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LECTURE 3

KINEMATICS

Kinematics uses a set of parameters (***time***, ***position, displacement, velocity and acceleration***) to describe the motion of an object in time.

1] THREE MAIN TYPES OF MOTION

- One may distinguish three basic ways of objects' motion: ***translation, rotation and vibration.*** During a translation and rotation there is *no deformation* of object; its *shape* and its *dimensions remain the same*. During a vibrational motion, the object experiences changes of one dimension or of its whole shape.

-**Translation**: All points of the object move in the same way and their *final position can be found by using the* ***same displacement vector****.*

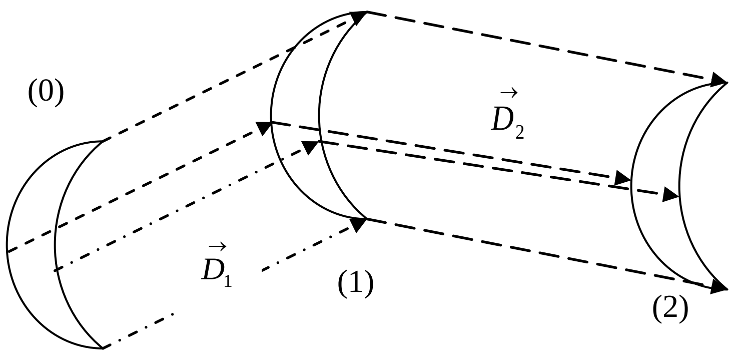

Figure 1 Translation

The same displacement vector $\vec{D}_1$ shifts each object point from position (0) to position (1). The same displacement vector $\vec{D}_2$ shifts each object point from position (1) to position (2).

-**Rotation**: The object changes orientation in space. Each object point has a ***different displacement vector***.

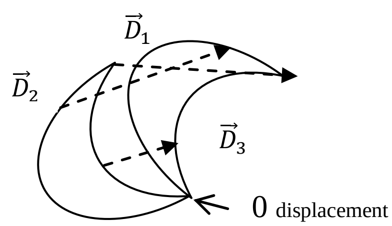

Figure 2 ***Rotation***

Each point of object has a specific displacement vector.

-***Vibration***: There is oscillations *in time of one* **,** *two or the three* **dimensions** of the *object.*

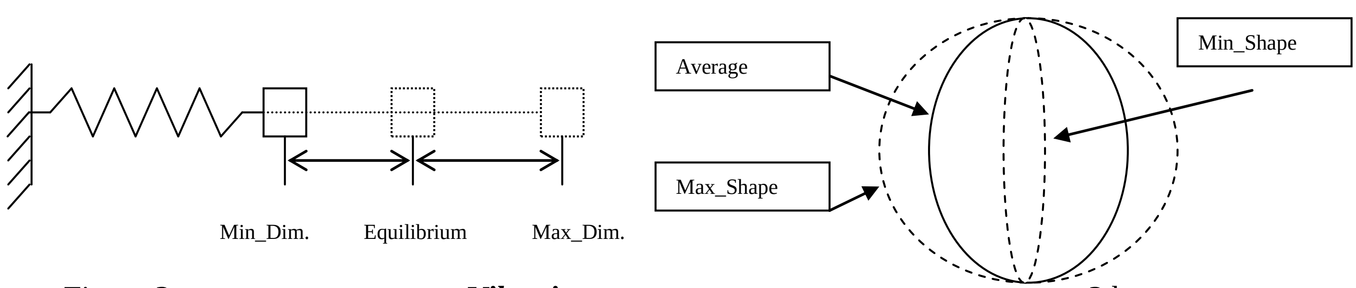

Figure 3.a ***Vibration*** 3.b

In fig.3.a, the object of study (*spring & block*) changes only one dimension in periodic way.
In fig.3.b, the object of study (an elastic ball) changes the shape (three dimensions) in time.

2] ONE DIMENSIONAL KINEMATICS

- Mechanics uses a specific ***physical model*** for the study of each type of motion. This chapter deals with the translation of an object along a fixed direction in space (***1-D translation***). In the next chapter, one will present the main steps for the study of the translation motion on a plane (***2-D translation***[1]).
As mentioned above, *in **a translation***, all object points are shifted by *the same displacement vector*.
This means that: " One can *study the movement of just one point of object and the results apply over the object as a whole***"**. So, one uses ***a point particle motion*** *as a **model** for the* ***object translation***.

