmable calculators are permitted.
- Give exact answers and reduce all fractions. $\sqrt{2}$ is exact, 1.41 is an approximation of $\sqrt{2}$. If using decimals, please give answers to four significant decimal places.

Note:

- Some questions will take more time, some less. Manage your time.
- Start by reading over the entire test.
- Start with a question you find easy.

Marks	
1	/
2	/
3	/
4	/
5	/
6	/
7	/
Total: / (%)	

Good Luck!

Cheating and plagiarism are serious academic offences. Anyone caught cheating, or aiding in the act of cheating, will immediately be given a mark of zero for this test, and a note will be placed in his or her file.

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1. **Multiple Choice**

Sheldon, Howard, and Raj are three students taking a multiple choice exam in Quantum Mechanics. Each question has five choices, but only one correct answer among them. For Question #1, Sheldon has no idea of the correct answer, Howard correctly identifies one answer that is wrong, and Raj correctly identifies two wrong answers. All three decide to guess at random from the answers they think stand a chance of being correct. Calculate the probability that

Solution

(a) none chose the correct answer

Let S = the event that Sheldon chooses the correct answer for Question #1
H = the event that Howard chooses the correct answer for Question #1
R = the event that Raj chooses the correct answer for Question #1

$$P(S) = \frac{1}{5} \qquad P(H) = \frac{1}{4} \qquad P(R) = \frac{1}{3}$$

$$P(S' \cap H' \cap R') = P(S') \cdot P(H') \cdot P(R') = \left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\left(\frac{2}{3}\right) = \frac{2}{5}$$

(b) Raj is the only person who chose the correct answer

$$P(S' \cap H' \cap R) = P(S') \cdot P(H') \cdot P(R) = \left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\left(\frac{1}{3}\right) = \frac{1}{5}$$

(c) Exactly one of the three chose the correct answer

$$\begin{aligned}
&P[(S \cap H' \cap R') \cup (S' \cap H \cap R') \cup (S' \cap H' \cap P(R))] \\
&\quad = P(S) \cdot P(H') \cdot P(R') + P(S') \cdot P(H^{)} \cdot P(R') + P(S') \cdot P(H) \cdot P(R') \\
&\quad = \left(\frac{1}{5} \cdot \frac{3}{4} \cdot \frac{2}{3}\right) + \left(\frac{4}{5} \cdot \frac{1}{4} \cdot \frac{2}{3}\right) + \left(\frac{4}{5} \cdot \frac{3}{4} \cdot \frac{1}{3}\right) \\
&\quad = \frac{13}{30}
\end{aligned}$$

2. **Rock, Paper, Scissors**

In 2005 a Japanese company wanted to auction off their priceless art collection. To choose an auction house, they invited Christie's and Sotheby's to take part in a Rock Paper Scissors battle. Christie's won.

A researcher asked 10 people to predict the winning move in a one-round Rock, Paper, Scissors match. Each participant selected one of the three options. It was found that 40% of the participants chose Rock, 30% chose Paper, and the remaining chose Scissors.

However, not all predictions were equally rational. Some participants were subconsciously influenced in their choices. The likelihood of a prediction being biased depended on the move selected. Among those who chose Rock, 5% were considered biased. Of those who chose Paper, 10% were biased. For Scissors, the rate of bias was 20%.

A person is selected at random.

Solution

Let $R =$ the person chose rock

$P =$ the person chose paper

$S =$ the person chose scissors

$B =$ the prediction is biased

$$P(R) = 0.4 \qquad P(P) = 0.3 \qquad P(S) = 0.3$$
$$P(B \mid R) = 0.05 \qquad P(B \mid P) = 0.10 \qquad P(B \mid S) = 0.20$$

(a) What proportion of all predictions were biased?

$$\begin{aligned} P(B) &= P(B|R) \cdot P(R) + P(B|P) \cdot P(P) + P(B|S) \cdot P(S) \\ &= 0.4(0.05) + 0.3(0.10) + 0.3(0.20) \\ &= 0.11 \end{aligned}$$

(b) If a prediction was biased, what's the chance it was Scissors?

$$P(S|B) = \frac{P(S \cap B)}{P(B)} = \frac{P(B \mid S) \cdot P(S)}{P(B)} = \frac{0.3(0.20)}{0.11} = 0.5455$$

(c) If a prediction was not biased, what's the chance it was Rock?

$$P(R|B') = \frac{P(R \cap B')}{P(B')} = \frac{P(B' \mid R) \cdot P(R)}{1 - P(B)} = \frac{0.4(0.95)}{1 - 0.11} = 0.4270$$

(d) What's the probability that a person made an unbiased Paper prediction?

$$P(P \cap B') = P(B' \mid R) \cdot P(R) = 0.3 \cdot (1 - 0.10) = 0.27$$

(e) What's the chance a person either made a biased prediction or chose Scissors?

$$
\begin{aligned}
P(B \cup S) &= P(B) + P(S) - P(B \cap S) \\
&= P(B) + P(S) - P(B \mid S) \cdot P(S) \\
&= 0.11 + 0.3 - 0.06 \\
&= 0.35
\end{aligned}
$$

3. **Urn**

An urn contains R red balls and W white balls, where R and W are strictly positive integers. Balls are drawn at random one after another. Every time a ball is drawn, it is replaced back in the urn together with N new balls of the same color.

