SN1 Practice Test 2

General Information and Recommendations

- It covers lectures:
  - L4. Probability Theory (Introduction to Probability)
  - L5. Probability Theory (Counting Techniques)
  - L6. Probability Theory (Rules of Probability)
  - L7. Probability Mass Functions
  - L8. Expected Value and Variance of a Discrete Random Variable
- It is strongly advised that you go **over all of the problems covered in class, the class exercises, examples on Probabilia, and the problems on this test**.

201-SN1-05 Probability and Statistics

Practice Test 2 - B

Winter 2025

Name:

This test consists of 7 questions.
You will have **1.5 hours** to complete this test.

Instructions:

- Write your answers directly on the questionnaire.
- Show all work. Your solutions will be scored on the correctness and completeness of your methods and use of proper notation as well as your answers. A final answer with no work, calculations, and/or explanations will result in a grade of zero for that questions - even if it is correct.
- Notation counts. Poor notation = Loss of marks.
- All cell phones and listening devices must be turned off. All unauthorized materials must be put away.
- Only non-graphing, non-programmable calculators are permitted.
- Give exact answers and reduce all fractions. $\sqrt{2}$ is exact, 1.41 is an approximation of $\sqrt{2}$. If using decimals, please give answers to four significant decimal places.

Note:

- Some questions will take more time, some less. Manage your time.
- Start by reading over the entire test.
- Start with a question you find easy.

Marks	
1	/
2	/
3	/
4	/
5	/
6	/
7	/
Total: / (%)	

Good Luck!

Cheating and plagiarism are serious academic offences. Anyone caught cheating, or aiding in the act of cheating, will immediately be given a mark of zero for this test, and a note will be placed in his or her file.

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1. **Ivan**

I have a colleague in the Math Department whose name is Ivan T.Ivanov. Suppose that the letters of his name IVANTIVANOV are arranged at random. What is the probability that

(a) the I's are together?

(b) the V's are not together?

(c) the letters A, N, and O appear together as a block, in that exact order (A-N-O)?

(d) the letters I-V-A-N appear together as a block, in that exact order?

2. **What's For Dinner?**

2013 is the first year in recorded history where consumption of fish has surpassed beef. In a survey, 100 people were asked if they ate fish, meat, both, or neither. You are told that 55 eat meat, 52 eat fish, and 21 eat neither. What is the probability that a randomly selected person

(a) eats either fish or meat?

(b) eats both?

(c) eats only meat?

(d) doesn't eat fish?

(e) eats meat if they eat fish?

3. **H** & **M**

A Swedish power plant is reducing its dependence on fossil fuels by burning clothes from H&M instead of coal.Some of these clothes are made of cotton, while others are made of synthetic materials. Cotton clothes burn more cleanly, whereas synthetic ones are more likely to produce pollution such as visible smoke.

Suppose that 60% of the clothing sent to the plant is synthetic, and the remaining 40% is cotton. When burned, 70% of the synthetic clothing produces visible smoke. In contrast, only 20% of the cotton clothing produces visible smoke.

(a) What is the probability that a piece of clothing produces visible smoke?

(b) What is the probability that a piece of clothing produces visible smoke given that it is synthetic?

(c) If a piece of clothing produced does not produce visible smoke, what is the probability that it was synthetic?

(d) What is the probability that a piece of clothing is either synthetic or produces visible smoke?

(e) What is the probability that a piece of clothing is either synthetic or produces visible smoke, but not both?

4. **Three Events**

Let A, B, and C be three events such that A is disjoint (mutually exclusive) from B and C. Suppose that $P(A) = 0.18$, $P(B) = 0.42$, $P(C) = 0.57$, and $P(B \cap C) = 0.26$. Compute

(a) $P\left((A \cup B') \cap (C \cup A')\right)$

(b) $P\left(A' \cap (B \cup C')\right)$

(c) $P\left((A \cup B \cup C')'\right)$

(d) $P\left((A \cup B') \cup (C \cap B)\right)$

(e) $P\left((B' \cap C') \cup (A \cap (B \cup C)')\right)$

5. **Game of Dice**

On a long train journey, a statistician is invited by a gambler to play a dice game. The game uses two ordinary dice which the statistician is to throw. If the total score is 12 , the statistician is paid \$6 by the gambler. If the total score is 8 , the statistician is to be paid \$3 by the gambler. However, if both or either dice shows a 1, the statistician pays the gambler \$2.

Let X be the amount paid to the statistician by the gambler after the dice are thrown once.

(a) Determine the probability mass function of X and organize the information into a table.

(b) Determine the amount of money that the statistician is expected to win or lose after playing 100 games.

(c) Determine the amount, k, that the 6 would have to be changed to in order to make the game unbiased (i.e. fair).

6. **Slot Machines**

Suppose that there are two slot machines, one of which pays out 10% of the time and the other pays out 20% of the time. Unfortunately, you have no idea which is which. Suppose you randomly choose a machine and put in a quarter.

(a) If you don't get a jackpot, what is the chance that you chose the machine that pays out 20% of the time?

(b) If you had instead gotten a jackpot, what would be the chance that you chose the one that pays out 20% of the time?

(c) What is the probability that you either chose the machine that pays out 20% of the time or you got a jackpot?

(d) Are the events "you chose the machine that pays out 20%" and "you got a jackpot" independent? Justify your answer with calculations.

(e) Three people independently choose a slot machine at random (each with a 50% chance of picking either machine), and each plays once. What is the probability that all three chose the machine that pays out 10% of the time and all got jackpots?

7. **It's Greek to Me**

The equivalents of the English saying "That's Greek to me" are: "This appears to be Spanish" (German), "This is Russian to me" (Dutch), 'It's German to me" (Philippines), "It's Hebrew" (Finnish), "It's Chinese to me" (Hebrew), "Sounds like Mars language/These are chicken intestines" (China).

A group of girls at a school are taking Advanced Cantonese which come in two modules: C1 and C2. Each girl takes only module C1, or only module C2, or both C1 and C2. The probability that a girl is taking C2 given that she is taking C1 is 0.2. The probability that a girl is taking C1 given that she is taking C2 is 0.33

Find the probability that a girl selected at random

(a) is taking both C1 and C2.

(b) is taking only C1.