

LECTURE 6

1] INERTIA AND THE FIRST LAW OF NEWTON

- Everyday experience tells that "if one does not push a body in motion , it will stop moving". So, one may say "the rest seems to be the natural state of objects". But, at this comment one forget the **action** of **friction with surrounding medium** (Ex. friction of a wooden block with the floor on which it is sliding on,..). What if one could avoid the friction? By using oil... the block would follow its motion a long way on. Galileo Galilei was the first to figure out that: *If there were **no friction** at all, **the block would keep moving at constant velocity** without slowing down.* This way one gets to the "Galileo's **principle of inertia**". He named as "**inertia**" the **propriety** of an object **to keep moving at constant velocity** as long as there is a **zero net force** applied on it.

- Assume that one places *carefully the block on a horizontal table*. It will not move even if there is oil on table. The block will follow to stay **at rest** on table unless a force acts on it. Therefore, if it is *initially at rest*, the object will follow to be **at rest** as long as "the **net force exerted on it is zero**".
By combining this observation with Galileo principle, Newton formulated his **first law** of mechanics:

A body remains at rest or keeps moving at constant velocity as long as there is a ZERO NET FORCE applied on it.

- Another qualitative *definition* for *inertia*, related only to the body itself, appeared with time:

"Inertia is the propriety of bodies to resist to changes in their state of motion"

To get to the *quantitative parameter that measures inertia* (labelled as **inertial mass**) one should refer to the second law of Newton (see the next lecture). For now, one should remember that:

a) The first Newton's law deals with situations where there is **zero net force** acting on the body.

Ex: You push an object so that it **slides** on the table at **constant velocity**. This means that **the sum of all forces** is

$$\vec{F}_{net} = \vec{F}_g + \vec{N} + \vec{F}_{app.} + \vec{F}_{fr.} = 0 \quad (1)$$

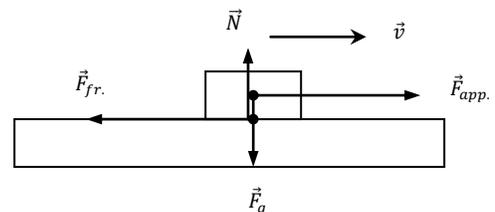


Figure 1

As the gravity force action is cancelled by normal force, it comes out that friction force cancels the action of applied force and they have equal magnitudes.

b) The inertia opposes any change¹ of velocity (not only in magnitude but in the direction, too)

Ex: In discus throwing (sport), *sportsman arm forces the discus to change the velocity direction. In the meantime, the sportsman feels on his arm the inertia opposition.*

When he leaves it free, *due to its inertia*, the discus follows its way straight along the direction of instantaneous velocity (tangent to its circular path).

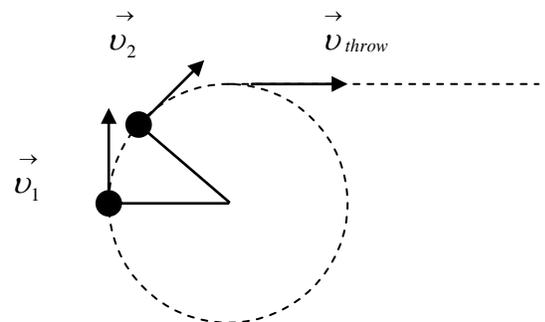


Figure 2

¹ Not just speed increase as it is considered in popular vocabulary.

2] INERTIAL REFERENCIAL FRAMES

-Any mechanics study starts by definition of a **reference frame** i.e. a **coordinative system** with its origin fixed **on an object**. Next, one introduces the kinematics parameters for a motion " *versus this frame* " .

Ex.: A man walks at 5km/h inside a train moving at 50 km/h with respect to the station. If one ties the frame origin to the station (earth) the results concern man's motion with respect to station (*reference frame* \equiv *station or earth*). If the man walks in the same direction as train movement versus earth, his velocity is 55km/h (*versus earth*). If one ties the *origin of reference frame* to the train, the results concern his motion with respect to the train. In this case, the man's velocity is 5km/h (*different for 55km/h*).

- Assume that a long transparent wagon with a frictionless floor moves at *constant velocity* 50 km/h versus station. Two observers, O -seated at the **station** and O' - seated in the **wagon**, observe a Plexiglas cube placed on the frictionless floor inside the wagon. Let's consider the following situations:

- The cube at rest on the wagon floor.* For the observer O' , the cube will be *all time at rest*. For him, "the cube **keeps its state of motion** (at rest - vs the wagon)". The observer O sees the cube *moving all time at 50km/h*. For O , too, the cube **keeps its state of motion** (*moving at 50 km/h - vs him*).
- The cube sliding at 5km/h on wagon floor* in the same direction as train movement versus earth. For the observer O' , "the cube **keeps its state of motion** (5km/h - vs the wagon)" all time. Observer O sees the cube *moving all time with same velocity 55km/h* . For observer O , too, the cube **keeps its state of motion** (*constant velocity 55 km/h - vs the station*) all time.

