LECTURE 7

REMEMBER
One uses the "***point particle***" model to study the motion of an object in translational motion. The objective of Kinematics is " ***the description of object motion*** *(without asking: why it happens?)*"
The main parameters used for description of point particle motion are the scalar ***time*** and the vectors of ***position***, ***displacement, velocity, acceleration.*** The method of 2D and 3D kinematics point particle motion consists in the separation *of motion along each coordinative axe* and the application of its ***basic equations; one uses the time to correlate the values of x, y, z coordinates at a given moment***.

DYNAMICS

- **Dynamics** answers the question: "Why ***an object moves or stays at rest "***. The dynamics of material point uses two main parameters " ***force*** and ***mass".*** *The* **force measures the action**, *which attempts to* ***change*** *the velocity of an object while the* **mass measures the object resistance against the change of its velocity.** *These two parameters make possible to explain why an object is in a given status of motion (example; a constant velocity i.e. zero acceleration or changing its velocity i.e. non zero acceleration).*

FORCE

- One introduced the parameter "***force"*** to measure a *push or pull* action exerted on a body. One could realize that a "*push or pull"* ***force tents*** *to* ***"change the velocity[1]"*** of the body, to "***deform*** *it"* or both.

- Initially, one considered only forces exerted on an object by using hands, ropes, springs, collisions. In all these situations there is a contact between the source *of action* and the *object that experiences the force*. The development of physics showed the existence of forces ***exerted without contact*** between the *source of the force* and the *object* where the force applies *(ex:* gravitational, magnetic, electrostatic,.). Meanwhile, no matter what is the *type* of exerted force (*contact* or *no contact*) one measures its effects on a body the same way. That is why one introduces the force concept from point of view of its effects; one says that ***a force is applied on a body if there is a change or a tendency for change of object motion / shape or both of them.***
Important Note: ***Dynamics neglects the deformations and considers only the changes of the motion***.

- Physics deals with measurable quantities. ***How to introduce a unit for measuring forces?*** One may:
a*) define a standard force unit; and b) define a procedure for comparison of different forces.*
A simple procedure would be using springs elasticity; *spring extension is proportional to the magnitude of force applied at its end*. One may select a ***standard*** *spring* and define the force unit **by referring to 1m** extension of this spring. Based on Hook's law proportionality, a *two unit's force* would produce a double extension on the standard spring. Therefore, in principle, one may introduce a *basic* unit for the magnitude of forces. However, the described method has a relevant drawback: It is impossible to produce *a standard spring that keeps its elasticity unchanged forever (a spring etalon).* Therefore, *one defines unit of force as a* **derived unit** *through the second law of Newton.* ***Is the force a vector or scalar quantity?***

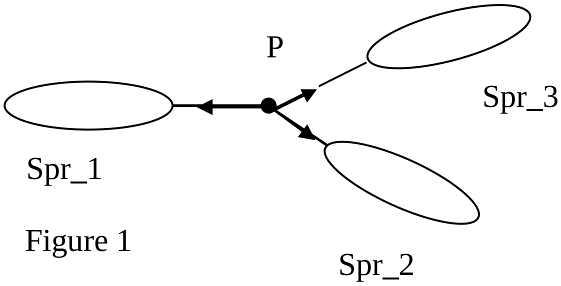


Figure 1

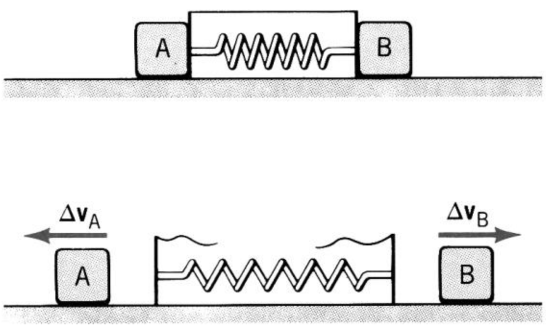
One can verify easily that the ***force is a vector parameter*** by using three or more spring scales with tied tails and oriented along different angles(fig.1). The vectors' rules applied for translational equilibrium of P-point at rest explain clearly the situation. (force table at Lab_4).

[1] The special case of a body at rest corresponds to velocity equal to zero.

