THE ESSENTIAL INFORMATIONS CONTAINED INSIDE A POTENTIAL ENERGY FUNCTION

- Any **potential** energy function is related to a given **conservative force**. We considered the problem in 1D space and we showed that a simple mathematical relation holds on between them $$f = -\frac{dU}{dx} \qquad (1)$$

We underlined that only the change of potential energy $\Delta U = U(x_f) - U(x_i)$ has a physical meaning. The choice of space location where U(x) is zero does not affect the result. In practice, one chooses the location where U(x) = 0 in the way that simplifies the math calculations for the solution of the considered problem.

Once the potential ***function U(x)*** (*potential energy*) is defined, one can write total mechanic energy of the system as function of "*x* ", i.e. ***ME(x) = K(x) + U(x).*** Then, if the system is ***isolated*** , the principle of mechanic energy conservation tells that ME is a constant(i.e. "***ME(x) = ME***") and it comes out that ***K(x) = ME – U(x)*** (2)

- The *potential function* ***U(x)*** contains some important information that are introduced by the following example. Assume that a 10 kg block of ice is sliding without friction on the sides of two icy hills which profile is shown in the figure. The *system block - earth* has constant ***ME*** for any position of block on hills because Wext = W_N = 0.

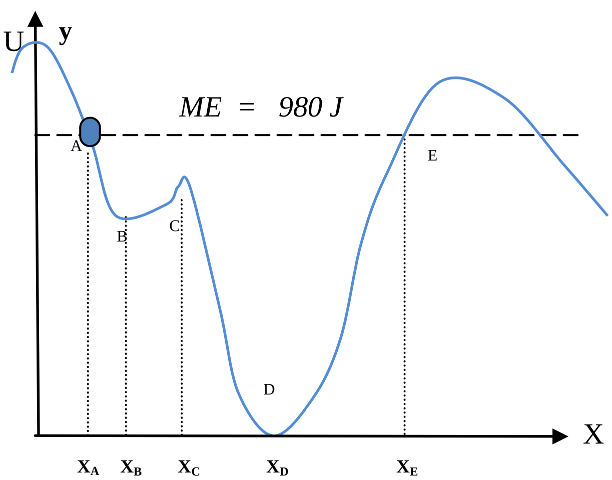

Fig.1

With Oy axe directed vertically up and origin at the lowest level(i.e. D- point), the *potential energy* is ***U = mgy*** ; Note that, the shape of ***U***-function is the same as y-values of hill; the difference is at numerical values of U which are in joules. So, if the block starts moving ***from rest*** at A point and A is ***10m*** higher than D, its kinetic energy is $K(x_A) = 0$ and $ME = U(x_A) = 10*10*9.8 = 980J.$ ***As ME*** is ***constant all time,*** its value remains equal to $U(x_A)$, i.e. ***maximum*** *U-* value. One says that during its motion the block cannot pass over the "***potential barrier"*** of ***980J***. Actually this *potential barrier* fixes the space region [x_A , x_E)] inside which the block can move.

Once arrived at "x_E", the kinetic energy of block becomes zero ($K(x_E) = 0$), i.e. $\upsilon_E = 0$ and it returns back; the point $x = x_E$ is "***a turning point "*** ; the point x_A is a turning point, too. It comes out that the block would move all time between the two *turning points* (**x_A, x_E**). One says that the ***particle*** *(material point model for block)* ***is trapped in a restricted space region*** *(due to potential barriers limiting it).*

- *At a* ***turning point*** the particle *changes the direction* of *motion* (***velocity changes the sign)***. If the block is *moving from "D versus E"*, the derivative $\frac{dU}{dx}$ *is* > 0 but the force is negative (see eq.1) and it is slowing down the block motion. At turning point "E" the velocity becomes zero and acceleration imposed by the internal force inverts the direction of motion. Next, the same force speedups motion untill it gets to point "D" where the kinetic energy gets its ***maximum*** value (***$K(x_E)$ = 980J)*** *while the particle moves with a* ***negative velocity***. On the left of D-point, $\frac{dU}{dx}$ *is* < 0 *i.e. "* $f > 0$ *"*. So, the ***acceleration is positive*** but the ***velocity is negative*** ; there is a slowing down. The slowing down motion follows till " C- point " where $\frac{dU}{dx} = 0$ and force becomes instantaneously zero but block follows moving left side because its velocity is not zero. From C-point to B -point, $\frac{dU}{dx} > 0$ *i.e.* $f < 0$ and the block speeds up because $a < 0$ & $\upsilon < 0$. Beyond B-point it slows down until A-turn point where it stops instantaneously and returns back.

