SN1 Practice Test 3

General Information and Recommendations

- Test 3 is scheduled to take place on **Monday, May 11**.
- It covers lectures:
  - L9. The Binomial Distribution
  - L10. The Normal Distribution
  - L11. The Normal Approximation to the Binomial
  - L12. Sampling Distributions and the Central Limit Theorem
  - L13. Confidence Interval on the Mean; Single Population, Variance Known
  - L14. Confidence Interval on the Mean; Single Population, Variance Unknown
  - L15. Confidence Interval on the Population Proportion
  - L16. Hypothesis Tests on the Mean; Single Population, Variance Known
  - L17. Hypothesis Tests on the Mean; Single Population, Variance Unknown
  - L18. Hypothesis Tests on the Population Proportion
- It is strongly advised that you go **over all of the problems covered in class**, the in class exercises, the examples on the course webpage, and the questions on the practice test.

201-SN1-05 Probability and Statistics

Practice Test 3 - B

Winter 2025

Name: **Solutions**

This test consists of 10 questions.
You will have **2 hours** to complete this test.

Instructions:

- Write your answers directly on the questionnaire.
- Show all work. Your solutions will be scored on the correctness and completeness of your methods and use of proper notation as well as your answers. A final answer with no work, calculations, and/or explanations will result in a grade of zero for that questions - even if it is correct.
- Notation counts. Poor notation = Loss of marks.
- All cell phones and listening devices must be turned off. All unauthorized materials must be put away.
- Only non-graphing, non-programmable calculators are permitted.
- Give exact answers and reduce all fractions. $\sqrt{2}$ is exact, 1.41 is an approximation of $\sqrt{2}$. If using decimals, please give answers to four significant decimal places.

Note:

- Some questions will take more time, some less. Manage your time.
- Start by reading over the entire test.
- Start with a question you find easy.

Good Luck!

Marks	
1	/
2	/
3	/
4	/
5	/
6	/
7	/
8	/
9	/
10	/
Total: / (%)	

Cheating and plagiarism are serious academic offences. Anyone caught cheating, or aiding in the act of cheating, will immediately be given a mark of zero for this test, and a note will be placed in his or her file.

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1. **Master Cleanse**

A popular weight loss method among celebrities these days is the “Master Cleanse”. The detox regime involves avoiding food for about 10 days, drinking a lemonade mixture flavoured with maple syrup and cayenne pepper, and consuming a gallon of saltwater on top of that. The only break from the monotony of this liquid diet is a delicious laxative before bed. Gwenyth Paltrow and Beyoncé swear by it.

“Master Cleanse” claims that you can lose an average of 10 pounds in 10 days on this diet. However, in a random sample of 14 dieters on this regime, it was found that their average weight loss in 10 days was only 8.5 pounds, with a sample standard deviation 1.75 pounds. Assume that the dieters were selected from a normally distributed population.

Solution

(a) At the 1% level of significance, does the data indicate that the amount of weight lost on Master Cleanse is less than 10 pounds in 10 days?

$H_0 : \mu = 10$
$H_1 : \mu < 10$
$\alpha = 0.01 \quad \Rightarrow \quad t_{0.01,13} = -2.650$

$$T = \frac{\overline{X} - k}{s/\sqrt{n}} = \frac{8.5 - 10}{1.75/\sqrt{14}} = \frac{-1.5}{0.4677} \approx -3.207$$

Reject H_0

Conclusion: At the 1% level of significance, there is strong evidence that the average weight loss on the Master Cleanse diet is less than 10 pounds in 10 days. $0.0005 < P - value < 0.005$

(b) Construct appropriate interval estimate for the actual average amount of weight lost in 10 days on this diet, and explain how it can be used to support the conclusion in (a).

UCB:

$$\begin{aligned} \overline{x} + t_{0.01,13} \cdot \frac{s}{\sqrt{n}} &\leq \mu \\ 8.5 + (-2.650) \cdot \frac{1.75}{\sqrt{14}} &\leq \mu \\ 7.2606 &\leq \mu \end{aligned}$$

Conclusion: With repeated sampling, we are 99% confident that the true average weight loss on this diet is at most 7.2606 pounds. Since this value is less than 10 pounds, this supports the conclusion from part (a) that the true average weight loss is significantly less than claimed.