2.a DISPLACEMENT VERSUS THE TRAVELLED DISTANCE

-Consider an object (*plane, car*..) moving **along a straight line**. One starts by modeling it as a particle in motion along a straight line. To define the ***position*** of this particle, *one draws one axe* and chooses :
a) its *origin* O; **b)** its p*ositive direction;* **c)** Length unit "***m***"; **d)**Time unit "***sec***"; **e)** *t=0 at initial location*.
Figure 4 presents 1-D motion of a ***point particle*** that starts the motion at $t_0 = 0sec$ from $X_{I(initial)} = +1m$, moves ***4m*** to the right, turns back at $X = +5m$ and stops at $X_{F(final)} = -3m$ after $\Delta t = t_F - t_o = 2s$.

-One defines the particle ***displacement*** as $\Delta x = X_F - X_I$ where X_F is the final location and X_I is the initial location. *The displacement "Δx"* may be **positive or negative**. If $\Delta x > 0$***, at the end of*** Δt ***,*** the particle ***is shifted on positive direction*** *and* ***if*** $\Delta x < 0$ ***it is shifted on negative direction of Ox axe***.

Remember: The **displacement** is *not the same as the* **travelled distance** (***always positive)***. In the case of Fig. 4, the *travelled distance* is **S** = 4 + 8 = 12m while the *displacement* is $\Delta x = (-3) - (+1) = -4m.$

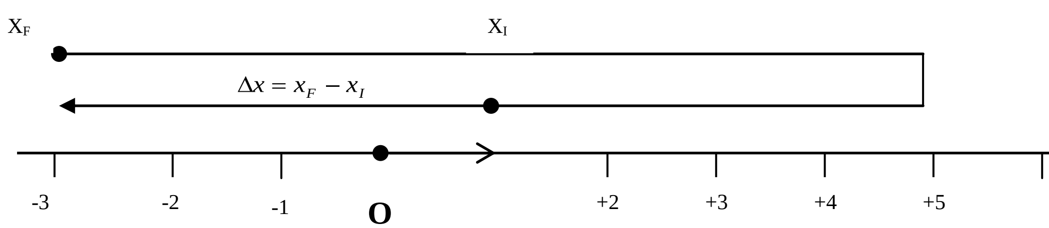

Figure 4

2.b VELOCITY VERSUS SPEED

How fast is moving the particle?

- In *everyday vocabulary* one uses ***the speed*** (***positive scalar***) to answer this question in case of cars, planes etc. If one does not need any information about the way the particle is moving in particular portions of its path, one refers to the

$$\textbf{average speed along the path} = sp_{av} = \textbf{travelled distance / time interval} = S / \Delta t \qquad (1)$$

*The travelled distance "S" and the average **speed** "sp_{Av}" along the path are **positive** (always) **scalars**.*

In the upper mentioned example, one would find: ***average speed,*** $sp_{av} = 12m / 2\ sec = 6\ m/sec$

[1] The movement in space, i.e. 3-D motion is not included in programme.

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-In physics, one uses mainly ***the velocity*** to describe the way a particle is moving. One starts by defining

the *average velocity as* $$average\_velocity = \upsilon_{Av} = \frac{\Delta x}{\Delta t} \qquad (2)$$

Note that the velocity may be ***negative or positive***; in our example $\boldsymbol{\upsilon_{av} = -4/2 = -2\ m/s}$ which is different from average speed (6 m/s).

Remember: ***Speed*** *and* ***velocity*** *are* ***different parameters*** *but they have the* ***same units****.*

Note: *The* ***displacement*** *and the* ***velocity*** *are* ***both positive*** *when the particle moves* ***along +Ox*** *direction.*
When the particle moves along opposite(-Ox) direction, they are ***both negative****.*
So, the ***sign of velocity*** *and* ***displacement*** *indicates the direction of motion.*

- One uses often the graph ***x = x(t)*** to present the *history of a particle motion*. The figures 5, 6 present two such graphs. The ***average velocity*** is calculated easily from the slope at these graphs. In the graph of figure 5, the slope (Δx/Δt) does not depend on the moment one starts counting the interval of time or the magnitude of Δt. In this case the motion follows *all time at the same velocity*; it is an ***uniform motion***. In the graph of fig.6, the ***average velocity*** depends on the initial moment and on the magnitude of the time interval **Δt**. This is not an uniform motion; $\boldsymbol{\upsilon_{Av}}$ does not offer enough information for this motion.

