LECTURE_10

1] INTRODUCTION OF POTENTIAL ENERGY CONCEPT

- Consider an object at rest on the floor. As long as the tail of forces exerted on it ($\vec{F}_G, \vec{N}$) is at the same location they do not produce work. If one shifts the object up (see fig.1) at height y = "h" and leaves it free, the object will fall *down* to *y = 0*. The *gravitation* force on object will *produce* a *positive work* because *its* tail is ***shifted*** along its own direction by $\vec{s}$ (see Fig.1).

$$W_G = \vec{F}_G * \vec{s} = mgh \qquad (1)$$

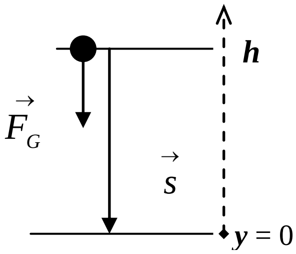

Figure 1

One can place the object at the height "***h***" by hand or by throwing it upward with the right initial velocity. No matter what way the ***object*** goes to the height "***h***", once there, "there is *a capacity for production* of ***<u>mechanical work</u>"***. One says that, just due to its **location**, the ***<u>object</u>*** possesses a ***<u>potential mechanical energy</u>***. Actually, this *potential energy is due to* the *gravitation attraction* of **earth** and the **location of object vs earth**. So, when talking about potential energy, one should refer to the **configuration** of ***<u>system</u>*** **earth - object** *(instead of just object location*). In more general terms:

Any kind of potential energy is due to a system configuration.

-***Block- spring <u>system</u>***. If one extends the spring end by "***x***", the ***restoring force*** produced by the spring will be directed versus the equilibrium position. If one leaves it free, when the block passes by its equilibrium position, *the elastic (or restoring) force has produced the positive work*

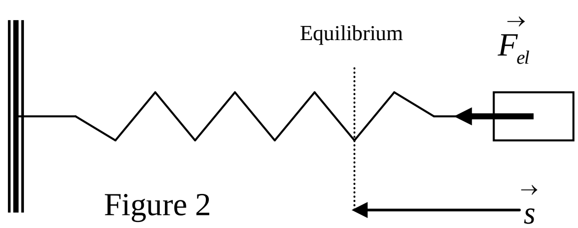

Figure 2

$$W_{el} = -(k/2)(0^2 - x^2) = kx^2/2 \qquad (2)$$

If the spring is *<u>at rest</u>*, at its equilibrium(x = 0), it does not possess any capacity to provide work. But, if it is extended or compressed (x ≠ 0), the spring " *is able* " to provide work; one says that it does ***possess energy***. The capacity of a spring to provide work depends only on its ***configuration*** (*extended* or *compressed*); *so this* is *a* ***potential energy***. The two main types of ***mechanical potential energy*** are***: gravitational*** *and* ***elastic***.

- In both cases, *initially*, an ***<u>external force</u>*** *(its source is not part of system; hand is not part of system*) ***<u>does a positive work</u>*** $\boldsymbol{W_{ext}}$ on the ***system*** and shifts it from an *initial configuration '* ***i*** *'* to a *final configuration '* ***f*** *'*. Next, just because of being at *'* ***f*** *'* configuration, the ***<u>system can</u>*** *provide work* and the amount of work it can produce (or *its work production* ***capacity)*** depends **only** on the ***configuration*** of the ***system***. This means that one may calculate this capacity by use of a *configuration function*[1] $\mathbf{U}_{conf.} \equiv \mathbf{U}$. If one refers to values of this function at *initial-final* configurations $\mathbf{U_i}$ and $\mathbf{U_f}$ and figures out that a ***positive*** *external work* $\boldsymbol{W_{ext}}$ brings *the system* from state *'* ***i*** *'* to state *'* ***f*** *'*, the simplest logical relation between **U** and $\boldsymbol{W_{ext}}$ would be

$$W_{ext} = U_f - U_i = \Delta U \qquad where \quad \boldsymbol{U_f} > \boldsymbol{U_i} \qquad (3)$$

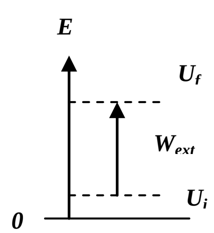

Figure 3

This relation fits perfectly with the logic that ***the energy of the system increases*** when an *external force* achieves ***positive external work*** $\boldsymbol{(W_{ext} > 0)}$ on it (see figure 3). Actually, this definition requires that **all** the ***exterior work*** *<u>goes only for configuration changes</u>* and ***not*** for the ***change of kinetic energy*** of system parts. So, the system must be shifted from the initial configuration $\boldsymbol{U_i}$ to the final configuration $\boldsymbol{U_f}$ **by keeping its kinetic energy constant** which means by a ***constant*** **speed (***in practice very slowly***)**.