2. **Sleepy Lions**

Lions don't sleep in the jungle, the mighty jungle at all. The word "jungle" means dense, tropical rainforest and although it only covers 6% of the Earth's land, nearly 57% of its species live there. Lions are not one of them - almost all lions live on the savannah of sub-Saharan Africa. Not only do lions not live in the jungle, "tonight" is the least likely time for them to be sleeping. Outside the mating season, they spend about 20 hours sleeping, two hours walking and just under an hour eating.

According to zoological records, the average lifespan of a male lion is 11.25 years, with a known population standard deviation of 2.89 years. However, some conservationists believe that lions in a particular region of South Sudan may live longer than average.

To investigate this, researchers took a random sample of 35 male lions from the area and found their mean lifespan to be 12.5 years.

Solution

(a) At the 2.5% level of significance, does the data indicate that male lions in this region tend to live longer than 11.25 years?

$H_0 : \mu = 11.25$
$H_1 : \mu > 11.25$
$\alpha = 0.025 \quad \Rightarrow \quad Z_{0.025} = 1.96$

$$Z = \frac{\overline{X} - k}{\sigma/\sqrt{n}} = \frac{12.5 - 11.25}{2.89/\sqrt{35}} = \frac{1.25}{0.4886} \approx 2.56$$

Reject H_0

Conclusion: At the 2.5% level of significance, there is sufficient evidence to suggest that male lions in this region of South Sudan tend to live longer than 11.25 years. P-value $= \mathbb{P}(Z > 2.56) \approx 0.0052$

(b) Construct a one-sided interval estimate and explain how it can be used to support the conclusion obtained in (a).
LCB:

$$\overline{x} - Z_{0.025} \cdot \frac{\sigma}{\sqrt{n}} \leq \mu$$
$$12.5 - 1.96 \cdot \frac{2.89}{\sqrt{35}} \leq \mu$$
$$11.5423 \leq \mu$$

Conclusion: With repeated sampling, we are 97.5% confident that the average lifespan of male lions in this region is at least 11.5423 years. Since this value is greater than 11.25 years, this supports the conclusion of the hypothesis test.

(c) What is the Type II error in the context of the problem?

Type II Error: Failing to conclude that lions in this region live longer than 11.25 years when they actually do.

3. **Liquid Soap**

A machine is set to fill 16-ounce bottle with liquid soap. When the machine has been properly calibrated, the average amount of soap dispensed into each bottle is 16 ounces, with a known standard deviation of 0.25 ounces. A quality control inspector selects 55 bottles from a production run and finds that the average volume of soap dispensed in the containers to be 15.93 ounces.

Solution

(a) At the 1% level of significance, does the data indicate that the average amount of soap dispensed into the containers by the machine is different from 16 ounces? What is the actual probability of committing a Type I error?

$H_0 : \mu = 16$
$H_1 : \mu \neq 16$
$\alpha = 0.01 \quad \Rightarrow \quad Z_{0.005} = \pm 2.575$

$$\overline{x} = 15.93, \quad \sigma = 0.25, \quad n = 55$$

$$Z = \frac{\overline{X} - k}{\sigma/\sqrt{n}} = \frac{15.93 - 16}{0.25/\sqrt{55}} = \frac{-0.07}{0.0337} \approx -2.077$$

Fail to reject H_0

Conclusion: At the 1% level of significance, there is insufficient evidence to conclude that the average amount of soap dispensed is different from 16 ounces.

(b) Construct and explain how an appropriate confidence interval/bound could be used to support the conclusion obtained in part (a).

CI:

$$\overline{x} \pm Z_{0.005} \cdot \frac{\sigma}{\sqrt{n}}$$
$$15.93 \pm 2.575 \cdot \frac{0.25}{\sqrt{55}}$$
$$15.8431 < \mu < 16.0169$$

Conclusion: With repeated sampling, we are 99% confident that the true mean amount of soap dispensed lies between 15.8431 and 16.0169 ounces. Since 16 ounces lies within this interval, it supports the conclusion from part (a) that the machine does not need recalibration.

