# Extra Practice L16 - L18

1. **Jellyfish**

Just like people, jellyfish sleep for around eight hours a day, and take more naps after a bad night's sleep. A marine biologist claims that a certain species of jellyfish sleeps an average of 8 hours per day. A random sample of 100 jellyfish showed that they slept an average of 8.4 hours per day. Suppose that the population standard deviation is known to be 1.6 hours.

(a) At the 0.05 level of significance, does the data indicate that the average amount of time this species of jellyfish sleeps is more than 8 hours per day? Compute a P-value and write a conclusion in the context of the problem.

**Solution**

$H_0 : \mu = 8$
$H_0 : \mu > 8 \quad \Rightarrow \quad RT \qquad Z_{0.05} = +1.645$

$$Z_T = \frac{\overline{X} - k}{\sigma/\sqrt{n}} = \frac{8.4 - 8}{1.6/\sqrt{100}} = 2.50 \quad \Rightarrow \quad Reject\ H_0$$

**Conclusion:** At the 0.05 level of significance, there is sufficient evidence to indicate that jellyfish sleep an average of more than 8 hours a day.

$P - value = P(Z > 2.50) = 0.0062 < \alpha = 0.05 \quad \Rightarrow \quad Reject\ H_0$

(b) Construct and interpret an appropriate confidence bound/interval to support the conclusion found in (a). Explain how this estimate could be used to corroborate the conclusion of the hypothesis test.

**Solution**

**LCB**

$$\overline{X} - Z_c \cdot \frac{\sigma}{\sqrt{n}} \leq \mu$$
$$8.4 - 1.645 \cdot \frac{1.6}{\sqrt{100}} \leq \mu$$
$$8.1368 \leq \mu$$

**Conclusion:** With repeated sampling we are 95% confident that jellyfish sleep an average of at least 8.1368 hours per day. Since the lower bound is greater than 8, reject $H_0$.

2. **Macho Man**

In 1978, The Village People recruited new members through a newspaper ad that read "Macho Types Wanted: Must Dance and Have a Moustache." A study claims that professional disco dancers in New York City spent an average of 6 hours per week practising their dance routines. A random sample of 40 professional disco dancers showed that they spent an average of 6.3 hours per week practising. Suppose that the population standard deviation is known to be 1.5 hours.

(a) At the 0.15 level of significance, does the data indicate that the average amount of time professional disco dancers spent practising their dance routines is different from 6 hours per week? Compute a $P$-value and write a conclusion in the context of the problem.

**Solution**

$$H_0 : \mu = 6$$
$$H_0 : \mu \neq 6 \quad \Rightarrow \quad DT \qquad Z_{0.075} = \pm 1.44$$

$$Z_T = \frac{\overline{X} - k}{\sigma/\sqrt{n}} = \frac{6.3 - 6}{1.5/\sqrt{40}} = 1.26 \quad \Rightarrow \quad \textit{Fail to Reject } H_0$$

**Conclusion:** At the 0.15 level of significance, there is insufficient evidence to indicate that professional dancers practising their dance routines is different from 6 hours.

$$P\text{−}value = 2 \cdot P(Z > 1.26) = 2(0.1038) = 0.2076 > \alpha = 0.15 \quad \Rightarrow \quad \textit{Fail to Reject } H_0$$

(b) Construct and explain how the question in (a) could be answered by constructing an appropriate confidence bound/interval.

**Solution**

**CI**

$$\overline{X} \pm Z_c \cdot \frac{\sigma}{\sqrt{n}}$$
$$6.3 \pm 1.44 \cdot \frac{1.5}{\sqrt{40}}$$
$$5.9585 < \mu < 6.6415$$

**Conclusion:** With repeated sampling we are 85% confident that the average amount of time that dancers spent practising their routine is between 5.9585 and 6.6415 hours per week. Since $6 \in$ CI; fail to reject the null hypothesis.

3. **Ambulance**

Until the 1960s, the US had no properly organised nationwide ambulance services. In case of an at-home emergency, many people had to ask the police, fire department, or a local funeral home to drive them to the hospital. A study claims that, before organized ambulance services became widely available, the average response time for emergency transportation to the hospital was 28 minutes. A random sample of 64 emergency transportation cases showed an average response time of 26.5 minutes. Suppose that the population standard deviation is known to be 6 minutes.

(a) At the 0.025 level of significance, does the data indicate that the average response time for emergency transportation to the hospital was less than 28 minutes?Compute a $P$-value and write a conclusion in the context of the problem.