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[1] Mathematical function that depends only on the location (**coordinates**) not on velocities or accelerations.

- To avoid the ambiguity related to the type of external forces, one ties the *definition of potential energy to* the ***work by internal forces***. The third law tells that during *system* transfer from U_i **to** U_f , the *work by internal forces* $W_{int} = -W_{ext}$. From relation (3), one gets to the **basic *definition*** for ***potential energy*** *as*

$$-W_{\text{int}} = \Delta U \qquad \text{or} \qquad \Delta U = (U_f - U_i) = -W_{\text{int}} \tag{4}$$

- Note that the work W_{in} in relation *(4) is* ***related to*** *the* ***difference*** $\Delta U = U_f - U_i$ *and* **not** *on the* ***U- values.*** *This means that* ***only*** ΔU ***has physical meaning*** *(<u>not U values</u>).* This definition for ***U*** *leaves* ***"free choice"*** *for the selection of* **configuration where** $U = 0$. *In practice***,** *one* **fixes** $U_i = 0$ *to an* **initial configuration which makes easier the solution of the considered problem.** Next, by considering the ***system*** *shifted from* $U_i = 0$ to U_f**,** it comes out that $(U_f - 0) = -W_{\text{int}}$. So, to calculate the numerical value of U- function, one starts by fixing U = 0 to a given system configuration and then calculates the **potential energy function** as

$$U_f \equiv U = -W_{int} \tag{5}$$

To ***<u>define a potential function U(x)</u>***: ***D****efine* the ***system; C****hoose a* ***coordinative axe Ox****; Note* x_0 *where* $U(x_0) = 0$***; C****alculate the* ***work*** W_{int} *by the* ***internal force*** *from* x_0 *till a* ***location*** *"x"; Use the* ***relation (5).***

Examples: 1) <u>System</u>: "***object - Earth***"; Chose Oy axe directed up. Take y = 0 at ground level and note $U_{ground} = U(o) = 0$ (or y= 0 at floor level and $U_{floor} = 0$). Refer to its displacement *<u>up</u>* by *s* = "*h* " *from ground*. As the ***internal force*** is $\vec{F}_g$, its work is $W_{in} = \vec{F}_g * \vec{s} = -mgh$. So, $U \equiv U_G = -W_{int} = -(-mgh) = mgh$
2) <u>System</u> "***spring- block***"; one selects $U = 0$ for *spring at equilibrium* ($x = 0$) and the Hook's force is the ***internal force***. Then, $W_{in}(0 \to x) = W_{elastic_{force}} = -k\left(\frac{x_f^2}{2} - \frac{0^2}{2}\right) = -k\frac{x^2}{2}$ and $U \equiv U_{elastic} = -W_{int} = k\frac{x^2}{2}$

NOTES: *A* ***potential energy*** *is* ***due*** *to the* ***interaction*** *between* ***system's constituents****.*
- So, *it does not make sense to talk about potential energy of a single object or particle alone*. When one says that the potential energy of an object with mass "***m***" at height "***h***" is $U_G = mgh$, actually, this means the gravitational energy U_G of the "***system object – earth***" which is due to their gravitational interaction.
-The **<u>mechanical</u> potential energy** is mainly due to **gravitational** and **restoring (***elastic***) forces**.
- From a general point of view, one ***may define*** a **potential energy only if** the force at origin of this energy is **conservative**. In the following one explains the *meaning of a* ***conservative force***.

2] CONSERVATIVE FORCES, POTENTIAL FUNCTION, SYSTEM – DEFINITIONS

-Some forces posses a *specific quality*: their ***work equals zero*** whenever their tail is shifted along a ***closed path*** (*the end point of path is the same as its initial point*). These are known as ***conservative forces***. Note that a *conservative force* does a *work* ***different from zero*** when its tail is shifted (fig.4) from point "1" to point "2" and when it is shifted from "2" to "1" **but** $W_{2-1} = -W_{1-2}$ and it comes out that $W_{1-1} = W_{1-2} + W_{2-1} = 0$. Three main examples of such forces are the ***gravitational***, ***restorative*** (*elastic*) and ***<u>static</u> electric*** forces.

