

# L15. Confidence Interval on the Population Proportion

## Example 1: Quality Control

The fraction of defective integrated circuits produced in a photolithography process is being studied. A random sample of 300 circuits is tested, revealing 15 defectives. Calculate a 90% two-sided confidence interval on the fraction of defective circuits produced in this particular process.

## Solution

**Example 2: Mail Orders**

A mail order company promises its customers that the products ordered will be mailed within 72 hours after an order has been placed. The quality control department checks from time to time to see if this promise is fulfilled. Recently the quality control department took a sample of 50 orders and found that 35 of them were mailed 72 hours after the order was placed.

- a. Construct a 98% confidence interval for the percentage of all orders that were mailed within 72 hours of their placement.
- b. Construct a 98% lower-confidence bound for the percentage of all orders that were mailed within 72 hours of their placement.
- c. Construct a 96% upper-confidence bound for the proportion of all orders that were mailed within 72 hours of the order being placed.

**Solution**

**Example 3: Research Grants**

In 2011, scientists in the US used \$660000 in federal research money to examine whether distant prayer could heal AIDS, \$406000 to see if squirting brewed coffee into someone's intestines would help with pancreatic cancer, and \$1.25 million to examine whether massages made people with advanced cancer feel better. The distant prayers and coffee enemas did not work, but the massages did. In a group of 200 cancer patients, all of who received regular massages, 165 of them reported feeling better after their treatment.

- a. Construct a 85% confidence interval for the true proportion of patients who felt better after receiving a massage.
- b. Suppose that a preliminary study has found that 84% of patients felt better after receiving a massage. What should the sample size be so that the 85% confidence interval for the population proportion has a margin of error 0.05?
- c. In the absence of a preliminary estimate, for  $\hat{p}$ , what sample size should be used so that the 85% confidence interval for the population proportion has a margin of error 0.05?

**Solution**

**Example 4: Helmets**

A random sample of 53 suspension helmets used by motorcyclists and race-car drivers was subjected to an impact test, and some damage was observed on 18 of these helmets.

- a. Find a 97% two-sided confidence interval on the true proportion of helmets that would show damage from this test.
- b. In a previous study, it was found that 37% of the helmets showed some damage after being subjected to the impact test. How many helmets must be tested in order to be 97% confident that the error in estimating  $p$  is at most 0.10?
- c. How large must the sample be if we wish to be at least 97% confident that the error in estimating  $p$  is less than 0.02 regardless of the true value of  $p$ ?
- d. Find a 97% lower-confidence bound for the true proportion of helmets that showed some damage from the impact test.

**Solution**