LECTURE 6

1] INERTIA AND THE FIRST LAW OF NEWTON

- Everyday experience tells that "*if one does not push a body in motion , it will stop moving"*. So, one may say "*the rest seems to be the natural state of objects*". But, at this comment one forget the ***action*** of ***friction with surrounding medium*** (Ex. friction of a wooden block with the floor on which it is sliding on,..). *What if one could avoid the friction?* By using oil… the block would follow its road a long way on. Galileo Galilei was the first to figure out that: *If there were* ***no friction*** *at all*, ***the block would keep moving at constant velocity*** *without slowing down. This way one gets to the* "Galileo's ***principle of inertia***". He named as "***inertia"*** *the* ***propriety*** *of an object* ***to keep moving at constant velocity*** as long as there is a ***zero net force*** applied on it.

- Assume that one places *carefully the block on a horizontal table*. It will not move even if there is oil on table. The block will follow to stay ***at rest*** on table unless a force acts on it. Therefore, if it is *initially at rest*, the object will follow to be ***at rest*** as long as "the *net force exerted on it is zero*".
By combining this observation with Galileo principle, Newton formulated his ***first law*** of mechanics:

A body remains at rest <u>or</u> keeps moving at same velocity as long as there is a ZERO NET FORCE applied on it.

- Another qualitative *definition* for *inertia*, <u>*related only to the body itself*</u>, appeared with time:

"Inertia is the propriety of bodies to resist to changes in their state of motion"

To get the *quantitative parameter that measures inertia* (which is called the ***inertial mass***) one refers to the second law of Newton (see the next lecture). For now, one should remember that:

a) The first Newton´s law refers to situations *where there is* **zero net force** *acting on the body.*

Ex: You push an object so that **it slides** on the table at an **<u>constant velocity</u>.** This means that **the sum of all forces** is

$$\vec{F}_{net} = \vec{F}_g + \vec{N} + \vec{F}_{app.} + \vec{F}_{fr.} = 0 \quad (1)$$

As the gravity force action is cancelled by normal force, it comes out that friction force cancels the action of applied force and they have equal magnitudes.

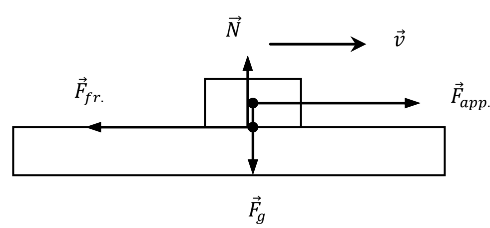

Figure 1

b) *The inertia opposes any change*[1] *in velocity (not only in magnitude but in the direction, too)*

Ex: In discus throwing (sport), *sportsman arm forces the discus to change the velocity direction. In the meantime,* the sportsman *feels on his arm the* ***<u>inertia opposition</u>***.

When he leaves it free, *due to its inertia*, the discus follows its way straight along the direction of instantaneous velocity (tangent to its circular path).

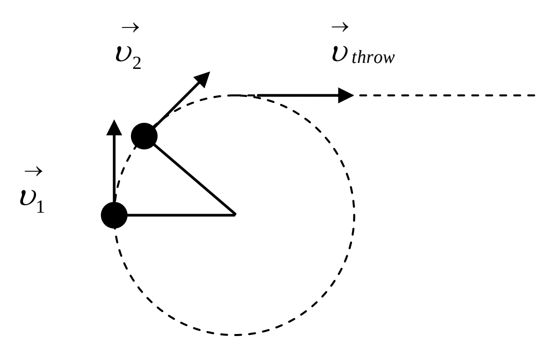

Figure 2

[1] Not just speed increase as it is considered in popular vocabulary.

