

L10. The Normal Distribution

Example 1

Suppose that X is a random variable that is normally distributed. Compute the following probabilities:

- a. $P(1.28 < Z)$
- b. $P(-0.54 > Z)$
- c. $P(-2.33 < Z)$
- d. $P(Z > 2.33)$
- e. $P(Z > 3.05)$
- f. $P(-1 < Z < 1)$
- g. $P(-2 < Z < 2)$
- h. $P(-3 < Z < 3)$

Solution

Example 2

Suppose that X is a random variable that is normally distributed. Find the value of k that satisfies the statement

- a. $P(k < Z) = 0.9$
- b. $P(k > Z) = 0.15$
- c. $P(Z < k) = 0.9451$
- d. $P(Z > k) = 0.6616$
- e. $P(-k < Z < k) = 0.95$
- f. $P(-k < Z < k) = 0.99$
- g. $P(-k < Z < k) = 0.80$

Solution

Example 3

Suppose that X is a random variable that is normally distributed with a mean of $\mu = 10$ and a standard deviation of $\sigma = 2$. Calculate the following probabilities:

- a. $P(X < 14)$
- b. $P(X \leq 8)$
- c. $P(X > 5)$
- d. $P(7.5 < X < 11.5)$
- e. $P(X < 0)$
- f. $P(X \geq 22)$

Solution

Example 4: Coffee vs. Tea

In 1743, King Gustav III of Sweden ordered one man to drink an excessive amount of coffee and his identical twin drink an excessive amount of tea to see which died first. Both men outlived the king.

The amount of time (in minutes) that customers spend in a coffee shop follows a normal distribution with a mean of 35 minutes and a standard deviation of 8 minutes.

- a. What is the probability that a randomly selected customer spends more than 45 minutes in the coffee shop?
- b. What is the probability that a customer spends between 30 and 40 minutes in the coffee shop?
- c. What percentage of customers spend less than 20 minutes in the coffee shop?
- d. 90% of customers spend at most how many minutes at the coffee shop?

Solution

Example 5: Have You Seen This Deer?

In 2020, a startled deer in the Czech Republic snagged a hunter's rifle with its antlers and bolted into the forest, leaving the hunter both disarmed and dumbfounded. The police were called, but the deer and the rifle have yet to be recovered.

The distance (in meters) a deer bolts when startled follows a normal distribution with a mean of 300 meters and a standard deviation of 40 meters.

- a. What is the probability that the deer ran more than 360 meters, making any attempt to recover the rifle even more hopeless?
- b. What is the probability that it ran a distance between 280 and 320 meters?
- c. Only the most elusive 5% of deer dash far enough to truly vanish without a trace. What is the minimum distance a deer must run to be in this elusive top 5%?

Solution

Example 6: Captain Morgan

The flag ship of Captain Morgan sank in 1671. It was discovered in 2011 by an expedition funded by the rum makers, Captain Morgan.

Let X be a normally distributed random variable representing the volume of liquor (in mL) in a standard bottle of Captain Morgan rum. According to quality control records, the mean fill volume is 750 mL , with a standard deviation of 6 mL .

- What is the probability that a randomly selected bottle contains less than 740 mL of rum? (i.e., a bottle that might make even Captain Morgan raise an eyebrow)
- Suppose there are 10 bottle of Captain Morgan rum on the shelves. What is the probability that fewer than three contain less than 740 mL of rum in them? What proportion of bottles contain between 745 mL and 755 mL ?
- The company promises that only the lowest 2.5% of under filled bottles should ever reach the shelves. What is the threshold volume (in mL) that separates the bottom 2.5% from the rest?

Solution