

L11. The Normal Approximation to the Binomial Distribution

Example 1: Gum Sales

Chewing gum sales have dropped by 15% over the past decade. One theory is that shoppers are too distracted by their smartphones to notice gum at the checkout. Let's suppose that the probability a shopper notices the gum is 0.4.

- a. In a small observational study, 6 shoppers are monitored. What is the probability that 4 or 5 shoppers notice the gum?
- b. In a different store, 10 shoppers were observed. What is the probability that at least 1 shopper notices the gum?
- c. In a busy super market, 70 shoppers were observed. What is the probability that
 - i. 30 or more shoppers notice the gum?
 - ii. less than 15 notice the gum?
 - iii. between 40 and 60 shoppers (inclusive) notice the gum?

Solution

Example 2: Amber Alert

In 2021, Texan authorities sent out an Amber alert asking citizens to look out for a 3ft1 28-year-old with red hair and blue eyes who is wearing denim overalls, a stripy shirt and carrying a large knife.



The probability that a random Texan received the Amber alert is 0.25.

- In a small town, 6 people were surveyed. What is the probability that 3 or 4 people received the alert?
- A larger sample of 80 people was taken in a suburban area. What is the probability that at least 20 people received the alert?
- In a state wide survey, 300 people were polled. Estimate the probability that between 65 and 90 people (exclusive) received the alert.
- In a different sample of 400 people, estimate the probability that fewer than 270 people did not receive the alert.

Solution

Example 3: AI Detection Test

When the U.S. military tested their new AI human-movement recognition robot, a group of Marines were asked to approach it undetected. Despite the robot's state-of-the-art sensors, all of them succeeded—one disguised himself as a fir tree, another somersaulted for 300 metres, and two hid under a cardboard box while giggling the entire way.

The distance a Marine could crawl before being detected by the robot follows a normal distribution with a mean of 240 metres and a standard deviation of 35 metres.

- a. What is the probability that a randomly selected Marine crawls more than 275 metres before being detected?
- b. Given that a Marine is detected before reaching 260 metres, what is the probability that he crawled between 200 and 230 metres?
- c. What crawling distance corresponds to the 99th percentile for Marines in this test?
- d. In a training camp of 150 Marines, each attempts to approach the robot once. Suppose the probability that a Marine remains undetected is 0.84. What is the probability that more than 135 Marines go unnoticed?
- e. In a smaller squad of 12 Marines, what is the probability that exactly 8 or 9 of them crawl more than 260 metres before being detected?

Solution

Example 4: Sunions

‘Sunions’ are a new type of genetically engineered onion that don’t cause tears when chopped. Marketed for their mild flavor and tear-free properties, they are now being tested in grocery stores across the country.

The circumference of Sunions follows a normal distribution with a mean of 18 cm and a standard deviation of 2.5 cm.

- a. What is the probability that a randomly selected Sunion has a circumference greater than 21 cm?
- b. If a Sunion has a circumference that is greater than 19 cm, what is the probability that it is larger than 22 cm?
- c. What circumference corresponds to the 95th percentile for Sunions?
- d. In a test panel of 120 people, each person is asked to chop a Sunion and report if it made them tear up. Suppose the probability a person does not cry is 0.92. What is the probability that more than 110 people report no tears?
- e. In a grocery shipment of 10 Sunions, what is the probability that 6 or 7 of them have a circumference greater than 20 cm?

Solution