2] INERTIAL REFERENCIAL FRAMES

-Any mechanics study starts by definition of a ***reference frame*** i.e. a ***coordinative system*** with its origin fixed ***on an object***. Next, one introduces the kinematics parameters for a motion *" versus this frame "* .
Ex.: A man walks at ***5km/h*** inside a train moving at ***50 km/h*** with respect to the station. If one ties the frame origin to the station (earth) the results concern man's motion with respect to station (*reference frame ≡ station or earth*). If the man walks in the same direction as train movement versus earth, his velocity is ***55Km/h*** (*versus earth*). If one ties the *origin of reference frame* to the train, the results concern his motion with respect to the train. In this case, the man's velocity is ***5Km/h*** **(***different for* ***55Km/h***).

- Assume that a <u>*long transparent wagon*</u> with a frictionless floor moves at *constant velocity* ***50 km/h*** versus station. Two observers, ***O***-seated at the ***station*** and ***O'***- seated in the ***wagon***, observe a Plexiglas cube placed on the <u>*frictionless floor*</u> inside the wagon. Let's consider the following situations:

a) *The <u>cube at rest</u> on the wagon floor*. For the observer ***O'***, the cube will be *all time* ***at rest***. For him, "the cube ***keeps its state of motion*** *(*at rest - *vs the wagon*)". The observer ***O*** sees the cube *moving all time at* ***50km/h***. For O, too, the cube ***keeps its state of motion*** *(moving at* ***50 km/h*** *- vs him*).
b) *The <u>cube sliding</u> at* ***5km/h*** *on wagon* floor in the same direction as train movement versus earth. For the observer ***O'***, "the cube ***keeps its state of motion*** *(****5Km/h*** *- vs the wagon*)" all time. Observer O sees the cube <u>*moving all time with same velocity* ***55km/h***</u> . For observer ***O***, too, the cube ***keeps its state of motion*** *(constant velocity* ***55 km/h*** *- vs the station*) all time**.**

So, the ***first Newton's law is*** **equally applied** for observers **in** ***two frames O, O'*** *which are in* ***uniform motion(****i.e. motion at constant velocity****) versus each other .***
Note: If the first Newton's law is valid in a given frame, this is called an <u>inertial reference frame</u>.

Two important conclusions come out of this "experiment ":
- ***If the <u>net force applied on an object is zero</u> and the object keeps its "status of motion"*** *(either at rest or moving at constant velocity)* ***vs a frame, this frame is an <u>inertial frame</u> .***
- ***If a frame F_1 is verified to be inertial, any other frame F_2 that is at rest or moving at constant velocity versus F_1, is also an inertial frame.***

-Let the cube be *at rest* on the floor while the wagon moves uniformly at 50km/h. If the train starts to speed up, the floor cannot pull the cube forward (***no friction force acts on the cube***). So, *"the cube will slide backward over the floor* ". Observer ***O'*** would say, "*The cube does not keep its state of motion even though there is not a force acting on it* (*for him, the cube was at rest and starts moving*)".
So, for the observer O´, *the first Newton's law does not work*.
At the same time, for the observer ***O***, the cube follows to move forward in the same way (at *constant speed* ***50km/h*** versus the earth). *Therefore, for him, the first Newton's law is still valid.*
This thought experiment shows that if ***one has a verified inertial frame F_1*** and another frame ***F_2 is moving with acceleration versus F_1, then F_2 is not an inertial frame****. One says that* ***F_2*** *is a* ***<u>non-inertial frame</u>****. The motion of objects versus non-inertial frames* **<u>does not obey</u>** *to Newton's laws.*

- How to check whether a frame is inertial or not? One must *proceed* ***by experiments****. Such* ***experiments show that the first law of Newton does apply on earth surface; this means*** *that the Earth is an inertial frame*. Subsequently, *any frame* ***at rest*** *versus earth* (building, mountain, table, tree,..) or *moving at* ***constant[2] velocity*** *versus earth* (train, plane, car, ..) *is equally an inertial system* for experiments done on earth surface. This conclusion shows that there is an infinity of inertial frames.

[2] As a vector, i.e. constant magnitude and constant direction, too.

