Lab 2 - Conditional Probability, Monty Hall, and the Value of Information

Overview

In this assignment, you will use an **AI agent as a simulation partner** to explore how information changes probability. In many problems, the important question is not just "What is the probability of an event?", but rather "How does the probability change once new information is revealed?" Through a series of investigations, you will examine how conditional probability can produce outcomes that seem surprising at first, and how the structure of a problem can dramatically affect the value of a strategy.

Your work in this lab will focus on variations of the Monty Hall problem. You will begin with the classic three-door version, then move to a generalized setting with more doors and more prizes, and finally investigate when switching ceases to be the better strategy.

The emphasis is on reasoning, interpretation, and mathematical justification. The AI can help you simulate cases and spot patterns, but the goal is for **you** to make sense of those patterns and explain what is going on.

AI Use Policy

When working with the AI on this lab, treat it as your simulation engine and brainstorming partner, not as a machine that does the thinking for you. You are expected to direct the AI carefully, test whether its outputs are sensible, and then verify the important ideas yourself. In particular, probability is an area where AI tools often sound convincing while making subtle logical mistakes. Your job is to remain skeptical, check the reasoning, and support your conclusions with proper arguments and calculations.

Deadline: April 8th; 4:00 pm (1 pdf file; Dropbox on Lea)

Note: It is your responsibility to ensure that the pdf is readable. Any empty or corrupted files will result in your lab receiving a grade of zero.

The Monty Hall Problem

The Monty Hall problem is based on a game show scenario. A contestant is presented with three closed doors. Behind one door is a prize, and behind the other two doors are non-prizes. The contestant first chooses one door, but it is not opened right away. The host, who knows where the prize is located, then opens one of the two remaining doors, always revealing a non-prize. The contestant is then given a choice: stay with the original door, or switch to the other unopened door.

At first glance, many people assume that once one non-prize door has been opened, the two remaining doors must each have probability $\frac{1}{2}$ of hiding the prize. One of the goals of this lab is to investigate whether that intuition is correct, and to understand the role that information plays in the problem.

Part 1 - The Monty Hall Simulation

Task: Use AI to simulate the classic Monty Hall problem (three doors, one prize). Your goal is to compare what happens when a contestant always stays with their original choice versus when they always switch after the host opens a door. You should then connect the simulation results to the underlying conditional probabilities.

What You Need to Submit

- The results of both simulations (Wins/Losses for "Stay" versus "Switch").
- A diagram of the sample space and the calculation of $P(\text{Win}|\text{Switch})$.
- A critique of the AI's explanation: Did it correctly identify the role of the host's knowledge? Did it properly account for how information changes the problem? Was the explanation logically sound?
- When first presented with the Monty Hall problem, many people assume that each remaining door has equal probability and conclude that switching does not matter. Explain what is missing in this line of reasoning. Back up your reasoning with a mathematical justification.

Part 2 - A Generalized Monty Hall Game

Task: In the classic Monty Hall problem, there are 3 doors and 1 prize. In this part, you will investigate a different version of the game. Suppose there are $2n+1$ doors and n prizes. A contestant chooses one door. The host, who knows where the prizes are, then reveals non-prize doors one at a time until only $n+1$ unopened doors remain. The contestant may then either stay with their original choice, or switch to *one single other unopened door*. Your goal is to investigate whether switching is still the better strategy in this setting.

Suggested steps:

- Ask the AI to simulate this generalized Monty Hall game for several values of n.

- For each case, compare the outcomes of two strategies: always staying with the original choice, and always switching to one specific other unopened door.
- Record what you observe and look for a pattern.
- Use probability and reasoning to explain why that pattern appears.
- Comment on how this version compares with the classic Monty Hall problem.

What You Need to Submit

- evidence from simulation for several values of the parameter
- a comparison of the outcomes for staying and switching
- a mathematical explanation of the pattern you observe
- a short discussion of why this version behaves differently from the classic case
- a paragraph in your on words summarizing your findings

Part 3 - When Is Switching Better?

Task: In Part 2, you explored a version of the Monty Hall problem with multiple prizes and multiple unopened doors remaining at the end. In that setting, "switching" meant switching to *one single other unopened door*. In this part, your goal is to determine more generally when switching is the better strategy, when it is no better than staying, and when staying is actually better.

Suppose a game begins with N doors and K prizes. A contestant chooses one door. The host, who knows where the prizes are, then reveals non-prize doors one at a time until exactly U unopened doors remain. At that point, the contestant may either stay with their original choice, or switch to *one specific other unopened door*.

Suggested steps:

- Begin by comparing the probability of winning by staying with the probability of winning by switching to one specific other unopened door.
- Test several examples with different values of N, K, and U.
- Look for a condition that tells you when switching is better, when the two strategies are equally good, and when staying is better.
- Use probability and clear reasoning to justify your conclusion.
- Check whether your result agrees with what you found in Parts 1 and 2.

What You Need to Submit

- a comparison of the probabilities of winning by staying and by switching
- a general pattern or condition supported by mathematical reasoning

- a short explanation in words of what your result means
- worked examples for at least three different choices of N, K, and U
- a paragraph in your own words interpreting the result strategically