Solution

(a) Compute the probability that a red ball is drawn on the first draw.

Let $R =$ a red ball is drawn
$W =$ a white ball is drawn

$$P(R) = \frac{R}{R+W}$$

(b) Compute the probability that a red ball is drawn on the second draw.

Let $R_i =$ a red ball is drawn on the i^{th} trial
$W_i =$ a white ball is drawn on the i^{th} trial

Want $P(R_2)$

$$\begin{aligned}
P\left(R_2\right) &= P\left(R_2 \cap R_1\right) + P\left(R_2 \cap W_1\right) \\
&= P\left(R_2 \mid R_1\right) P\left(R_1\right) + P\left(R_2 \mid W_1\right) P\left(W_1\right) \\
&= \frac{R+N}{R+W+N} \cdot \frac{R}{R+W} + \frac{R}{R+W+N} \cdot \frac{W}{R+W} \\
&= \frac{R}{(R+W+N)(R+W)} \cdot (R+N+W) \\
&= \frac{R}{R+W}.
\end{aligned}$$

4. **Who?**

In a 2008 survey, nearly a quarter of Britons polled thought Winston Churchill was a fictional character, and 58% thought Sherlock Holmes was real. Suppose that we randomly select four British citizens.

Solution

(a) Let X denote the number of British citizens in the sample who believe that Sherlock Holmes is a real person. Determine the probability mass function of X, and present your solution in a clearly organized table.

If $X =$ the number of people who believe that Sherlock Holmes is real with $p = 0.58$, then $X = 0, 1, 2, 3, 4$

X	$P(X = x)$
0	$C_0^4\,(0.58)^0 \cdot (0.42)^4 = 0.0311$
1	$C_1^4\,(0.58) \cdot (0.42)^3 = 0.1719$
2	$C_2^4\,(0.58)^2 \cdot (0.42)^2 = 0.3560$
3	$C_3^4\,(0.58)^3 \cdot (0.42) = 0.3278$
4	$C_4^4\,(0.58)^4 \cdot (0.42)^0 = 0.1132$

(b) What is the probability that at least 1 person in the group believe he is real?

$$\begin{aligned} P(X \geq 1) &= P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) \\ &= 1 - P(X = 0) \\ &= 1 - C_0^4\,(0.58)^0 \cdot (0.42)^4 \\ &= 1 - 0.0311 \\ &= 0.9689 \end{aligned}$$

(c) What is the probability that at most 2 people in the group does not believe that Sherlock Holmes is real?
At most 2 people do not believe that Holmes is real $\Rightarrow [P(X \leq 2)]'$

$$\begin{aligned} P(X \leq 2)]' = 1 - P(X < 2) &= 1 - [P(X = 0) + P(X = 1)] \\ &= 1 - [C_0^4\,(0.58)^0 \cdot (0.42)^4 + C_1^4\,(0.58) \cdot (0.42)^3] \\ &= 1 - [0.0311 + 0.1719] = 0.797 \end{aligned}$$

Alternatively, let $Y =$ the number of people who believe that Sherlock Holmes is **not** real with $p = 0.42$, then $Y = 0, 1, 2, 3, 4$

Y	$P(Y = y)$
0	$C_0^4\,(0.42)^0 \cdot (0.58)^4 = 0.1132$
1	$C_1^4\,(0.42) \cdot (0.58)^3 = 0.3278$
2	$C_2^4\,(0.42)^2 \cdot (0.58)^2 = 0.3560$
3	$C_3^4\,(0.42)^3 \cdot (0.58) = 0.1719$
4	$C_4^4\,(0.42)^4 \cdot (0.58)^0 = 0.0311$

$$\begin{aligned} P(Y \leq 2) &= P(Y = 0) + P(Y = 1) + P(Y = 2) \\ &= C_0^4\,(0.42)^0 \cdot (0.58)^4 + C_1^4\,(0.42) \cdot (0.58)^3 + C_2^4\,(0.42)^2 \cdot (0.58)^2 \\ &= 0.1132 + 0.3278 + 0.3560 \\ &= 0.7970 \end{aligned}$$

5. **Bulgarian Lottery Mystery**

n 2009, the same six numbers were drawn twice in a row in Bulgaria's national lottery. The odds were 1 in 4 million. No one won the first round, but 18 people picked those same numbers in the second and each won 10,164 leva ($\approx 10,129$ CAD).

Five thousand lottery tickets are sold for \$10 each. One ticket will win \$2,500, two tickets will win \$800 each, and five tickets will win \$50 each. Let X denote the net gain from the purchase of a randomly selected ticket.

Solution

(a) Create a probability distribution for X.

