LECTURE 8

1] THE NORMAL FORCE

-Let's consider a body at rest on a horizontal floor. The gravitational field of the earth attracts this body by a force $\vec{F}_G$, known as body's weight . If one applies the 2nd law without being cautious, one would write $\vec{F}_G = m\vec{a} = 0$ (1) because the body is at rest, so $\vec{a} = 0$. But, as $\vec{F}_G \neq 0$ there is another force acting on the body which is missing in relation (1). The third law helps to find the origin of this force.

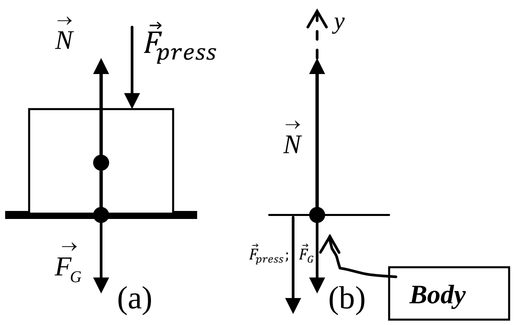


Figure 1

As the body *acts* (press) on the support (floor) by a force equal to $\vec{F}_{Fl-B} = \vec{F}_G$, the support *reacts* on the body (fig.1) by a force $\vec{N}$ such that

$$\vec{F}_{B\_Fl} = -\vec{F}_{Fl\_B} \equiv \vec{N} \qquad (2)$$

Note: The 2nd law applied on the body shows that the *magnitude* of $\vec{N}$ is *equal* to that of $\vec{F}_G$. If one pushes on the body vertically down by a force $\vec{F}_{press}$, the magnitude of normal force N (action by the support on the body) becomes $F_{press}+F_G$; i.e. the reaction of the support on the body is larger than Fg (Fig 1.b). $\vec{N} = -(\vec{F}_{pres} + \vec{F}_G)$ (3)

- The *elasticity* of the support material is at the origin of force $\vec{N}$. Under the effect of force exerted by the body on it (*generally body weight)*, the support is deformed locally (*even though not seen by free eye*). This deformation raises elastic forces which sum is directed ***vertically up***. One may model the surface of support as a set of straight strings deformed(fig.2) due to the weight of a body hanged there. By referring to the same magnitude of rope tension, one get that the sum tension forces (*at origin of* $\vec{N}$) is ***perpendicular to the support* surface**.

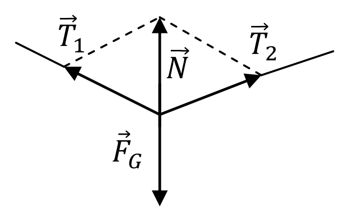


Figure 2

- Assume that you are trying to slide a couch on a horizontal floor by pushing it and even if you apply a considerable force $\vec{F}_{push}$ (fig.3), the couch does not move. If one applies the second law without being cautious, one could write $\vec{F}_{push} + \vec{N} + \vec{F}_G = m\vec{a} = 0$ (4) as the body is at rest ($\vec{a} = 0$).

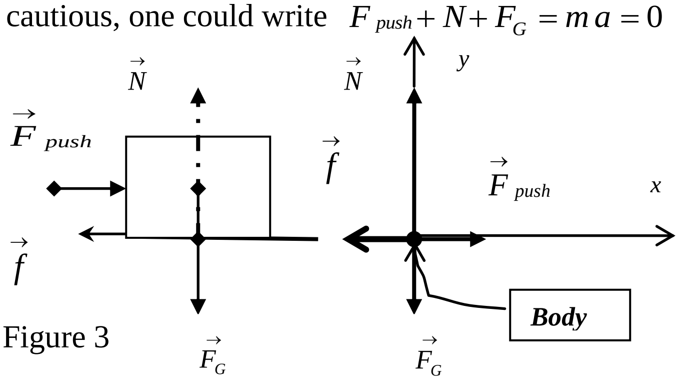


Figure 3

By projecting eq.(4) on *Ox* axe, one gets $F_{push} = 0$ but one knows that $F_{push} \neq 0$. This contradiction is solved by introducing another force acting along the opposite direction and having the same magnitude as F_{push} . The only place this force can be ***produced is at the interface*** between the couch and the floor. This force is called friction force $\vec{f}$. Its direction is *opposite to the expected direction of body motion*.