(c) Based on your results from (a) and (b), does the machine require recalibration? Explain in one or two sentences.

Conclusion: No, the machine does not require recalibration. The sample evidence does not show a statistically significant difference from the target of 16 ounces.

4. **Dinosaurs**

In the 1940s, some scientists believed Venus was home to dinosaurs because it was at an earlier stage of evolution than Earth.

A science historian claims that more than 60% of science fiction articles from the 1940s mentioned prehistoric creatures when describing life on Venus. To investigate this claim, a researcher randomly selects 200 articles from that decade and finds that 135 of them mention prehistoric creatures.

Solution

(a) At the 5% level of significance, test the historian's claim. What is the $P - value$ of this test?

$H_0 : p = 0.60$
$H_1 : p > 0.60$

$\hat{p} = \frac{135}{200} = 0.675, \quad ; \quad n\hat{p} = 135 > 5 \quad ; \quad n(1-\hat{p}) = 65 > 5$
$\alpha = 0.05 \quad \Rightarrow \quad Z_{0.05} = 1.645$

$$Z = \frac{\hat{p} - k}{\sqrt{\frac{k(1-k)}{n}}} = \frac{0.675 - 0.60}{\sqrt{\frac{0.60 \cdot 0.40}{200}}} = \frac{0.075}{0.0346} = 2.167$$

Reject H_0

Conclusion: At the 5% level of significance, there is sufficient evidence to support the historian's claim that more than 60% of 1940s science fiction articles about Venus mentioned prehistoric creatures. $P(Z > 2.17) = 0.0151$

(b) Construct and explain how an appropriate confidence interval/bound could be used to support the conclusion of the test obtained in part (a).

LCB:

$$\hat{p} - Z_{0.05} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p$$
$$0.675 - 1.645 \cdot \sqrt{\frac{0.675 \cdot 0.325}{200}} \leq p$$
$$0.6205 \leq p$$

Conclusion: With repeated sampling, we are 95% confident that the true proportion of articles mentioning prehistoric creatures is at least 62.05%. Since this is greater than 60%, this supports the conclusion of the hypothesis test.

(c) What is the minimum sample size required to estimate the true proportion of science fiction articles mentioning prehistoric creatures, if we want to test the claim at the 5% level of significance but do not have a preliminary estimate for p, and we want the margin of error to be no more than 15%?

$$n = p(1-p)\left(\frac{Z_{\alpha/2}}{E}\right)^2 = 0.5(1-0.5)\left(\frac{1.96}{0.15}\right)^2 = 42.7$$

Conclusion: A minimum sample size of 43 articles is required to achieve a margin of error no greater than 15% with 95% confidence and no prior estimate for p.

5. **Charles Darwin**

When Charles Darwin published a diary of his baby son's milestones, he started a trend among naturalists who were also fathers to take more of an interest in their children's development.

Records indicate that the birth weights of Victorian-era babies were approximately normally distributed with a mean of 3.45 kg and a standard deviation of 0.56 kg

Solution

(a) What is the probability that a randomly selected baby from the Victorian-era weighs between 2.8 kg and 3.8 kg?

Let X = the birth weight of a baby
X is normally distributed with $\mu = 3.45$ and $\sigma = 0.56$

$$\begin{aligned} P(2.8 < X < 3.8) &= P\left(\frac{2.8 - 3.45}{0.56} < Z < \frac{3.8 - 3.45}{0.56}\right) \\ &= P(-1.16 < Z < 0.63) \\ &= P(Z < 0.63) - P(Z < -1.16) \\ &= 0.7357 - 0.1230 \\ &= 0.6127 \end{aligned}$$

(b) What is the probability that out of 6 randomly selected Victorian-era babies, at least 2 of them weigh less than 3.85 kg?