Let X = the net gain from the purchase of a ticket.
The net gain is the prize minus the cost of the \$10 ticket.

Net Gain X	2492	790	40	-10
$P(X = x_i)$	$\frac{1}{5000}$	$\frac{2}{5000}$	$\frac{5}{5000}$	$\frac{4992}{5000}$

(b) Find the expected value $E(X)$ and interpret the meaning of this value in the context of the problem.

$$\begin{aligned}\mathbb{E}[X] &= \sum x_i \cdot P(X = x_i) \\ &= 2490 \cdot \frac{1}{5000} + 790 \cdot \frac{2}{5000} + 40 \cdot \frac{5}{5000} + (-10) \cdot \frac{4992}{5000} \\ &= -\frac{45650}{5000} \\ &= -9.13\end{aligned}$$

Interpretation: On average, a person will **lose \$9.13** per ticket purchased.

(c) Calculate the variance and standard deviation of X. Include units in your answers.
$\mathbb{V}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$

$$\begin{aligned}E(X^2) &= \sum x_i^2 \cdot P(X = x_i) \\ &= 2490^2 \cdot \frac{1}{5000} + 790^2 \cdot \frac{2}{5000} + 40^2 \cdot \frac{5}{5000} + (-10)^2 \cdot \frac{4992}{5000} \\ &= \frac{7942500}{5000} \\ &= 1588.5\end{aligned}$$

$$\begin{aligned}\mathbb{V}[X] &= \mathbb{E}[X^2] - (\mathbb{E}[X])^2 \\ &= 1588.5 - (-9.13)^2 \\ &= 1505.13 \ \$^2\end{aligned}$$

$$\mathbb{S}[X] = \sqrt{\mathbb{V}[X]} = \sqrt{1505.13} = \$\,38.80$$

6. **Yahtzee**

In the game of Yahtzee, five balanced dice are rolled simultaneously. Find the probability of getting

Solution

(a) four of a kind (ie. exactly four of the five die are show the same number of dots).

Let $E =$ the event that we roll a four of a kind.

$$P(E) = C_1^6 \cdot C_1^5 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right) = \frac{150}{7776}$$

First choose the number on the die that will appear four times: C_1^6. The probability of rolling this number on four of the dice is $\left(\frac{1}{6}\right)^4$. Next, there are 5 remaining possible faces for the last die, and we must choose one of them: C_1^5, and the probability that this final die shows a number different from the one forming the four-of-a-kind is $\frac{5}{6}$

(b) a full house (three of a kind and a pair).

Let $F =$ the event that we roll a full house.

$$P(F) = C_1^6 \cdot C_1^5 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^2 = \frac{300}{7776}$$

Similar calculation as in (a).

(c) three of a kind (exactly three of the five die show the same number of dots, and the other two show exhibit different numbers from each other and the triple).

Let $G =$ the event that we roll a three of a kind.

$$P(G) = C_1^6 \cdot C_2^5 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right) \left(\frac{4}{6}\right) = \frac{1200}{7776}$$

First choose the number on the die that will appear three times: C_1^6. The probability of rolling this number on three of the dice is $\left(\frac{1}{6}\right)^3$. Next, there are 5 remaining possible faces for the two remaining die, and we must choose two of them: C_2^5. The probability that one of these dice shows a number different from the one forming the three-of-a-kind is $\frac{5}{6}$, and the probability that the other die shows a number different from both the three-of-a-kind and the first distinct number is $\frac{4}{6}$

7. PIN Number

You are creating a 4-digit pin code. How many choices are there in the following cases?

Solution

(a) With no restriction.

Let $n(S)$ = the total number of 4 digit PINs.

Each digit can be from 0 to 9 (10 choices), and repetition is allowed:

$$n(S) = 10 \times 10 \times 10 \times 10 = 10^4 = 10000$$

(b) No digit is repeated.

There are 10 digits (0–9), and order matters with no repetition:

$$n(S) = 10 \times 9 \times 8 \times 7 = P_4^{10} = 5040$$

(c) No digit is repeated, digit number 3 is a 0.

We fix the 3rd digit to be 0, and choose 3 other distinct digits from the remaining 9 (excluding 0), and place them in the 3 other positions. Here is what the PIN should look like:

$$\_\_\ \_\_\ 0\ \_\_$$

$$n(S) = 9 \times 8 \times 1 \times 7 = P_3^9 = 504$$

(d) No digit is repeated, and they must appear in increasing order.

We are selecting 4 different digits from 10 (0–9), and sorting them in increasing order (only 1 such arrangement per selection):

$$C_4^{10} = 210$$

(e) No digit is repeated, 2 and 5 must be present.

Here is what a hypothetical PIN should look like:

$$2\ \_\_\ 5\ \_\_$$

We must choose 2 more digits from the remaining 8 (excluding 2 and 5), and arrange all 4 digits:

Choose 2 digits from remaining $8 = C_2^8 = 28$

Arrange all 4 digits (including 2 and 5) $= 4!$

$$n(S) = C_2^8 \cdot 4! = 672$$