-The everyday experiments show that, to keep a body in uniform motion (i.e. at *constant velocity*) on a horizontal support, one must apply a horizontal force on it. But the first law tells that, in uniform motion

($\vec{a} = 0$) the net force on body is zero. So, even in the case of a **uniform motion** one has to accept the presence of a *friction force in opposite direction and equal magnitude to that of applied force*.
The friction force is acting even when the body is moving in accelerated way. So, there is a friction force applied[1] on a body: a) when it is moving ; b) if it is at rest but there is an action tenting to make it move.

- The experiments show a *different behaviour of* ***static friction*** *and* ***kinetic friction*** versus the ***applied force***. In case of the static friction, the body does not move; the second law tells that the magnitude of friction force is equal to the magnitude of applied force $\left|\vec{f}_s\right| = \left|\vec{F}_{push}\right|$. This holds on till a certain value $F_{push}^{\max}$ of applied force magnitude and for larger magnitude of pushing force, the body starts moving. The ***magnitude*** of ***maximum static friction***, i.e. $f_{s\_\max} = \left|F_{push}^{\max}\right|$ depends on the nature of materials in contact, on the roughness of surface in contact, temperature, but in all cases it is **proportional** to the ***magnitude of the normal force*** $\vec{N}$;

$$f_{s\_\max} = \mu_s \left|\vec{N}\right| \qquad (5)$$

μ_s is the coefficient of static friction [no units]

Note: Only the ***maximum value of static friction*** is equal to (5). In any other situation, the magnitude of static friction force is equal to the magnitude of the applied force that tents to move the object.

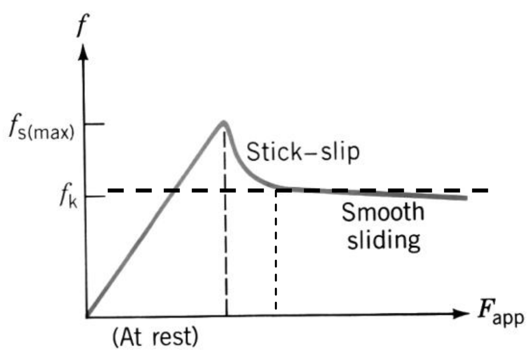


Figure 4

The straight line (at angle 45^0 for same magnitude of unit on axes) shown in figure 4 describes the evolution of magnitude of friction force with the increase of applied force on a body at rest; $f_{s\_\max}$ value depends on the coefficient μ_s and the magnitude of normal force at interface.
For $F_{app} > f_{s-\max}$ the body start to move but there is still a friction force. Meanwhile, the relation between the friction force and applied force changes.

- Initially, there is a so called "*stick-slip*" region, where the object starts to slide and stops several times and the friction acts as a *static - kinetic mixture*. For larger magnitudes of applied force, the ***object slides*** without stopping and the ***magnitude of friction force*** remains *almost* ***constant***, no matter what is the magnitude of applied force. As it is referred to a motion state (*uniform or accelerated*), one calls it ***kinetic friction force*** $\vec{f}_k$. The ***magnitude*** (6) of *kinetic friction force* is proportional to the normal force , too, but the ***kinetic friction*** coefficient μ_k is ***smaller*** than ***static friction*** coefficient μ_s.

$$f_k = \mu_k \left|\vec{N}\right| \qquad (6)$$

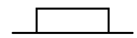
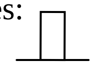
- Assume that an object is moving with acceleration (i.e. $F_{app} > f_k$). If, after a while, the ***magnitude*** $\boldsymbol{F_{app}}$ decreases and becomes ***equal to*** $\boldsymbol{f_k}$, the net force on the object becomes zero and the object starts to move at constant ***velocity***. If the applied force decreases more, the kinetic friction takes over, decelerates the object and makes it stop. After this the friction becomes zero. The described phenomena happen for "**a smooth sliding**" but *without lubrication. If there is a lubricant between contact surfaces, the friction depends in a more complicated way from* $\vec{N}$ *- magnitude and it depends from the object speed, too.*

[1] One can diminish or avoid friction only by applying special measures.