Let X = the birth weight of a baby

$$P(X < 3.85) = P\left(Z < \frac{3.85 - 3.45}{0.56}\right) = P(Z < 0.71) = 0.7629$$

Let Y = the number of babies out of 6 weighing less than 3.85 kg
Y is binomially distributed with $n = 6$, $p = 0.7629$

$$\begin{aligned} P(Y \geq 2) &= 1 - [P(Y = 0) + P(Y = 1)] \\ &= 1 - \left[C_0^6 \cdot (0.2371)^6 + C_1^6 \cdot (0.7629)^1 \cdot (0.2371)^5\right] \\ &= 1 - (0.00018 + 0.0035) \\ &= 0.9963 \end{aligned}$$

(c) If 25 Victorian-era babies are selected, what is the probability that their average weight is below 3.25 kg?
Let $\overline{X}$ = the average birth weight of the 25 babies
$\overline{X}$ is normally distributed with $\mu_{\bar{x}} = 3.45$ and $\sigma_{\bar{x}} = \frac{0.56}{\sqrt{25}}$

$$P(\overline{X} < 3.25) = P\left(\frac{3.25 - 3.45}{0.56/\sqrt{25}}\right) = P(Z < -1.79) = 0.0370$$

(d) If 99 Victorian-era babies are selected, what is the probability that their average weight is more than 3.575 kg?
Let $\overline{X}$ = the average birth weight of the 99 babies
$\overline{X}$ is normally distributed with $\mu_{\bar{x}} = 3.45$ and $\sigma_{\bar{x}} = \frac{0.56}{\sqrt{99}}$

$$P(\overline{X} > 3.575) = P\left(\frac{3.575 - 3.45}{0.56/\sqrt{99}}\right) = P(Z > 2.22) = 1 - 0.9868 = 0.0132$$

6. **ADHD**

A recent study claims that American doctors are 14% more likely to diagnose children with ADHD on Halloween, raising concerns about the influence of timing and social expectations on medical decisions.

The historical proportion of ADHD diagnoses in children is approximately 10%. On Halloween, a random sample of 300 medical records shows that 39 children were diagnosed with ADHD.

Solution

(a) At the 1% level of significance, test the claim that the proportion of diagnoses is different from 10% on Halloween.

$H_0 : p = 0.10$
$H_1 : p \neq 0.10$

$\hat{p} = \frac{39}{300} = 0.13, \quad ; \quad n\hat{p} = 39 > 5 \quad ; \quad n(1-\hat{p}) = 261 > 5$

$$\alpha = 0.01 \quad \Rightarrow \quad Z_{0.005} = \pm 2.575$$

$$Z = \frac{\hat{p} - k}{\sqrt{\frac{k(1-k)}{n}}} = \frac{0.13 - 0.10}{\sqrt{\frac{0.10 \cdot 0.90}{300}}} = \frac{0.03}{\sqrt{0.0003}} = 1.734$$

Fail to reject H_0

Conclusion: At the 1% level of significance, there is insufficient evidence to conclude that the proportion of ADHD diagnoses on Halloween differs from 10%.

(b) What is the highest level significance for which you would be able to reject the null hypothesis.

This is the $P - value$

$$P - value = 2 \cdot P(Z > 1.73) = 2 \cdot 0.0416 = 0.0832$$

Conclusion: The null hypothesis could be rejected at any significance level greater than 8.32%.

(c) Construct and explain how an appropriate confidence interval/bound could be used to support the conclusion of the test obtained in part (a).

$$\hat{p} = 0.13, \quad Z_{0.005} = 2.575, \quad \text{SE} = \sqrt{\frac{0.13 \cdot 0.87}{300}} \approx 0.0194$$

$$\hat{p} \pm Z \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.13 \pm 2.575\sqrt{\frac{0.13 \cdot 0.87}{300}}$$

$$0.0800 < p < 0.1800$$

Conclusion: With 99% confidence, the true proportion of Halloween ADHD diagnoses is between 8.00% and 18.00%. Since 10% is contained in this interval, the result supports the conclusion to fail to reject H_0.