**Solution**

$$H_0 : \mu = 28$$
$$H_0 : \mu < 28 \quad \Rightarrow \quad LT \qquad Z_{0.025} = -1.96$$

$$Z_T = \frac{\overline{X} - k}{\sigma/\sqrt{n}} = \frac{26.5 - 28}{6/\sqrt{64}} = -2.00 \quad \Rightarrow \quad Reject\ H_0$$

**Conclusion:** At the 0.025 level of significance, there is sufficient evidence to indicate that the average transport time to the hospital is less than 28 minutes.

$$P - value = P(Z < -2.00) = 0.0028 < \alpha = 0.025 \quad \Rightarrow \quad Reject\ H_0$$

(b) Construct and explain how the question in (a) could be answered by constructing an appropriate confidence bound/interval.

**Solution**

**LCB**

$$\mu \leq \overline{X} + Z_c \cdot \frac{\sigma}{\sqrt{n}}$$
$$\mu \leq 26.5 + 1.96 \cdot \frac{6}{\sqrt{64}}$$
$$\mu \leq 27.97$$

**Conclusion:** With repeated sampling we are 97.5% confident that the average amount of time that it takes to the hospital is at most 27.97 minutes. Since this upper bound is smaller than 28; we reject the null hypothesis.

4. **Library Fees**

For one month in 2024, Worcester Public Library in Massachusetts waived fees for overdue and damaged books for anyone who showed staff a picture of a cat. A study claims that library patrons with overdue or damaged books owe an average of \$12 in library fees. A random sample of 25 patrons who participated in the cat-picture fee waiver program showed that they owed an average of \$13.40 in library fees, with a sample standard deviation of \$4.20.

(a) At the 0.05 level of significance, does the data indicate that the average amount owed in library fees by patrons who participated in the cat-picture fee waiver program is different from \$12? Estimate a $P$-value and write a conclusion in the context of the problem.

Solution

$$H_0 : \mu = 12$$
$$H_0 : \mu \neq 12 \quad \Rightarrow \quad DT \qquad t_{0.025,24} = \pm 2.064$$

$$T_T = \frac{\overline{X} - k}{s/\sqrt{n}} = \frac{13.40 - 12}{4.20/\sqrt{25}} = 1.67 \quad \Rightarrow \quad \textit{Fail to Reject } H_0$$

**Conclusion:** At the 0.05 level of significance, there is insufficient evidence to indicate that the average library fee is different from 12.

$$2 \cdot (0.05) < P - value < 2 \cdot (0.075)$$
$$\alpha = 0.05 < 0.10 < P - value < 0.015 \quad \Rightarrow \quad \textit{Fail to reject } H_0$$

(b) Construct and explain how the question in (a) could be answered by constructing an appropriate confidence interval/bound.

Solution

**CI**

$$\overline{X} \pm t_{c,n-1} \cdot \frac{s}{\sqrt{n}}$$
$$13.40 \pm 2.064 \cdot \frac{4.20}{\sqrt{25}}$$
$$11.6662 < \mu < 15.1338$$

**Conclusion:** With repeated sampling we are 95% confident that the average library fees owed is between \$11.66 and \$15.13. Since $12 \in$ CI; fail to reject $H_0$

(c) State the Type I and Type II errors in the context of the problem.

**Type I:** Concluding that the average library fees owed is different from \$12, when in fact it is not.

**Type II:** Concluding that the average library fees owed is \$12, when in fact it is not.

5. **Science Project**

The largest vinegar-and-baking-soda 'volcano' was made in a British school in 2015. It stood 8.62 metres tall and used 100 litres of vinegar. A study claims that students spend an average of 3.5 hours preparing a science project. A random sample of 30 students who built vinegar-and-baking-soda volcanoes showed that they spent an average of 4.1 hours preparing their projects, with a sample standard deviation of 1.4 hours.

(a) At the 0.05 level of significance, does the data indicate that the average amount of time students spend preparing vinegar-and-baking-soda volcano projects is more than 3.5 hours? Compute a $P$-value and write a conclusion in the context of the problem.

**Solution**

$$H_0 : \mu = 3.5$$
$$H_0 : \mu > 3.5 \quad \Rightarrow \quad RT \qquad t_{0.05,29} = 1.699$$

$$T_T = \frac{\overline{X} - k}{s/\sqrt{n}} = \frac{4.1 - 3.5}{1.4/\sqrt{30}} = 2.347 \quad \Rightarrow \quad Reject\ H_0$$

**Conclusion:** At the 0.05 level of significance, there is sufficient evidence to indicate that the average spent on preparing a baking soda and vinegar volcano is greater than 3.5 hours.

$$0.01 < P - value < 0.025 < \alpha = 0.05 \quad \Rightarrow \quad Reject\ H_0$$

(b) Construct and explain how the question in (a) could be answered by constructing an appropriate confidence interval/bound.