So, the **first Newton's law is equally applied** for observers **in two frames O, O'** which are in **uniform motion**(i.e. *motion at constant velocity*) **versus each other** .

Note: *If the first Newton's law is valid in a given frame, this is called an inertial reference frame.*

Two important conclusions come out of this "experiment ":

- **If the net force applied on an object is zero and the object keeps its "status of motion" (either at rest or moving at constant velocity) vs a frame, this frame is an inertial frame .**
- **If a frame F_1 is verified to be inertial, any other frame F_2 that is at rest or moving at constant velocity versus F_1 , is also an inertial frame.**

-Let the cube be *at rest* on the floor while the wagon moves uniformly at 50km/h . If the train starts to speed up, the floor cannot pull the cube forward (*no friction force acts on the cube*). So, "*the cube will slide backward over the floor*". Observer O' would say, "*The cube does not keep its state of motion even though there is not a force acting on it (for him, the cube was at rest and starts sliding backward)*".

So, for the observer O' , *the first Newton's law does not work*.

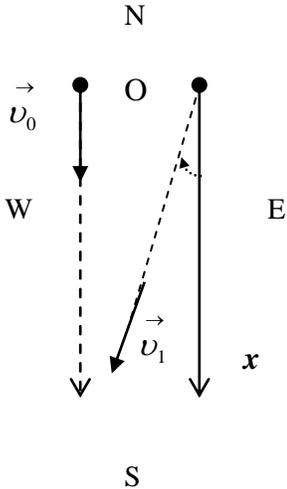
At the same time, for the observer O , the cube follows to move forward in the same way (at *constant velocity 50km/h* versus the earth). *Therefore, for him, the first Newton's law is still valid.*

This thought experiment shows that if one has a verified inertial frame F_1 and another frame F_2 is moving with acceleration versus F_1 , then F_2 is not an inertial frame. One says that F_2 is a non-inertial frame. The motion of objects versus non-inertial frames does not obey to Newton's laws.

- How to check whether a frame is inertial or not? One must **proceed by experiments**. **Such experiments show that the first law of Newton does apply on earth surface; this means that the Earth is an inertial frame.** Subsequently, **any frame at rest versus earth** (building, mountain, table, tree,..) or **moving at constant² velocity versus earth** (train, plane, car, ..) **is equally an inertial system** for experiments done on earth surface. This conclusion shows that there is an infinity of inertial frames.

² As a vector, i.e. constant magnitude and constant direction, too.

- The assertion "the earth is an inertial frame" is based on lab experiments. Actually, it is proved to be true for " *small areas over the earth, like labs* ". But, in a larger scale experiment, the earth is not " *a perfect inertial frame* ". Let's explain " *why* ". Consider a small cube sliding over an oil film (*no friction*) on a horizontal flat plane at constant velocity \vec{v}_0 (versus this plane) in direction North-South (figure 3).



To verify the inertia law, one observer at rest on the earth ties the origin O to his position, draws the Ox axe along the direction N-S and measures the *direction of velocity vector in time*. As the *earth rotates* in direction W vs E, the Ox axe shifts versus E and the sliding ball on the flat plane " *finds itself to the west of the reference line Ox* ". For an observer

(*tied to plane and earth frame*), the velocity has changed the direction from \vec{v}_0 to \vec{v}_1 .

So, the cube " **is not keeping the same state of motion at constant velocity.**

This means that the reference frame (i.e. earth) is not an inertial frame".

But, the daily rotation of earth is quite slow: there is one turn per day which means a rotation $360^0 / (24 * 60 * 60) = 0.00416$ degrees per second: *that is why we cannot feel earth rotation*. However, if we would observe the motion over several minutes (*say 10min*), the effect would be visible (angle = $10 * 60 * 0.00416 = 2.5$ degrees).

Conclusion; the earth is **not a perfect inertial frame, but it is a very good approximation for the majority of experiments performed on its surface.**

For the interplanetary studies, a frame with *origin on the sun* is very **good approximation of an inertial frame**. For the time being, no 100% real inertial frames are found, yet.