(d) A researcher wants to estimate the true proportion of ADHD diagnoses on Halloween with a margin of error no greater than 3% at the 99% level of confidence. A preliminary study suggests the proportion may be around 13%. What is the minimum sample size the researcher should use to achieve this level of precision?

$$n = p(1-p)\left(\frac{Z_{\alpha/2}}{E}\right)^2 = 0.13 \cdot 0.87 \left(\frac{2.575}{0.03}\right)^2 = 838.3$$

Conclusion: The researcher should sample at least 839 medical records to estimate the proportion within 3% at the 99% confidence level.

7. **Confiscated Crops**

In the past three years, Essex Police have been growing high-quality weed. When they seize undeveloped cannabis plants, they often grow them to maturity — otherwise, the plants may be considered worthless and not justify the confiscation of a drug dealer's assets. An internal review claims that the average yield per mature plant is 52 grams. A random sample of 10 plants grown to maturity produced the following yields (in grams):

47	55	60	49	53	57	51	58	48	54

Solution

(a) Calculate the mean and the standard deviation of the sample data.

x	x^2
47	2209
55	3025
60	3600
49	2401
53	2809
57	3249
51	2601
58	3364
48	2304
54	2916
532	**28478**

$$\overline{x} = \frac{\sum x}{n} = \frac{532}{10} = 53.2 \text{ grams}$$

$$s^2 = \frac{1}{n-1}\left[\sum x^2 - \frac{(\sum x)^2}{n}\right] = \frac{1}{10-9}\left[28478 - \frac{(532)^2}{10}\right] = \frac{175.6}{9} = 19.5111\text{grams}^2$$

$$\begin{aligned} s &= \sqrt{19.5111} \\ &= 4.416 \text{ grams} \end{aligned}$$

(b) Using your results from (a), test the claim that the average yield per plant is 52 grams. Use a significance level of $\alpha = 0.05$ and estimate the P-value.

$\alpha = 0.05 \quad \Rightarrow \quad t_{0.05,9} = 1.833$

$$T = \frac{\overline{X} - k}{s/\sqrt{n}} = \frac{53.2 - 52}{4.416/\sqrt{10}} = \frac{1.2}{1.3965} \approx 0.859$$

Fail to reject H_0

Conclusion: At the 5% level of significance, there is insufficient evidence to conclude that the average yield per cannabis plant is greater than 52 grams; $0.125 < P - value < 0.25$

(c) Construct and explain how a confidence interval or bound could support the conclusion from part (b).

LCB (lower confidence bound):

$$\begin{aligned} \overline{x} - t_{0.05,9} \cdot \frac{s}{\sqrt{n}} &\leq \mu \\ 53.2 - 1.833 \cdot \frac{4.416}{\sqrt{10}} &\leq \mu \\ 50.641 &\leq \mu \end{aligned}$$

Conclusion: With repeated sampling, we are 95% confident that the average yield per plant is at least 50.641 grams. Since this is less than 52 grams, this supports our conclusion to fail to reject the null hypothesis.

(d) Clearly state what a Type I and Type II error would mean in the context of this problem.

Type I Error: Concluding that the average yield per cannabis plant is greater than 52 grams when in fact the true average yield is exactly 52 grams.

Type II Error: Failing to conclude that the average yield per cannabis plant is greater than 52 grams when in fact the true average yield is higher than 52 grams.

8. For each question below, select **all** the statements that are **correct**. Each question has **at least one correct answer, but not necessarily all options are correct**. You will receive **full credit** if and only if you select all correct answers and **no incorrect answers**. Selecting an incorrect option or missing a correct option may result in **partial credit or no credit**.

(a) Suppose a medical test is used to determine whether a person has a certain disease. The null hypothesis H_0, is that the person does not have the disease. Which of the following statements are correct?

✓ A Type I error occurs if the test concludes the person **has** the disease when they actually do **not**.

✓ A Type II error occurs if the test concludes the person does **not** have the disease when they actually **do**.

✓ Reducing the significance level α decreases the probability of a Type I error.

☐ Reducing the significance level α always decreases the probability of a Type II error.

☐ It is possible to simultaneously reduce both Type I and Type II error rates to zero by choosing a very low significance level.

(b) Suppose a 95% confidence interval for the mean weight of a certain species of bird is between 2.4 kg and 3.0 kg. Which of the following statements are correct?