-Not all the forces are conservative. The ***normal*** force is always *perpendicular to displacement* and its work is *always zero,* no matter what is the path. The net work by ***friction*** force on an object sliding on the floor is **negative** even when object is shifted along a closed path. Forces that depend on the velocity (*the drag force in fluids, the magnetic force,..*) are not conservative forces, too.

Figure 4

-In the case of a ***conservative force***:

a) If the path followed by the tail of conservative force is *not closed*, its work depends only on the *initial* and *final* ***locations*** of its tail and ***not*** *on* the ***shape of the path*** from initial to final location.

b) One can define a ***potential U(or potential energy)*** *which is a function of space coordinates.* The history of physics showed that this function is very useful for solving difficult problems.

- Any *force* is produced by a "*source*" and is applied over an "*object"*. So, formally, one may always refer to the *system source-object*. However, only if this *force* is ***conservative***, one can *define the* ***potential energy of the system*** *(due to this force)* and use the *concept of* ***system*** for energy calculations.

Related Notes:

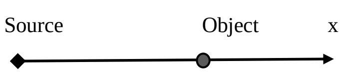
1-Any two or more ***objects with mass*** undergo their ***gravitational*** attraction **forces**. As ***gravitational forces are conservative***, one may ***always*** define a *system* and a ***gravitational potential energy*** U_G for this *system*. If an *elastic* (restoring) ***force*** acts over one of system components, one deals with an additional conservative force. So, one can include the source of elastic force into the system and add the corresponding term U_{El} of elastic potential energy to the function of potential energy of system and it can be written as $U = U_G + U_{El}$.

2-If one of conservative forces of system is much larger than the others, at a first step approximation, one may consider that the potential function of system is equal to its corresponding term and neglect the smaller terms.

3-As ***non-conservative forces cannot provide a potential function*** their sources ***are not part of*** *the system.* So, the ***non conservative forces are considered as external forces to the system***.

3] INTERNAL FORCES. TOTAL MECHANICAL ENERGY OF AN ISOLATED SYSTEM

-If one has identified a *conservative force* and defined a simple mechanical **system** (one object & one source), the ***source*** of the *force* is *part of the* ***system*** and there is an **internal force** for this ***system***. Next, one may *fix the origin of an axe Ox* at "***source***" of the force and align it along the direction " *source - object* ", calculate W_{int} and get $U(x)$ expression. This *potential energy* function $U(x)$ depends only on x-coordinate and one may show that, the ***internal force*** acting on the "***object"***, is related to the *potential function* $U(x)$ as follows



$$F_{\text{int}} = -\frac{dU}{dx} \qquad (6)$$

Once precisely defined (math.expression and 0-value location), $U(x)$ **function** (or **PE** - *potential energy*) becomes a ***parameter*** tied to the ***system***. If the "***system"*** contains several "*sources*" and "*objects*", there are several internal forces and the *potential function* will depend on a *set of coordinates* $U = U(x_1, x_2, .. y_1, y_2 .. z_1, z_2 ..)$.

Exemple: An aeroplane with mass ' ***m*** ' is flying at height ' ***y*** ' from earth surface. The *gravitational force* acting on the plane (*weight)* is a *conservative force*. The ***source*** of the *weight* is the ***earth.*** One defines the ***system earth-plane***, selects *Oy-axe* along vertical and fixes the *origin O on earth surface (or its center).* The potential energy of this system (*we are used to call it ´potential energy of aeroplane´*) is $U(y) = PE = mgy$

Then, the plane weight (***internal*** *force of system*) can be calculated by use of (6) as $F_G = F_{\text{int}} = -\frac{dU}{dy} = -mg$

The "–" sign shows that the weight direction is opposite to selected Oy positive direction.