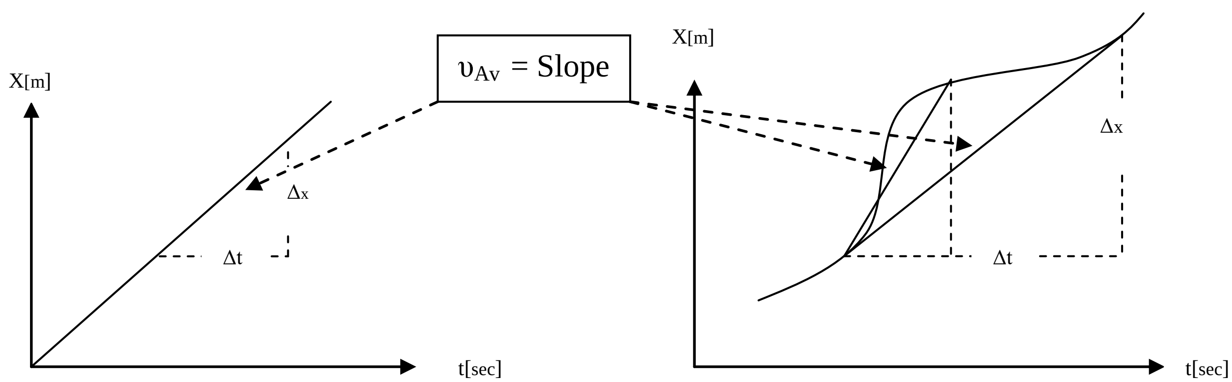


Figure 5 Figure 6

2.c INSTANTANEOUS VELOCITY

- A simple observation of figure 7 shows that, for right information about the *particle velocity* at point P one must refer to the *shortest time interval* Δt counted from the moment t = t_P .

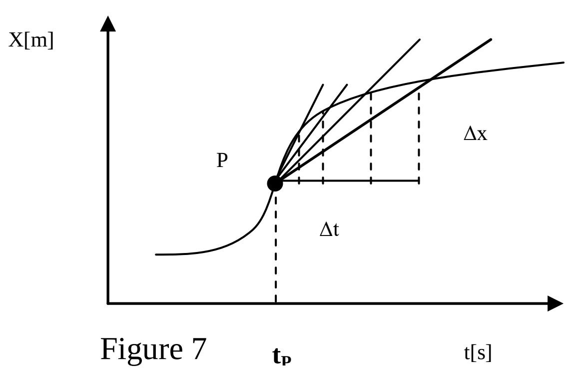


Figure 7

One may figure out that the best estimation of velocity close to point P is the limiting value of average velocity when the time interval Δt goes versus zero. In fact, this is the definition of ***instantaneous velocity*** at point P

$$\upsilon^P = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\Big|_{t_P} = \frac{dx}{dt}\Big|^P \qquad (3)$$

Note_1: As this derivative is equal to the slope of ***tangent of*** curve ***at point P***, one may find the velocity from the function ***x = x(t)***.
Note_2: From this definition it comes out that the **instantaneous speed is equal to absolute value of instantaneous velocity**.

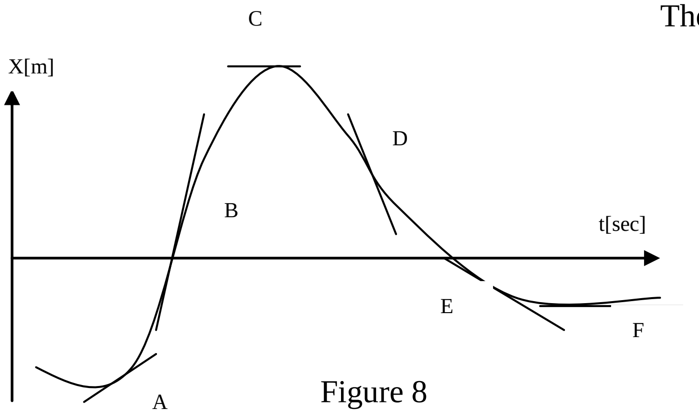


Figure 8

The graph in fig.8, presents a motion along Ox axe.