- The interaction between the atoms and molecules on the surfaces of two objects in contact creates some *molecular bonds* that are at the origin of the friction. There is not an unified physical model that explains completely the microscopic mechanism of friction, yet.
Meanwhile, the upper presented formulas apply in majority of real life situations where one deals with its characterisation from macroscopic point of view.

- Remember that for <u>non lubricated surfaces</u>, the <u>magnitude</u> of $\vec{f}_{s-max}$ and $\vec{f}_k$ forces:

a) are proportional to " N ", i.e. to the "*load on the support*",

b) do not depend on the area of contact (*same friction force for*  *and* )

Also, the magnitude of $\vec{f}_k$ is independent on speed of object.

3] THE MOTION OF A BODY IN A RESISTIVE MEDIA

-When a body moves through a fluid (*liquid or gas*) the molecules on its surface interact with molecules of fluid and this produces a ***drag force*** (*friction over all body surface*). For small relative velocity of body versus fluid (*laminar flow of fluid around body*), the magnitude of this force is proportional to the magnitude of velocity, i.e. the drag force is given by expression

$$\vec{F}_D = -\gamma * \vec{\upsilon} \qquad (7)$$

γ is the *viscosity constant* and $\vec{\upsilon}$ is the velocity of the body versus the fluid.

For turbulent flow of fluid around the object (*high velocities* and large bodies) the drag force is

$$\vec{F}_D = -k\upsilon * \vec{\upsilon} \qquad (8)$$

i.e. its magnitude is $F_D = k\upsilon^2$ ***k*** is another drag coefficient

4] THE DYNAMICS OF CIRCULAR MOTION OF A PARTICLE(object modelled as a single particle)

- In kinematics we found that a particle moving at **constant speed** "$\boldsymbol{\upsilon}$" on a circle with radius "$\boldsymbol{r}$" has an acceleration. This acceleration is directed all time versus the center of circle and has constant magnitude

$\boldsymbol{a_c = \upsilon^2 / r}$. In vector form, one gets

$$\vec{a}_c = -\frac{\upsilon^2}{r}\hat{r} \qquad (9)$$

where $\hat{r} = \frac{\vec{r}}{r}$ is the ***unit*** *radial vector* (*tail at center*) defining the direction from origin. The vector $\vec{a}_c$ is called ***centripetal acceleration*** because it is directed <u>*all time versus the center of circle*</u>. The 2nd law tells that, if a ***particle*** with mass $\boldsymbol{m_p}$ is moving with centripetal acceleration, the ***net force*** on it has the same directions as $\vec{a}_c$ (see fig.5)

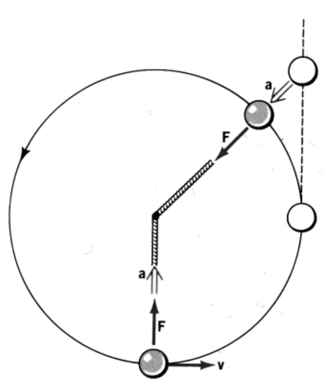


Figure 5

$$\vec{F}_{NET} = \vec{F}_c = m_p \vec{a}_c = -\frac{m_p \upsilon^2}{r}\hat{r} \qquad (10)$$

This ***net force***, has magnitude $\frac{m_p \upsilon^2}{r}$ and it is called ***centripetal force*** because is directed all time versus the circle center. *Note that it **is named only from its direction and not from its physical origin***. A force of gravity, friction, electric …or their sum can be in the role of a ***centripetal net force***.

