Class Exercise 3 - Solution

1. **Multiple Choice**

   For each question below, select **all** the statements that are **correct**. Each question has **at least one correct answer, but not necessarily all options are correct**. You will receive **full credit** if and only if you select all correct answers and **no incorrect answers**. Selecting an incorrect option or missing a correct option may result in **partial credit or no credit**.

   A. Which of the following statements about the **Empirical Rule** are true?

   a. The Empirical Rule applies only to distributions that are symmetric and unimodal.
   b. If a data set follows the Empirical Rule, then roughly two-thirds of the data values are clustered close to the mean.
   c. For an approximately normal distribution, about 95% of observations lie within two standard deviations of the mean.
   d. ~~Observations more than 3 standard deviations from the mean are impossible.~~
   e. ~~The Empirical Rule applies to all data sets, regardless of their shape.~~

   B. Which of the following statements about the **mean, median, and mode** and the **shape of a distribution** are true?

   a. ~~In a left-skewed distribution, the mean is typically greater than the median.~~
   b. In a perfectly symmetric distribution, the mean and median are equal.
   c. In a right-skewed distribution, the mean is typically greater than the median.
   d. ~~If the mean is larger than the median, the distribution must be symmetric.~~
   e. In a symmetric distribution with one peak, the mean, median, and mode are all equal.

   C. Which of the following statements about **measures of dispersion** are true?

   a. ~~Adding the same constant to every data value changes the value of the standard deviation.~~
   b. ~~The range provides a reliable summary of variability for skewed distributions.~~
   c. ~~A data set with a larger standard deviation is necessarily more variable than a data set with a smaller standard deviation.~~
   d. The variance is zero if and only if all data values are equal.
   e. Multiplying every data value by a constant k multiplies the standard deviation by k.

2. **Shampoo**

Scented shampoo can sometimes make people feel that their hair is smoother and silkier than when using an unscented version. The table below shows the number of different hair products kept in the bathrooms of several people.

Number of Hair Products	**Number of People**			
x_i	f_i	$f_i x_i$	$f_i x_i^2$	$LTCF$
3	15	45	135	15
4	17	68	272	32
5	19	95	475	51
7	11	77	539	62
8	24	192	1536	86
	86	477	2957	

(a) Calculate the average (mean) number of hair products.

Solution

$$\overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{477}{86} = 5.5465 \text{ products}$$

(b) Calculate the sample variance and the sample standard deviation. State the units for each.

Solution

$$s^2 = \frac{1}{n-1}\left[\sum (f_i x_i^2) - \frac{(\sum f_i x_i)^2}{n}\right] = \frac{1}{86-1}\left[2957 - \frac{(477)^2}{86}\right] = 3.6625 \text{ products}^2$$

$$s = \sqrt{s^2} = \sqrt{3.6625} = 1.9138 \text{ products}$$

(c) Multiply each data value, x_i, by 10 and recalculate the mean, sample variance, and sample standard deviation. How does multiplying every data value by 10 affect the mean, variance, and standard deviation?

Solution

Number of Hair Products	**Number of People**		
x_i	f_i	$f_i x_i$	$f_i x_i^2$
30	15	450	13500
40	17	680	27200
50	19	950	47500
70	11	770	53900
80	24	1920	153600
	86	4770	295700

$$\overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{4770}{86} = 55.465 \text{ products}$$

Comment: Multiplying each data value by 10 increases the mean by a factor of 10

$$s^2 = \frac{1}{n-1}\left[\sum(f_i x_i^2) - \frac{(\sum f_i x_i)^2}{n}\right]$$
$$= \frac{1}{86-1}\left[295700 - \frac{(4770)^2}{86}\right] = 366.2517 \text{ products}^2$$

$$s = \sqrt{s^2} = \sqrt{366.25} = 19.138 \text{ products}$$

Comment: Multiplying each data value by 10 increases the variance by a factor of 10^2 and the standard deviation by a factor of $\sqrt{10^2} = 10$

(d) Use the LTCFs to estimate the value of P_{65}

Solution

$$L = \frac{(N+1)P_i}{100} = \frac{(86+1)65}{100} = 56.55 \qquad \Rightarrow \qquad P_{65} = 7 \text{ hair products}$$