MASS

- The inertia is the propriety of bodies to resist to the changes on their velocity. ***The mass*** is the ***physical parameter*** that ***measures the inertia of a body***. The ***mass*** of a body *tells how difficult is to* ***change its velocity*** (*magnitude, direction or both*). So, larger mass means larger inertia, i.e. larger effort is needed to get the same *change* of *velocity*. The experiments show that a constant effort is needed to keep an object at *uniform rotation*. This shows that there is the *same inertial resistance* by the body if the *external* action *changes only the direction of its velocity*. Therefore, the mass of an object is a *parameter* that *does not depend on the direction* of its motion *in space*. Consequently, the mass is a ***scalar parameter.***

-***How to measure the magnitude of mass***? By use of the standard method; ***a) define a mass unit.***
The standard in the International Office for Measurement Units in Paris is chosen as the unit mass 1kg.
b) Define a procedure for comparison of different masses.
The following is a direct procedure for mass measurement. One fixes two objects with masses $\boldsymbol{m_A}$, $\boldsymbol{m_B}$ *on two equal massless*[2] *pucks at rest on two sides of a spring kept compressed by use of a string. This set stays on an air table. The air cushion action avoids the friction and balances the gravity. Once one cuts the string, the spring extends and produces* **two forces with the same magnitude** *on its both sides.*



The pucks, that were initially ***at rest*** *(*$\upsilon_o=0$*), start to move with velocities* $\vec{\upsilon}_A, \vec{\upsilon}_B$. One can measure the magnitudes of these velocities; they are *equal to the change of velocity itself, which is inverse proportional to mass.*
So, one can measure the speed of two objects and get

$$\left|\vec{\upsilon}_A\right| \sim 1/\boldsymbol{m_A}\ ; \quad \left|\vec{\upsilon}_B\right| \sim 1/\ \boldsymbol{m_B} \quad \textit{from which} \quad \frac{m_A}{m_B} = \frac{\left|\vec{\upsilon}_B\right|}{\left|\vec{\upsilon}_A\right|} \qquad (1)$$

Figure 2

By using the standard unit mass $\boldsymbol{m_B} = 1kg$, the described method and the expression $m_A = m_B \dfrac{\left|\vec{\upsilon}_B\right|}{\left|\vec{\upsilon}_A\right|}$ *(2)*

in principle, one can measure *the* ***inertial mass*** *of different objects.* But, this method is not practical.

In practice, one measures the *inertial mass* of objects by comparing their weight (measured by use of a balance). As we will see, this is not a very accurate method but is simple and its accuracy is good enough for general-purpose measurements.

SECOND LAW OF NEWTON

- The first Newton's law (***inertia law***) says: A ***zero net force on a body does not change its velocity***[3].
Consider that a puck is moving at constant velocity (so, $F_{NET} = 0$) on an air track. If, at a given moment, one applies a horizontal force on it (so, F_{Net} becomes $\neq 0$), its velocity will change and this means presence of *acceleration*. How big will be its acceleration? This depends on the ***mass*** of the body. So, during an accelerated motion the ***net force on a body, its mass and its acceleration*** must be related to each other.

[2] Insignificant with respect to $\boldsymbol{m_1}$, $\boldsymbol{m_2}$
[3] The velocity may be constant or zero.

-Assume now that the puck is at rest on horizontal plan air track. If one applies a net horizontal ***force*** $\vec{F}$ on it, it will start moving with ***acceleration*** $\vec{a}$. One may change $\vec{F}$ (*direction or magnitude*) and show by measurements that: *a) the* ***direction*** *of* ***vector*** $\vec{a}$ *is the same as that of* ***vector*** $\vec{F}$.

b) The ***magnitude*** *of* $\vec{a}$ *changes* ***proportionally*** *with the* ***magnitude*** *of* $\vec{F}$.

We express mathematically these *experimental facts* through the expression $\vec{a} = k_1 \vec{F}$ (3)

Now, consider the same ***force*** $\vec{F}$ exerted on a set of pucks with ***different masses***. One may verify by measurements that the ***magnitude*** of produced acceleration ***is inverse proportional*** to the ***mass*** of the puck. So, mathematically expressed $|\vec{a}| = k_2 / m$ (4)

One can combine the expressions (3-4) as $\vec{a} = \frac{k_1}{k_2} * \frac{\vec{F}}{m}$ or $\vec{a} = k * \vec{F}/m$ and get $\vec{F} = \frac{1}{k} m \vec{a}$ (5)

($k = \frac{k_1}{k_2}$ is a new constant). *Remember that one has defined the etalon of mass unit 1kg.* So, one can get an object with mass 1kg, exert on it different magnitudes of $\vec{F}$ and measure the magnitudes of accelerations.