Note: ***<u>Don't draw the centripetal force in a free body diagram</u>***

(One never draws $\vec{F}_{net}$ in a FBD) .

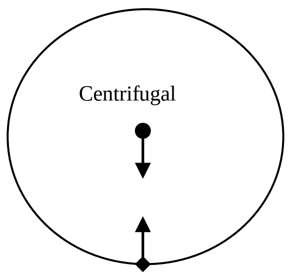


Figure 6 An object in uniform circular motion

– The action of centripetal force consists in not allowing the particle to follow its path along the tangent "*as required by its inertia* ". As the third law affirms, the particle reacts on this action by a force of equal magnitude but opposite direction and *applied "to the source"* of centripetal force (figure 6). This is called *centrifugal force*

$$\vec{F}_{Centrifug} = -\vec{F}_{Centripet} = \frac{m\upsilon^2}{r}\hat{r} \qquad (11)$$

Ex_a; An object fixed at end point of a wire rotates uniformly on a ***horizontal floor without friction*** due to rotation of an axe at the other end wire. The ***net force*** acting on the object is ***equal to*** the wire ***tension*** (because its weight is cancelled by normal force)***. This net force acts as a centripetal force exerted on the object.*** The object reacts (via the wire tension) by a force with equal magnitude and opposite direction ***applied on the rotating axe; this is centrifugal force acting on the axe***.

Ex_b; A car travels at high speed around a horizontal curved path. A center seeking **static *friction force*** exerted by the road on the tires plays the role of ***centripetal force*** (its source is the road). An equal force with opposite direction, the ***centrifugal force***, is ***exerted on the road by tires***.

Note: Sometimes, by error, one talks about the centrifugal force exerted on the rotating body. The example of a not seated passenger inside the bus moving around a curved road that feels himself pushed "*out of circle*" is mentioned often. The truth is that this passenger is ***not hard bounded*** with bus to **get the centripetal force** applied on him straight away. So, due to his inertia he may follow his way along the tangent….until he touches the interior of the bus. Next, the bus interior transmits to him the centripetal force and after that he follows the same circular motion as the bus.
Remember: *There is **no centrifugal force exerted on the rotating particle..***only..centripetal force.

5] SATELLITE MOTION

- A satellite is an object with mass "***m"*** that turns uniformly around a central body with larger mass "***M"*** due to ***gravitation***. As the *net force* on the satellite is a centripetal force and there is only a gravitational force applied on the satellite, one can equalize the expressions for their magnitudes

$$G\frac{mM}{r^2} = \frac{m\upsilon^2}{r} \qquad (12)$$

and find out that

$$\upsilon_{orb} = \sqrt{\frac{GM}{r}} \qquad (13)$$

So, the ***orbital speed*** of a satellite rotating uniformly on an orbit with radius "r":

a) ***Depends*** only from central ***mass M*** and not from satellite mass;
b) ***Decreases*** with the ***increase of*** orbit ***radius***.

- The period of satellite on a fixed (***r value***) orbit is

$$T = \frac{2\pi r}{\upsilon_{orb}} = \frac{2\pi}{\sqrt{GM}} r\sqrt{r} = \frac{2\pi}{\sqrt{GM}} r^{3/2} \qquad (14)$$

and

$$T^2 = \frac{4\pi^2}{GM} r^3 \equiv \kappa r^3 \qquad (15)$$

The expression (15) known as ***Kepler's third law*** shows that the period of rotation around a given body (i.e. fixed κ-value) depends only from distance of satellite from the body.
This expression is very useful because it allows to calculate the mass of the central body by measuring the radius of orbit and the period of rotation of its satellite.
For example, one may calculate the earth's mass by using moon's period and its distance from the earth.
Similarly one may calculate the mass of sun by using the periods and distances of its satellites (*planets*).