-Assume that ***several forces*** are ***applied*** on an object but ***only*** one is ***conservative force.*** Then one *ties* a *frame* ***Ox*** *to the source* of this force, ***defines a system***, and get the function of potential energy $U(x)$. The conservative force is an ***internal force*** for this system. If the object get shifted by displacement "Δx " under the action of all applied forces, this produces a change ΔU of potential and an amount of work $W_{int} = -\Delta U$ done by the ***internal force***. Note that one can calculate the change of **kinetic energy** K of the object with respect to frame Ox by applying the theorem work-energy for the object as: $W_{net} = K_{fin} - K_{in} = \Delta K$

If one calls W_{ext} the work done by all external forces, then $W_{net} = W_{int} + W_{ext}$ and one get to the relations

$$W_{net} = W_{int} + W_{ext} = -\Delta U + W_{ext} = \Delta K \qquad \text{and} \qquad W_{ext} = \Delta U + \Delta K = \Delta(U + K) \qquad (7)$$

If the ***system is isolated*** (no external forces), $\boldsymbol{W_{ext} = 0}$, $\Delta(U + K) = 0$ and one gets $\boldsymbol{U + K = constant}$ (8)
So, the sum of kinetic energy ***K*** of the "*object*" and "*its potential energy*" ***U*** remains constant in time.
In fact, the *potential energy* belongs to the object **and** the **"source"** of conservative force; also, the kinetic energy K of object is the same as kinetic energy of the whole system (*the frame is tied to source and* $K_{source}=o$).
So, the sum ***K + U*** presents actually the **total mechanical energy** E_{mech} (or ***ME***) of the ***system***.
The expression (8) tells that, even if there are external forces, provided that their net work is zero ($W_{ext}= 0$)

$$ME=KE+PE= const \quad \text{or} \quad \mathbf{E_{mech} = K + U = } \boldsymbol{const} \tag{9}$$

The **<u>total mechanical energy of a system is conserved if there is zero external work on it.</u>**

4] MECANICAL ENERGY FOR NON ISOLATED SYSTEMS

How to deal with situations where *conservative and non-conservative* forces apply on the same object while it is shifted from an " *initial* " location to " *final* " location of space ?

Step_1 *Identify* the *conservative forces(weight, restoring,..)* acting on it. *Define the system constituents*.

Step_2 *Define* a *reference frame* and a *potential function* for *each conservative force*. Define clearly the locations <u>*where*</u> each potential is zero. Take their **sum** as common **U** ***<u>potential</u>*** for **system.**

Step_3 Divide the set of all acting forces on the object into **internal (** ***they contribute to total U*** **)** and **external** (***they do not contribute to U*** **).**

Step_4 Note that <u>*if*</u> there are <u>*no external forces*</u> or if they do zero work, $W_{ext} = 0$ and the <u>*total mechanical energy*</u> of the system remains <u>*constant*</u>; $\boldsymbol{E_{mech\text{-}fin} = E_{mech\text{-}in}}$.
If the ***external*** forces are doing work $\boldsymbol{W_{ext\_net}}$ on the object, this work will change the mechanical **energy of the system** and this change will be equal to $\boldsymbol{W_{ext\_net}}$;

$$\boldsymbol{\Delta E_{mech} = E_{mech\_fin} - E_{mech\_in} = W_{ext\_net}} \tag{10}$$

Notes: - If the work by ***external*** forces is ***negative*** there is a **decrease** of mechanical energy of the system.
- Expression (10) presents the <u>*general form*</u> of the ***principle of mechanical energy conservation.***
It is valid for any kind of mechanical system (*isolated* $\boldsymbol{W_{ext\_net} = 0}$ *or not isolated* $\boldsymbol{W_{ext\_net} \neq 0}$ ***)***.

REMEMBER: The *kinetic, potential and total mechanical energies of a system are mathematical functions <u>defined at a reference frame</u>*.

- While the definition of kinetic energy does not need the concept of system, the ***potential and total mechanical energy functions cannot be defined without*** going through *system* definition. The values of ***KE, PE, ME*** functions do not have any precise physical meaning by themselves because ***they depend on the choice of reference frame*** (the change of reference frame changes the value of those functions). But, the ***change*** of those functions **has a very precise physical meaning**. It is related to ***the net external work achieved on the system and this quantity does not depend on the selected frame for calculations***. This is the basic issue one must not forget when dealing with energy related problems.

- One choses the location of *zero value for U-functions (or potential energy functions)* in such a way that makes easier the mathematical solution of the related problem.