A-point; $\upsilon > 0$; <u>Motion in (+) Ox direction</u>.
B-point; ***larger*** $\upsilon > 0$; <u>Faster motion along (+) Ox</u>

C-point; $\upsilon = 0$; I<u>nstantaneous rest, turn back point.</u>

D-point; $\upsilon < 0$; <u>Motion in (-) Ox direction.</u>
E-point; ***smaller*** $\upsilon < 0$; <u>Slower motion along (-) Ox</u>

F-point; $\upsilon = 0$ ***for a while*** ;
<u>Long rest before turning back along (+)Ox direction.</u>

2.d ACCELERATION

-In the popular vocabulary the word "**acceleration**" means *speed increase*. In physics, it means a ***change of velocity*** *(magnitude, direction or magnitude & direction simultaneously)*. If one knows the velocity at each moment of time one may build the graph of instantaneous velocity versus time $\upsilon = \upsilon(t)$. If this graph is a *straight line*, the **acceleration** $a = \Delta\upsilon/\Delta t$ is *constant* all time. If the graph $\upsilon = \upsilon(t)$ is curved one has to introduce the ***instantaneous*** **acceleration**. The *average acceleration is* $a_{Av} = \Delta\upsilon/\Delta t$ is the *ratio (change of velocity / time interval)* ***and*** *the* ***instantaneous acceleration at point D*** *is* $a_D = \lim_{\Delta t \to 0} \frac{\Delta \upsilon}{\Delta t}\Big|_{t_D} = \frac{d\upsilon}{dt}\Big|_{t_D}$ (4)

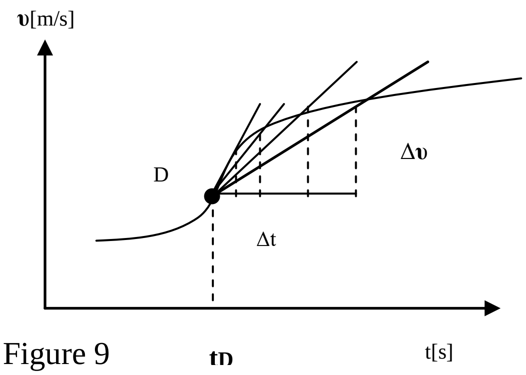


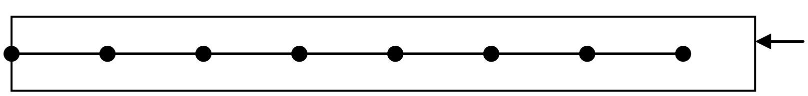
Figure 9

The **acceleration** at any point on the graph $\upsilon = \upsilon(t)$ ***is*** *equal to the* ***slope*** *of graph at that point.*
The *acceleration may be positive or negative*. It's important to note that acceleration sign alone is not sufficient to find whether the particle is *speeding up* or *slowing down* .To get this information, one must <u>*compare the **sign of "a"** to **sign of "υ"***</u>

Notes: a) If ***a and υ*** have the <u>***same sign***</u> the particle is <u>***speeding up***</u> (or accelerating).
b) If ***a and υ*** *have* <u>***opposite sign***</u> *the body is* <u>***slowing down***</u> (or decelerating).

Remember: The <u>***direction of motion***</u> is shown by <u>*sign of displacement and **the sign of velocity***</u>.
If $\upsilon > 0$ the motion is ***along*** + ***Ox direction***; if $\upsilon < 0$ the particle moves in ***opposite*** direction to + Ox.

- How to distinguish an <u>*accelerated*</u> motion ($a \neq 0$) ***from*** a uniform motion (<u>*constant velocity*</u> ,$a = 0$)?
Fix an interval of time (say 1sec) and note the position of object for several consecutive time intervals. Then, if all **displacements** are **equal**, there is a motion at constant velocity (fig 10.a $\upsilon = 2m/s$, $a = 0m/s^2$).
If the consecutive **displacements change (increase** or decrease) there is an <u>***accelerated** motion*</u>.