- The assertion "the earth is an inertial frame" is based on lab experiments. Actually, it is proved to be true for " *small areas over the earth, like labs* ". But, in a larger scale experiment, the earth is not "*a perfect inertial frame*". Let's explain " *why* ". Consider a small cube sliding over an oil film (*no friction)* on a horizontal flat plane at constant velocity $\vec{\upsilon}_0$ (versus this plane) in direction North-South (figure 3).

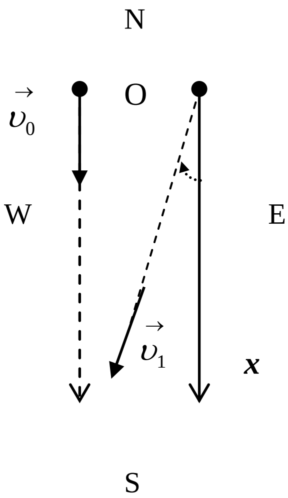

Figure 3

To verify the inertia law, one *observer at rest on the earth* ties the origin ***O*** to his position, draws the ***Ox*** axe along the direction N-S and measures the *direction of velocity vector in time*. As the *earth rotates* in direction *W vs E*, the Ox axe shifts versus E and the sliding ball on the flat plane "*finds itself to the west of the reference line Ox"*. For an observer (*tied to plane and earth frame*), the velocity has changed the direction from $\vec{\upsilon}_0$ to $\vec{\upsilon}_1$. So, the cube "**is not keeping the same state of motion at constant velocity. This means that the reference frame (i.e. earth) is not an inertial frame"**.

Note that the *daily rotation of earth is quite slow*: there is one turn per day which means a rotation $360^0/(24*60*60) = 0.00416$ degrees per second: *that is why we cannot feel earth rotation*. However, if we would observe the motion over several minutes (*say 10min*), the effect would be visible (angle =10*60*0.00416 = 2.5degrees). Conclusion; the earth is ***not a perfect inertial frame, but it is a very good approximation for the majority of experiments performed on its surface***.

For the interplanetary studies, a frame with *origin on the sun* is very ***good approximation of an inertial frame***. For the time being, no 100% real inertial frames are found, yet.

3] GALILEAN TRANSFORMATIONS

(*TRANSFORMATION OF KINEMATIC PARAMETERS BETWEEN TWO INERTIAL FRAMES*)

- Let the **position vector** $\vec{r}$ (***x***, ***y***, ***z***) define the location of a particle in ***Oxyz*** inertial frame**.** The location of same particle in ***O'x'y'z'*** frame *(* ***moving*** at constant velocity versus O "*at rest* ") is defined by position vector $\vec{r}\,'$(***x'***, ***y'***, ***z'***) (see fig. 4).

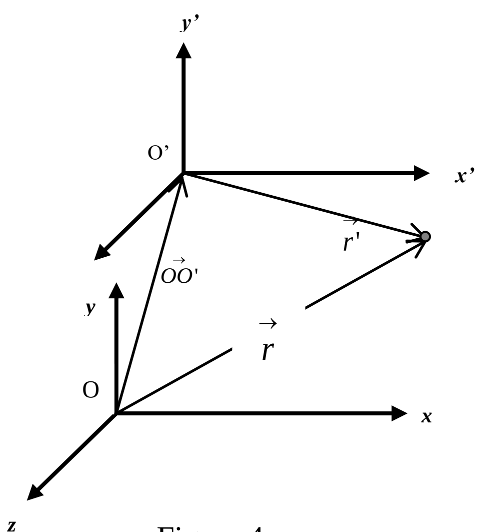

Figure 4

As one may see from the figure 4

$$\vec{r}\,' = \vec{r} - \vec{OO'} \qquad (2)$$

Assume now that *at* ***t = 0,*** $\vec{OO'} = 0$ and the ***frame O'*** is moving at a ***constant velocity*** $\vec{u} \uparrow\uparrow \vec{OO'}$ versus frame O. So, $\vec{OO'} = \vec{u} * t$ (3)