When the mass 1kg gets the acceleration 1m/s^2, one consider that the magnitude of applied force is the unit force and one calls it 1 Newton (N). This definition fixes the constant at equation (5) to k =1 and one gets the well known mathematical expression for the second law of Newton $\vec{F} = m\vec{a}$ (6)

- Actually, Newton formulated the second law for the **net force** (vector sum $\sum_i \vec{F}^i$) applied on a ***point particle with mass m;*** so, one must write it as $\vec{F}_{Net} = \sum_i \vec{F}^i = m\vec{a}$ (7)

The *direction of* ***acceleration vector*** $\vec{a}$ *is always the* ***same*** *as that of the* ***net force*** *exerted on the body*.

Note: The ***direction*** *of body* **motion** is ***defined*** by ***velocity vector*** $\vec{\upsilon}$ *and* ***not by acceleration vector*** $\vec{a}$.

- The Galileo transformations show that the **acceleration is the same in all-inertial systems**. As the body's **mass** and the **force** *do not depend on the reference frame*, it comes out that the product " $m\vec{a}$ " is the same in all *inertial frames* and it is equal to the net force. Therefore, the Newton's law is valid in all *inertial frames* and in each of them it is expressed by an expression with the same form as (7).

THIRD LAW OF NEWTON

-The third law of Newton says: ***If the body A is exerting the force*** $\vec{F}_{BA}$ ***on the body B, than the body B is exerting the force*** $\vec{F}_{AB}$ ***on the body A such that*** $\vec{F}_{AB} = -\vec{F}_{BA}$ (8)

Note that from equation (8) one get $|\vec{F}_{AB}| = |\vec{F}_{BA}|$ (9)

Note: The first label shows the object on which force applies; the second label shows the source of force.

One can observe easily a qualitative demonstration of the third law of Newton by pushing the wall. More you push, larger opposing reaction (*force exerted by wall on you*) you feel.

-The third law is known as the **"action – reaction"** law, too. One must pay special attention to the fact that these *two forces are different* and *applied on different bodies*. This is an important issue when drawing the ***isolation diagram if there is contact forces.*** One must start by *isolating mentally only the object of interest and follow by drawing carefully only forces acting on it, before* drawing FBD of the material point.

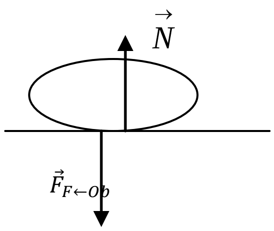


Figure 3.a

Example: *An object is at rest on the floor. The object is pushing on the floor with a force $\vec{F}_{F\leftarrow Ob}$ (Fig. 3.a). This force (ACTION) is exerted on the floor. According to the third law, the floor will react by a force $\vec{N}$ (REACTION exerted on the object) such that $\vec{N} = -\vec{F}_{F\leftarrow Ob}$. Figure 3.b shows the free-body diagram for the object. In this diagram one* ***includes all the forces that act only on the considered body i.e. $\vec{F}_G$ and $\vec{N}$. No other objects (****like floor****) should be included in this diagram***.

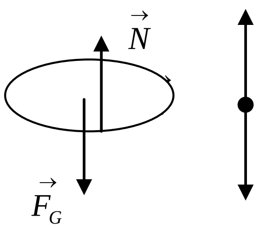


Figure 3.b

Note*: The magnitude of reaction is the same as that of action. But, it is another issue what way the interacting bodies modify the motion of each other.*

Example: Let's consider an apple in free fall. The earth is acting on the apple with a force $\vec{F}_{a-earth} = \vec{F}_{a-gravity} = m_a * \vec{g}$. The apple "reacts" by exerting ***on the earth*** *a* ***force*** *$\vec{F}_{earth-a} = -\vec{F}_{a-earth} = m_{earth} * \vec{a}_{earth}$. This force has* ***equal magnitude*** *but the effect of its action on earth motion , i.e.* acceleration $\vec{a}_{earth}$ *is practically ≈ 0.*

$$m_a * |\vec{g}| = m_{earth} * |\vec{a}_{earth}| \rightarrow a_{earth} = g * {}^{m_a}/_{m_{earth}} \cong 0$$

- In mechanics, one uses the word *tension* for the *force exerted between the two parts of rope(or string)* on different sides of any section. The tension transmits forces from one end to other end of rope and makes it an important tool in the practice of pulling objects. Consider that one applies F_{ext} on a rope end to pull an *object tied at the other end but "the set rope - object" is at* ***rest****, yet.* The rope piece to the left of shown section attracts the piece on the right by force $\vec{F}_{21}$ and the piece on the right reacts by attracting the left piece by the force