**Solution**

**LCB**

$$\overline{X} - t_{c,n-1} \cdot \frac{s}{\sqrt{n}} \leq \mu$$
$$4.1 - 1.699 \cdot \frac{1.4}{\sqrt{30}} \leq \mu$$
$$3.6657 \leq \mu$$

**Conclusion:** With repeated sampling we are 95% confident that the average time spent on making a vinegar and baking soda volcano is at least 3.6657 hours. Since the lower bound is greater than 3.5, reject $H_0$.

6. **Gifts**

In a recent survey, 10% of pet-owning Brits said they were disappointed when guests didn't bring a Christmas present for their animal. A study claims that pet-owning Brits spend an average of $35 on Christmas gifts for their pets. A random sample of 110 pet-owning Brits showed that they spent an average of $31.50 on Christmas gifts for their pets, with a sample standard deviation of $9.20.

(a) At the 0.005 level of significance, does the data indicate that the average amount spent by pet-owning Brits on Christmas gifts for their pets is less than $35? Compute a $P$-value and write a conclusion in the context of the problem.

**Solution**

$$H_0 : \mu = 35$$
$$H_0 : \mu < 35 \quad \Rightarrow \quad RT \qquad t_{0.005,100} = -2.626$$

$$T_T = \frac{\overline{X} - k}{s/\sqrt{n}} = \frac{31.50 - 35}{9.20/\sqrt{110}} = -3.99 \quad \Rightarrow \quad Reject\ H_0$$

**Conclusion:** At the 0.005 level of significance, there is sufficient evidence to indicate that the average amount spent on gifts for pets is less than $35.

$$P - value < 0.0005 < \alpha = 0.005 \quad \Rightarrow \quad Reject\ H_0$$

(b) Construct and explain how the question in (a) could be answered by constructing an appropriate confidence interval/bound.

**Solution**

**UCB**

$$\mu \leq \overline{X} + t_{c,n-1} \cdot \frac{s}{\sqrt{n}}$$
$$\mu \leq 31.50 + 2.626 \cdot \frac{9.20}{\sqrt{110}}$$
$$\mu \leq 33.8034$$

**Conclusion:** With repeated sampling we are 99.9995% confident that the average amount of money spent on gifts for pets is at most \$33.80. Since the upper bound of this interval is less than 35, reject $H_0$.

(c) State the Type I and Type II errors in the context of the problem.

**Type I:** Concluding that the average amount spent on gifts for pets is less than \$35, when in fact it is not.

**Type II:** Concluding that the average amount spent on gifts for pets is \$35, when in fact it is not.

7. **Conspiracy Theories**

In a conspiracy theory study, 10% of respondents claimed to believe that the Canadian Armed Forces were secretly developing an army of super intelligent giant raccoons to invade neighbouring countries (a theory invented by the researchers). A researcher claims that 10% of Canadians believe this giant-raccoon conspiracy theory. A random sample of 400 Canadians showed that 32 of them believed the conspiracy theory.

(a) At the 0.03 level of significance, does the data indicate that the proportion of Canadians who believe this giant-raccoon conspiracy theory is less than 10%? Compute a $P$-value and write a conclusion in the context of the problem.

**Solution**

Let $X$ = the number of adults who believe in the conspiracy.

$\hat{p} = \frac{x}{n} = \frac{32}{400} = 0.08 \qquad n\hat{p} > 5 \quad n(1-\hat{p}) > 5$

$H_0 : p = 0.10$

$H_0 : p < 0.10 \quad \Rightarrow \quad LT \qquad Z_c = -1.88$

$$Z_t = \frac{\hat{p} - k}{s/\sqrt{\frac{k(1-k)}{n}}} = \frac{0.08 - 0.10}{\sqrt{\frac{0.10(1-0.10)}{400}}} = -1.33 \quad \Rightarrow \quad \textit{Fail to Reject } H_0$$

**Conclusion:** At the 0.03 level of significance, there is not enough evidence to indicate that the percentage of adults who believe in the conspiracy is less than 10%

$$P - value = P(Z < -1.33) = 0.0918 > 0.03 = \alpha \quad \textit{Fail to reject } H_0$$

(b) Construct and explain how the question in (a) could be answered by constructing an appropriate confidence interval/bound.