✓ If we constructed many such intervals from repeated samples, about 95% of them would contain the true mean.

☐ There is a 95% chance that the true mean is in the interval $(2.4, 3.0)$.

✓ The confidence interval provides a plausible range for the population mean.

☐ A larger sample size would result in a wider confidence interval.

✓ We are 95% confident that the true mean weight of the species lies between 2.4 kg and 3.0 kg.

(c) Suppose the distribution of weights for a species of fish is right-skewed with a mean of 4.5 kg and a standard deviation of 1.2 kg. A biologist takes random samples of size $n = 40$ and calculates the sample mean weight $\bar{x}$ for each sample. Which of the following statements are correct?

☐ The standard deviation of the sampling distribution of $\bar{x}$ is 1.2.

☐ The sampling distribution will be skewed right, just like the population distribution.

☐ Increasing the sample size would increase the variability of the sample means.

✓ The distribution of the sample means $\bar{x}$ will be approximately normal.

✓ The mean of the sampling distribution of $\bar{x}$ will be approximately 4.5 kg.

(d) A recent study reports that 60% of all adults in a city recycle regularly. A researcher believes the true proportion has increased. They collect a random sample and conduct a hypothesis test with $H_0 : p = 0.60$ and $H_1 : p > 0.60$. The test yields a p-value of 0.03. Assume a significance level of $\alpha = 0.05$. Which of the following statements are correct?

✓ The null hypothesis should be rejected.

✓ There is statistically significant evidence that the true proportion of adults who recycle is greater than 60%.

☐ The result is not significant at the 5% level, so we fail to reject the null hypothesis.

☐ The p-value of 0.03 means there's a 3% chance the null hypothesis is true.

✓ If the true proportion were actually 60%, there would be a 3% chance of getting a sample result this extreme or more by random chance.

9. **Traffic Tickets**

When traffic cameras were first installed in California, a man received a speeding ticket in the mail, complete with a photo as evidence. Upset, he mailed back a photo of the fine instead of the actual money. Weeks later, he received another photo — this time of handcuffs.

Suppose that 15% of drivers who speed past a traffic camera receive a ticket in the mail.

Solution

(a) What is the probability that out of twelve drivers who sped past a camera, three or four will receive a ticket in the mail?

Let X = the number of drivers who receive a ticket

X is a binomial random variable with $p = 0.15$

$$\begin{aligned} P(X = 3 \cup X = 4) &= P(X = 3) + P(X = 4) \\ &= C_3^{12}(0.15)^3(0.85)^9 + C_4^{12}(0.15)^4(0.85)^8 \\ &= 0.1720 + 0.0683 \\ &= 0.2403 \end{aligned}$$

(b) What is the probability that out of fifteen drivers who sped past a camera, at least two will receive a ticket in the mail?

$$\begin{aligned} P(X \geq 2) &= P(X = 2) + P(X = 3) + P(X = 4) + \cdots + P(X = 15) \\ &= 1 - [P(X = 0) + P(X = 1)] \\ &= 1 - [C_0^{15}(0.15)^0(0.85)^{15} + C_1^{15}(0.15)(0.85)^{14} \\ &= 1 - [0.0874 + 0.2312] \\ &= 0.6814 \end{aligned}$$

(c) What is the probability that out of 1000 drivers who sped past a camera, at least 870 will not receive a ticket in the mail?

Let Y = the number of drivers who will not receive a ticket.

Y is a binomial random variable with $p = 0.85$

$\because \quad np = 1000(0.85) = 850 > 5 \quad \Rightarrow \quad$ normal approximation to the binomial

$$\mu = np = 1000(0.85) = 850, \qquad \sigma = \sqrt{np(1-p)} = \sqrt{1000(0.85)(0.15)} = \sqrt{127.5}$$

$$P(Y \geq 870) = P\left(Z > \frac{869.5 - 850}{\sqrt{127.5}}\right) = P(Z > 1.73) = 0.0418$$

(d) What is the probability that out of 1000 drivers who sped past a camera, at between 820 and 860 (exclusive) will not receive a ticket in the mail?