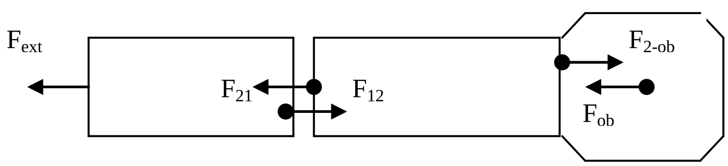


Figure 4

$\vec{F}_{12} = -\vec{F}_{21}$. So, the *magnitude of the tension* inside the rope is $T = \left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right|$ (10)

As the left piece is at rest, the net force on it must be zero; $\vec{F}_{ext} + \vec{F}_{12} = 0; \rightarrow \left|\vec{F}_{12}\right| = \left|\vec{F}_{ext}\right| = T$ (11)

Similarly, for the piece on right we have; $\vec{F}_{21} + \vec{F}_{2-ob} = 0; \rightarrow \left|\vec{F}_{2-ob}\right| = \left|\vec{F}_{21}\right| = T$ (12)

By comparing (12) to (11) one finds out that $\left|\vec{F}_{2-ob}\right| = T = \left|\vec{F}_{exert}\right|$ (13)

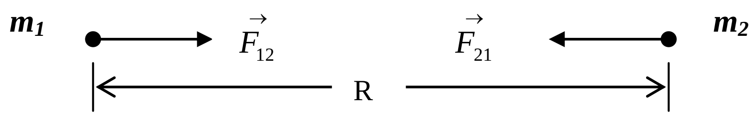
Then, based on third law, the magnitude of force exerted on the object is $\left|\vec{F}_{ob}\right| = \left|\vec{F}_{2-ob}\right| = T = \left|\vec{F}_{exert}\right|$ (14)

So, the rope transfers the applied force on its left end to the object tied to its other end.

GRAVITATION FORCE AND THE WEIGHT

- The study of motion of planets was a main objective for scientists in "Newton's time". Newton dealt with this problem and discovered that planets' motion is governed by the ***law of universal gravitation***:

Two point[4] particles with masses m_1, m_2 at distance R attract each other by a force with magnitude



$$F_{12} = F_{21} = G\frac{m_1 m_2}{R^2} \qquad (15)$$

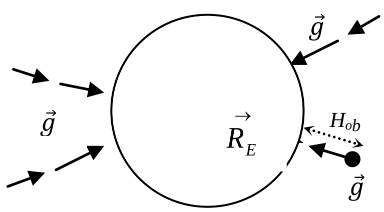
Figure 5

The universal gravitational constant is ***$G = 6.67*10^{-11}\ Nm^2/kg^2$***

Newton ***modeled*** the ***planets as material points*** with mass equal to that of the planet.

Note: $\vec{F}_{12}$ *and* $\vec{F}_{21}$ *have* ***equal magnitudes***, ***opposite directions*** *and are* ***applied on different objects***.

- One has ***defined*** the ***weight*** $(\vec{F}_g)$ of an *object* as the ***net gravitational force*** acting on it; this means that one has to take into account *all gravitational forces* exerted on the object. If some of them are very small, one can neglect their action. For an object ***close to the earth surface***, *the main gravitational action is that due to the earth*. So, one *models* the object as a point with mass "m_{ob}" under the gravitational attraction of earth modelled as a point with mass "m_E" located at earth center. There is a distance $R_E + H_{ob}$ between the *point _object* and the *point_earth* ; R_E is the *average radius* of earth and H_{ob} presents the ***height of object from earth surface.*** Then, one applies the law of universal gravitation to find the object weight as



$$\vec{F}_{ob-E} \equiv \vec{F}_{g(ob)} = G\frac{m_{ob}m_E}{(R_E+H_{ob})^2}\left(-\frac{\vec{R}_E}{R_E}\right) \cong m_{ob}\left[G\frac{m_E}{R_E^2}\left(-\frac{\vec{R}_E}{R_E}\right)\right] \equiv m_{ob}\vec{g} \qquad (16)$$

Figure 6. *Gravitational field strength* (g- vector) of earth in different locations around it.

In the last term one has neglected H_{ob} ($H_{ob} << R_E$) and arrived at expression $\vec{F}_{g(ob)} = m_{ob}\vec{g}$ (17)

where

$$\left|\vec{g}\right| = G\frac{m_E}{R_E{}^2} = 6.67*10^{-11}[Nm^2/kg^2]\frac{5.98*10^{24}[kg]}{(6.37*10^6)^2[m^2]} \approx 9.8N/kg \qquad (18)$$

Note that $\vec{g}$ is a **vector quantity** and it presents the "***strength of gravitational field near earth surface".*** *Actually it is the* ***gravitation force exerted on 1kg on earth surface at different locations*** (see fig.6).

