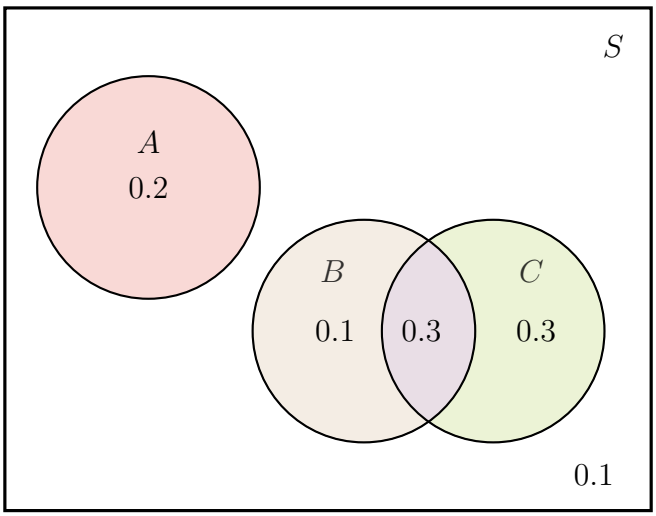
Class Exercise 6 - Solutions

1. **Three events**

Let A, B and C be three events such that A is mutually exclusive from both B and C, $P(A) = 0.2$, $P(B) = 0.4$, $P(C) = 0.6$ and $P(B \cap C) = 0.3$. Determine the following probabilities

Venn Diagram



In the following equations we automatically take into account the disjointness of the event A from the events B and C.

(a) $P(A \cup B \cup C)$

Solution

$$\begin{aligned} P(A \cup B \cup C) = P(A) + P(B \cup C) &= P(A) + P(B) + P(C) - P(B \cap C) \\ &= 0.2 + 0.4 + 0.6 - 0.3 \\ &= 0.9 \end{aligned}$$

(b) $P(A' \cap B \cap C)$

Solution

Observe that $B \cap C$ is a subset of A' and therefore $A' \cap B \cap C = B \cap C$. Hence,

$$P(A' \cap B \cap C) = P(B \cap C) = 0.3$$

(c) $P(A' \cap B' \cap C')$

Solution

Here we will use De Morgan's law:

$$\begin{aligned} P(A' \cap B' \cap C') = P(A \cup B \cup C)' &= 1 - P(A \cup B \cup C) \\ &= 1 - 0.9 \\ &= 0.1 \end{aligned}$$

(d) $P((A \cup B) \cap C)$

Solution

$$P((A \cup B) \cap C) = P((A \cap C) \cup (B \cap C)) = P(B \cap C) = 0.3$$

(e) $P((A \cup B') \cap C')$

Solution

Note that $A \cup B' = B'$. Hence

$$\begin{aligned} P((A \cup B') \cap C') = P(B' \cap C') = P(B \cup C)' &= 1 - P(B \cup C) \\ &= 1 - (0.4 + 0.6 - 0.3) \\ &= 0.3 \end{aligned}$$

(f) $P(C' \cup (A' \cap B'))$

Solution

$$P(C' \cup (A' \cap B')) = P((C \cap (A \cup B))') = 1 - P(C \cap (A \cup B)) = 1 - 0.3 = 0.7$$

(g) $P(A \cup B' \cup C')$

Solution

$$P(A \cup B' \cup C') = 1 - P(A' \cap B \cap C) = 1 - 0.3 = 0.7$$

(h) $P((A \cup B) \cap (A' \cup C))$

Solution

$A' \cup C = A'$.

$$\therefore \quad P((A \cup B) \cap (A' \cup C)) = P((A \cup B) \cap A') = P(B \cap A') = P(B) = 0.4$$

(i) $P((B \cup C) \cap A')$

Solution

Since both B and C are subsets of A', we have $(B \cup C) \cap A' = B \cup C$. Thus,

$$\begin{aligned} P((B \cup C) \cap A') &= P(B \cup C) \\ &= P(B) + P(C) - P(B \cap C) \\ &= 0.4 + 0.6 - 0.3 \\ &= 0.7 \end{aligned}$$

(j) $P((A \cap B') \cup (B \cap C'))$

Solution

Since A is mutually exclusive with B, we have $A \subseteq B'$, so

$$A \cap B' = A$$

and therefore

$$P(A \cap B') = P(A) = 0.2$$

Also,

$$P(B \cap C') = P(B) - P(B \cap C) = 0.4 - 0.3 = 0.1$$

Since A and B are mutually exclusive, the events $A \cap B'$ and $B \cap C'$ are disjoint. Hence,

$$P((A \cap B') \cup (B \cap C')) = P(A \cap B') + P(B \cap C') = 0.2 + 0.1 = 0.3$$