$$\begin{aligned} P(821 \leq Y \leq 859) = P(820.5 < Y < 859.5) &= P\left(\frac{820.5 - 850}{\sqrt{127.5}} < Z < \frac{859.5 - 850}{\sqrt{127.5}}\right) \\ &= P(-2.61 < Z < 0.84) \\ &= P(Z < 0.84) - P(Z < -2.61) \\ &= 0.7995 - 0.0045 \\ &= 0.7950 \end{aligned}$$

10. **A Beetle Called Hitler**

Anophthalmus hitleri is a blind beetle found only in five caves in Slovenia. Named after Hitler in 1933, it is now endangered not because of pollution or pressure from predators, but from collectors of Nazi memorabilia3. Suppose that the lifespan of these beetles are normally distributed with a mean of 90 days and a standard deviation of 15 days.

Solution

(a) What is the probability that a randomly selected Anophthalmus hitleri will live longer than 140 days?

Let X = the lifespan of an Anophthalmus hitleri beetle in days

X is normally distributed with $\mu = 90$ days and $\sigma = 15$ days.

$$Z = \frac{140 - 90}{15} = \frac{50}{15} = 3.33$$

$$P(X > 140) = P(Z > 3.33) \approx 1 - 0.9996 = 0.0004$$

(b) What is the probability that a randomly selected Anophthalmus hitleri will survive longer than 70 days but not more than 110 days?

$$Z_1 = \frac{70 - 90}{15} = -1.33, \quad Z_2 = \frac{110 - 90}{15} = 1.33$$

$$P(70 < X < 110) = P(-1.33 < Z < 1.33) \approx 0.9082 - 0.0918 = 0.8164$$

(c) The bottom 5% of beetles will survive for how many days.

We want the value x such that $P(X < x) = 0.05$

$$z = -1.645, \quad x = \mu + z\sigma = 90 + (-1.645)(15) = 90 - 24.675 = 65.33$$

So, approximately **65.3 days**.

(d) Out of 22 beetles, what is the probability that at least 19 of them will survive more than 100 days?

Calculate the probability that a beetle will survive more than 100 days

$$P(X > 100) = P\left(Z > \frac{100 - 90}{15}\right) = P(Z > 0.67) = 0.2514$$

Let Y = the number of beetles our of 22 that survive for more than 100 days.

Y is a binomial random variable with $p = 0.2514$

$$\begin{aligned}P(Y \geq 19) =& P(Y = 19) + P(Y = 20) + P(Y = 21) + P(Y = 22)\\ &= C^{22}_{19}(0.2514)^{19}(0.7486)^3 + C^{22}_{20}(0.2514)^{20}(0.7486)^2\\ &\quad + C^{22}_{21}(0.2514)^{21}(0.7486) + C^{22}_{22}(0.2514)^{22}\\ &= 2.6134 \times 10^{-9} + 1.3165 \times 10^{-10} + 4.2106 \times 10^{-12} + 6.4274 \times 10^{-14}\\ &= 2.749 \times 10^{-9}\end{aligned}$$

(e) If a beetle lives at least 70 days, what is the probability that it lives between 80 and 100 days?

We want: $P(80 < X < 100 \mid X > 70)$

$$P(80 < X < 100 \mid X > 70) = \frac{P(80 < X < 100)}{P(X > 70)}$$

$$Z_1 = \frac{80 - 90}{15} = -0.67, \quad Z_2 = \frac{100 - 90}{15} = 0.67$$

$$\begin{aligned} P(80 < X < 100) &= P(-0.67 < Z < 0.67) \\ &= P(Z < 0.67) - P(Z < -0.67) \\ &= 0.7486 - 0.2514 \\ &= 0.4972 \end{aligned}$$

$$\begin{aligned} P(X > 70) &= P\left(Z > \frac{70 - 90}{15}\right) \\ &= P(Z > -1.33) \\ &= 0.9082 \end{aligned}$$

$$\therefore P(80 < X < 100 \mid X > 70) = \frac{P(80 < X < 100)}{P(X > 70)} = \frac{0.4908}{0.9082} = 0.5404$$