(x=2*t) Figure 10.a

Δx = 2 2 2 2 2 2 2

Acc. motion (Δx increased by 4)

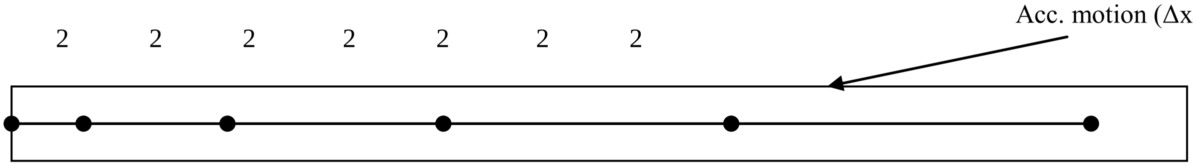


Figure 10.b
($x=2*t^2$)

Δx = 2 6 10 14 18

-The graphs in Figure 11 show how to follow the signs of ***υ(t)*** and ***a(t)*** at different motion situations .

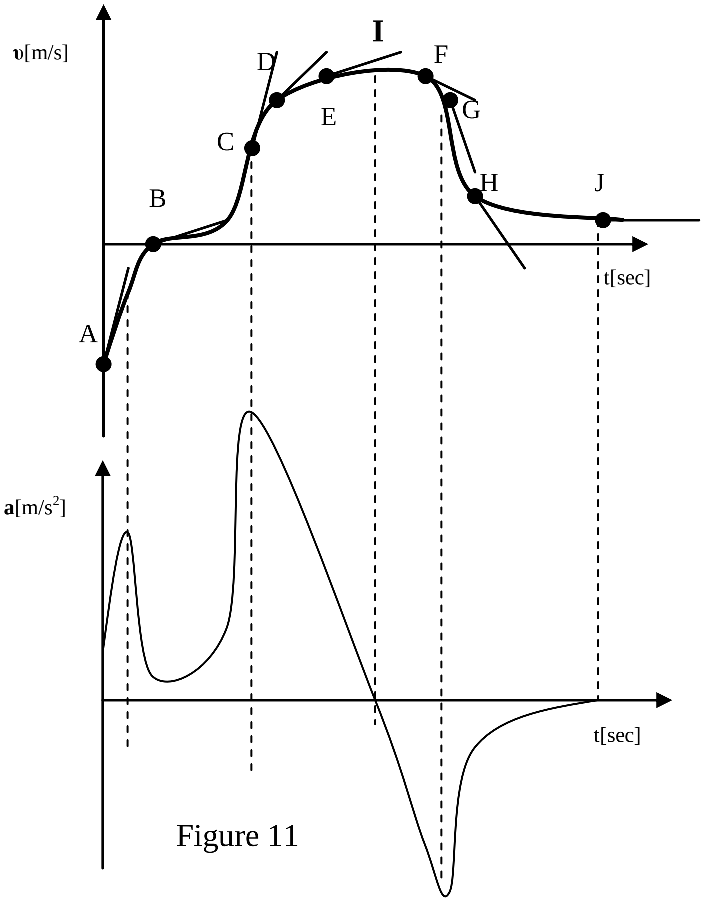


Figure 11

A-point; ***υ < 0*** means *motion along* (**–) Ox**,
As a > 0 *(opposite sign****) there is*** **"slowing down"**.

B-point; ***υ = 0 but a ≠ 0 ;*** *this means* ***instantaneous rest***,
As a > 0 means Δ***υ*** > **0; So,** ready to speed up vs (+**Ox**).

Between B&C; ***υ > 0*** means motion ***along +Ox,***
as ***a > 0*** *i.e. speeding up ;* ***C****-point max_acceler.*

Between C&E; ***υ > 0*** *motion along* ***+Ox,***
As a > 0 but decreasing means *slight* ***speeding up****;*

I*-point =* ***zero_acceleration***

Between I &J; ***υ > 0*** *means motion along* ***+Ox,***
As a < 0 but υ > 0 *the particle is slowing down.*

major slowing down between G and H.

Beyond H-point, there is a slight slowing down until a=0 at point J (constant velocity)

Notes: *In real life the velocity cannot change instantaneously and the acceleration (a =Δυ/Δt) has always a finite value (no infinite).*