Solution

**UCB**

$$P \leq \hat{p} + Z_c\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$
$$P \leq 0.08 \pm 1.88\sqrt{\frac{0.08(1-0.08)}{400}}$$
$$P \leq 0.1055$$

**Conclusion:** With repeated sampling we are 97% confident that the actual percentage of Canadians who believe in this conspiracy is at most 10.55%. Since 10.55% is in the confidence bound, it does not support the alternate hypothesis. Therefore, fail to reject $H_0$.

8. **Premature Aging**

Having difficult relatives or colleagues could age you 1.5% faster. A study claims that 42% of adults report having at least one difficult relative or colleague who causes them regular stress. A random sample of 500 adults showed that 230 of them reported having at least one difficult relative or colleague who causes them regular stress.

(a) At the 0.02 level of significance, does the data indicate that the proportion of adults who report having at least one difficult relative or colleague who causes them regular stress is different from 42%? Compute a $P$-value and write a conclusion in the context of the problem.

Solution

Let $X$ = the number of adults who believe that colleagues and relatives cause stress.
$\hat{p} = \frac{x}{n} = \frac{230}{500} = 0.46 \qquad n\hat{p} > 5 \quad n(1-\hat{p}) > 5$

$H_0 : p = 0.42$
$H_0 : p \neq 0.42 \quad \Rightarrow \quad DT \qquad Z_c = \pm 2.33$

$$Z_t = \frac{\hat{p} - k}{s/\sqrt{\frac{k(1-k)}{n}}} = \frac{0.46 - 0.42}{\sqrt{\frac{0.42(1-0.42)}{500}}} = 1.812 \quad \Rightarrow \quad \textit{Fail to Reject } H_0$$

**Conclusion:** At the 0.02 level of significance, there is not enough evidence to indicate that the percentage of adults who believe that a difficult relative or colleague causes them stress is different from 42%.

$$P - value = 2 \cdot P(Z > 1.81) = 2(0.0351) = 0.0702 > 0.02 = \alpha \quad \textit{Fail to reject } H_0$$

(b) Construct and explain how the question in (a) could be answered by constructing an appropriate confidence interval/bound.

Solution

**CI**

$$\hat{p} \pm Z_c\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$
$$0.46 \pm 2.33\sqrt{\frac{0.46(1-0.46)}{500}}$$
$$0.3981 < p < 0.5019$$

**Conclusion:** With repeated sampling we are 98% confident that the actual percentage of people who believe that their colleagues or relatives cause them stress is between 39.81% and 50.19%. Since 42% is in the CI, fail to reject $H_0$.

9. **Crying at the Movies**

In a survey, 15% of men said they were more likely to cry while watching a film on a flight than at home. Only 6% of women said the same.A study claims that 15% of men are more likely to cry while watching a film on a flight than while watching the same film at home. A random sample of 300 men showed that 57 of them said they were more likely to cry while watching a film on a flight.

(a) At the 0.05 level of significance, does the data indicate that the proportion of men who are more likely to cry while watching a film on a flight is greater than 15%?Compute a $P$-value and write a conclusion in the context of the problem.

Solution

Let $X$ = the number of men who cry watching movies.

$\hat{p} = \frac{x}{n} = \frac{57}{300} = 0.19 \qquad n\hat{p} > 5 \quad n(1-\hat{p}) > 5$

$H_0 : p = 0.15$

$H_0 : p > 0.15 \quad \Rightarrow \quad RT \qquad Z_c = 1.645$

$$Z_t = \frac{\hat{p} - k}{s/\sqrt{\frac{k(1-k)}{n}}} = \frac{0.19 - 0.15}{\sqrt{\frac{0.15(1-0.15)}{300}}} = 1.94 \quad \Rightarrow \quad Reject\ H_0$$

**Conclusion:** At the 0.05 level of significance, there is enough evidence to indicate that the percentage of men who cry watching movies is greater than 15% .

$$P - value = P(Z > 1.94) = 0.0262 < 0.05 = \alpha \quad reject\ H_0$$

(b) Construct and explain how the question in (a) could be answered by constructing an appropriate confidence interval/bound.

**Solution**

**LCB**

$$\begin{aligned}\hat{p} - Z_c\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} &\leq P \\ 0.19 - 1.645\sqrt{\frac{0.19(1-0.19)}{300}} &\leq P \\ 0.1527 &\leq P\end{aligned}$$

**Conclusion:** With repeated sampling we are 95% confident that the actual percentage of men who cry while watching movies is at least 15.27%. Since this lower bound is greater than 15, reject $H_0$.