and $$\vec{r}\,' = \vec{r} - \vec{u} * t \qquad (4)$$

By projecting relation (4) on the Ox, Oy, Oz (same as Ox´, Oy´, Oz´ directions), one get the set of ***Galilean transformation for coordinates***

$$\begin{cases} x' = x - u_x t \\ y' = y - u_y t \\ z' = z - u_z t \end{cases} \quad (5) \qquad \textit{If O' moves along Ox direction} \qquad \begin{cases} x' = x - u_x t \\ y' = y \\ z' = z \end{cases} \quad (6)$$

As the ***time flows equally***[3] ***in both frames,*** one can take the derivative of (4) versus ***t***-variable

$$\frac{d\vec{r}\,'}{dt} = \frac{d\vec{r}}{dt} - \frac{d\vec{OO'}}{dt} \Rightarrow\Rightarrow \vec{\upsilon}\,' = \vec{\upsilon} - \vec{u} \qquad (7)$$

Note that, from relation (3), one get $\frac{d\vec{OO'}}{dt} = \vec{u}$ and this velocity has components (u_x,u_y,u_z) vs O frame.

By projecting relation (7) on the Ox, Oy, Oz directions one get ***Galilean transformation for velocities***

$$\begin{cases} v_x' = v_x - u_x \\ v_y' = v_y - u_y \\ v_z' = v_z - u_z \end{cases} \quad (8) \qquad \textit{If O' moves along Ox}, \; u_y = 0, \; u_z = 0 \qquad \begin{cases} v_x' = v_x - u_x \\ v_y' = v_y \\ v_z' = v_z \end{cases} \quad (9)$$

Next, one can take another time derivative on (7) and, knowing that[4] $\frac{d\vec{u}}{dt} = 0$ one finds out that

$$\frac{d\vec{\upsilon}\,'}{dt} = \frac{d\vec{\upsilon}}{dt} - \frac{d\vec{u}}{dt} \Rightarrow\Rightarrow\Rightarrow\Rightarrow\Rightarrow \vec{a}\,' = \vec{a} \qquad (10)$$

Expression (10) shows that the acceleration of the point particle versus any inertial frame is the same. This expression is at the origin of the ***Galilean principle of relativity*** that essentially says:

Mechanics' laws *apply equally and are expressed in the same mathematical form in all inertial frames*.

Important note: The net force applied on a particle in frame O' is equal to the net force applied on that particle in O frame because $\vec{F}'_{Net} = m\vec{a}' = m\vec{a} = \vec{F}_{Net}$.

Note: Meanwhile, the velocity is not the same in all inertial frames. So, when using the velocity, it is important to mention always " *relative to* " what frame. Remember that $\vec{\upsilon}\,'$ is the velocity of the point particle versus the " ***moving frame*** " and $\vec{\upsilon}$ is its velocity versus the " ***rest frame*** ".
In several problems one gives the " velocity $\vec{\upsilon}\,'$ versus a frame moving at velocity $\vec{u}$ " and asks for the velocity versus a frame "*at rest*". In these situations, one may calculate $\vec{\upsilon}$ from expression (7) in form

$$\vec{\upsilon} = \vec{\upsilon}\,' + \vec{u} \qquad (11)$$

Notes:
- The vector relations 4, 7, 10 are known as the Galileo transformation rules for ***position, velocity*** and ***acceleration*** between two inertial frames.
- When deriving those relations, one ***assumes*** that the ***time flows equally into the two inertial frames.***
- We have aligned the unit vectors of frame O' parallel to those in frame O; this means $\hat{i}' = \hat{i};\ \hat{j}' = \hat{j};\ \hat{k}' = \hat{k}$ Otherwise they would be related by a multiplying constant factor.

[3] This is an assumption based on our everyday experience.
[4] Inertial frames do not accelerate with respect to each other.