Figure 3

3] GALILEAN TRANSFORMATIONS

(TRANSFORMATION OF KINEMATIC PARAMETERS BETWEEN TWO INERTIAL FRAMES)

- Let the **position vector** $\vec{r}(x, y, z)$ define the location of a particle in $Oxyz$ inertial frame. The location of same particle in $O'x'y'z'$ frame (**moving** at constant velocity versus O "at rest") is defined by position vector $\vec{r}'(x', y', z')$ (see fig. 4). As one may see from the figure 4

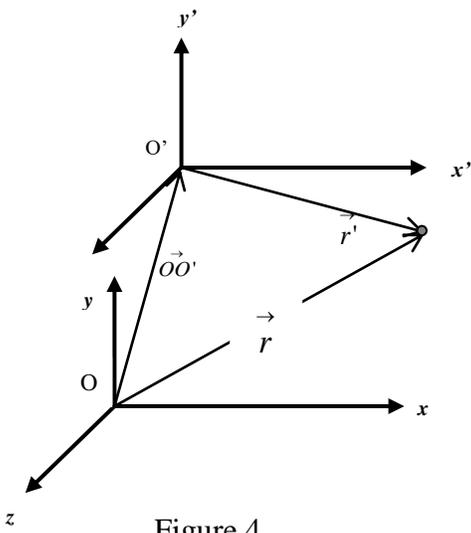


Figure 4

$$\vec{r} = \vec{r}' + \vec{OO}' \tag{2}$$

Assume now that *at $t = 0$* , $\vec{OO}' = 0$ and the **frame O'** is moving at a

constant velocity $\vec{u} \uparrow \vec{OO}'$ versus frame O . So, $\vec{OO}' = \vec{u} * t$ (3)

and
$$\vec{r} = \vec{r}' + \vec{u} * t \tag{4}$$

By projecting relation (4) on the Ox, Oy, Oz (same as Ox', Oy', Oz' directions), one get the set of **Galilean transformation for coordinates**

$$\begin{cases} x' = x - u_x t \\ y' = y - u_y t \\ z' = z - u_z t \end{cases} \tag{5} \text{ If } O' \text{ moves along } Ox \text{ direction}$$

$$\begin{cases} x' = x - u_x t \\ y' = y \\ z' = z \end{cases} \tag{6}$$

As the *time flows equally³ in both frames*, one can take the derivative of (4) versus t -variable

$$\frac{d\vec{r}'}{dt} = \frac{d\vec{r}}{dt} - \frac{d\vec{OO}'}{dt} \Rightarrow \vec{v}' = \vec{v} - \vec{u} \quad (7)$$

Note that, from relation (3), one get $\frac{d\vec{OO}'}{dt} = \vec{u}$ and this velocity has components (u_x, u_y, u_z) vs O frame.

By projecting relation (7) on the Ox, Oy, Oz directions one get *Galilean transformation for velocities*

$$\begin{cases} v'_x = v_x - u_x \\ v'_y = v_y - u_y \\ v'_z = v_z - u_z \end{cases} \quad (8) \quad \text{If } O' \text{ moves along } Ox, u_y=0, u_z=0 \quad \begin{cases} v'_x = v_x - u_x \\ v'_y = v_y \\ v'_z = v_z \end{cases} \quad (9)$$

Next, one can take another time derivative on (7) and, knowing that⁴ $\frac{d\vec{u}}{dt} = 0$ one finds out that

$$\frac{d\vec{v}'}{dt} = \frac{d\vec{v}}{dt} - \frac{d\vec{u}}{dt} \Rightarrow \vec{a}' = \vec{a} \quad (10)$$

Expression (10) shows that the acceleration of the point particle versus any inertial frame is the same. This expression is at the origin of the *Galilean principle of relativity* that essentially says:

Mechanics' laws apply equally and are expressed in the same mathematical form in all inertial frames.

Important note: The net force applied on a particle in frame O' is equal to the net force applied on that particle in O frame because $\vec{F}'_{Net} = m\vec{a}' = m\vec{a} = \vec{F}_{Net}$.

Note: Meanwhile, the velocity is not the same in all inertial frames. So, when using the velocity, it is important to mention always "relative to" what frame. Remember that \vec{v}' is the velocity of the point particle versus the "moving frame" and \vec{v} is its velocity versus the "rest frame".

In several problems one gives the "velocity \vec{v}' versus a frame moving at velocity \vec{u} " and asks for the velocity versus a frame "at rest". In these situations, one may calculate \vec{v} from expression (7) in form

$$\vec{v} = \vec{v}' + \vec{u} \quad (11)$$

Notes:

- The vector relations 4, 7, 10 are known as the Galileo transformation rules for *position, velocity* and *acceleration* between two inertial frames.
- When deriving those relations, one *assumes* that the *time flows equally into the two inertial frames*.
- We have aligned the unit vectors of frame O' parallel to those in frame O; this means $\hat{i}' = \hat{i}; \hat{j}' = \hat{j}; \hat{k}' = \hat{k}$ Otherwise they would be related by a multiplying constant factor.

³ This is an assumption based on our everyday experience.

⁴ Inertial frames do not accelerate with respect to each other.