-The value g = 9.8N/kg is a first-order approximation for magnitude of $\vec{g}$ -vector. If the object is located at a considerable height, one has to take into account H_{ob} *and will find* a smaller g-value. Most important, this model considers the earth as a *sphere with uniform mass density*. Actually, the earth is not a sphere with uniform density and the measurements of gravitational field of earth near its surface show different "***g-values"*** at different locations but the accepted value " 9.8N/kg " is reasonable for most situations.

[4] The maximum dimension of bodies is much smaller than the distance R between them

- When writing the second law for an object *close to earth* surface in ***free fall***, the only force exerted on the object is its ***weight***. So, $\vec{F}_{NET} = \vec{F}_{G-ob} = m_{ob} * \vec{a}$ and as $\vec{F}_{G-ob} = m_{ob}\vec{g}$ it comes out that $\vec{a} = \vec{g}$ (19)

i.e. the ***acceleration of free fall*** is equal to $\vec{g}$; $a_{free-fall} = g = 9.8[N/kg] = 9.8m/s^2$ but don't forget that ***g-quantity is essentially the strength of gravitational field of earth and not an acceleration***.

- In commercial activities people talk about the ***weight*** *of objects* **in kilogram** but this is wrong. In fact, the ***kg is an unit of the mass*** (a ***scalar*** *quantity) while the* ***weight*** (*unit N*) ***is a force vector***. Also, the *mass of a body does not depend on the location* where one measures it while the *weight does depend on location (through vector $\vec{g}$)*. The process of "weight measuring" by an arm balance ***compares the weight*** of the object with the ***weight of a standard mass*** and when the balance is at equilibrium, one get:

$$F_{G-ob} = F_{G-st} \Rightarrow m_{ob}g = m_{st}g \Rightarrow m_{ob} = m_{st} \qquad (20)$$

So, one *compares the weights and derive a result for the mass*. If one weighs a body by use of a *spring scale*, one must be aware that the result is right only around the location where the spring was calibrated because only at the same location there is really equal ***g***-values and one can cancel them in eq. (20).

APPARENT WEIGHT

- A person at rest acts (pushes) on the floor with a force equal to his weight. From the third law, it comes out that the floor reacts on him by a force $\vec{N}$ with the same magnitude as $\vec{F}_G$ but opposite direction. One feels his own weight through $\vec{N}$ force. For this reason, the force $\vec{N}$ is often labelled as apparent weight. These two forces ($\vec{F}_G$ and $\vec{N}$) produce a ***zero net force*** on the person(*the person does not move vertically*).

The apparent weight of a body is the force exerted on it by the surface on which it stands .

Calc. in Earth Frame

- Now, consider yourself inside an elevator. When the elevator is moving:

a) *Up or down* at **constant velocity** (***a = 0***) you *feel* the ***same weight as at rest***.

The second law explains this: $\vec{F}_G + \vec{N} = m\vec{a} = 0; Oy :\Rightarrow N - F_G = 0; N = F_G$

b) *Upward* with acceleration (***a > 0***) you *feel* larger ***weight (N-force*** *on feet)* ***than at rest***.

The second law explains this: $\vec{F}_G + \vec{N} = m\vec{a}; \_Oy :\Rightarrow N - F_G = ma; N = F_G + ma$

c) *Downward* with acceleration (***a < 0***) you feel ***lesser weight (N-force*** *on feet****) than at rest***.

The second law explains this: $\vec{F}_G + \vec{N} = m\vec{a}; \_Oy \Rightarrow N - F_G = -ma; N = F_G - ma$

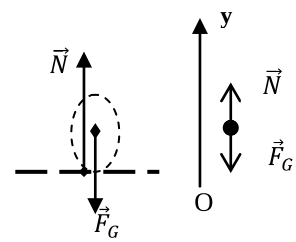


Figure 7

If the elevator cable get broken, you are in *free fall*; "***a"*** becomes equal to "***g"*** and *N = mg-mg = 0. The apparent weight becomes zero and you cannot feel your weight.*

- These *situations* show that the ***apparent weight*** *depends on the* motion of frame *tied to support surface.* ***The apparent weight equals the true weight only if the support frame is inertial****(moving at constant velocity versus earth)****.*** *The magnitude of the* **true weight** *of a body (Fg) is defined by its mass and magnitude of local* $\vec{g}$ field vector . The issue of the apparent weight is one example of the general rule telling that:

THE THREE DYNAMICS' LAWS APPLY ONLY IN INERTIAL FRAMES.