- If the block is left **at rest** at "A" this force will make it slide down as $\frac{dU}{dx} < 0$ **and** $\boldsymbol{f > 0}$ (*directed along +x*). If the block is left **at rest** at points "B or C or D" it will not move because $\frac{dU}{dx} = 0$ and consequently $\boldsymbol{f = 0}$. ***The particle*** *(i.e. block) will be* ***at equilibrium*** *if it is* ***placed at rest at any location where*** $\frac{dU}{dx} = \mathbf{0}$. Assume that the particle is placed at equilibrium point " D " and next, an external force (*exerted say by our hand*) moves it slightly on the right. Once the external action is removed the particle will return to its equilibrium position (*D-point*). This happens because on the right side of D-point, $\frac{dU}{dx} > 0$ and **f** < **0** (directed versus –x) will push it to D. If the external action moves the particle left side, once it is removed $\frac{dU}{dx} < 0$ and $\boldsymbol{f > 0}$ will drive it to D point, too. The same happens to B-point. If the particle is placed to an ***equilibrium point*** such that the ***internal forces*** around this location ***tent to keep the particle there,*** one says that this is a location of ***stable equilibrium of system***.
The configuration with ***lowest U-value*** is the ***most stable configuration*** of the system (D-point in fig.1).

-Assume now that the particle is placed at equilibrium at C-point and an external force (*say by our hand*) moves it slightly on the right. The particle will slide down versus D-point and will not return to C. This happens because on the right of D-point $\frac{dU}{dx} < 0$ and **f** > **0** (directed versus +x) will push it versus E-point. If the particle would be moved to the left of C-point it would slide down to B-point. C-point is a point of ***unstable equilibrium of system.*** The internal *forces of system* around this location tent to remove the particle from this equilibrium location.

- The shape of ***U-function*** graph is a characteristic of the system under study (block-earth, block-spring, ….). If one knows it , one can use it very efficiently to get essential information on the motion of a particle in that space. Let's consider a particle located inside a space region where the profile of the potential function is that shown in fig.2.

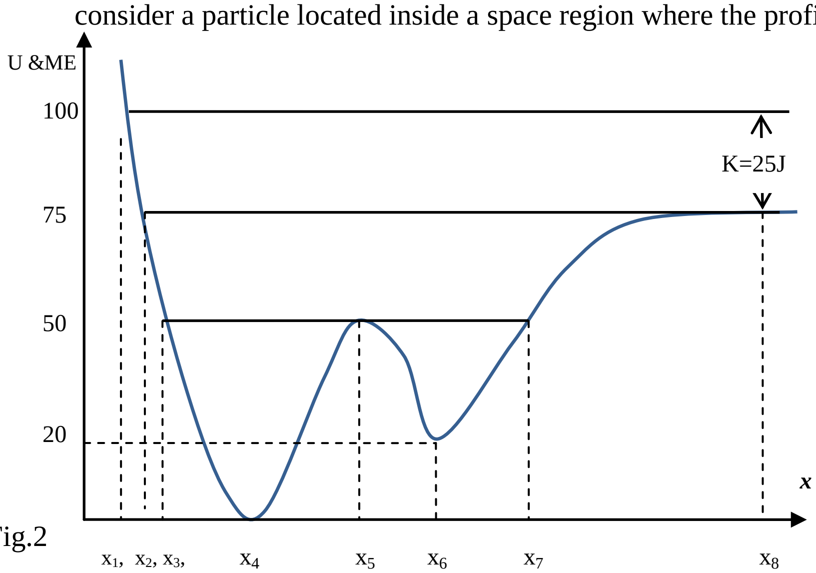

Fig.2

If the particle is placed *at rest* at x_1, it will posses U=100J, ME=U=100J and K= 0J. Also x_1 is a **turning point**. As $\frac{dU}{dx} < 0$ and **f** > **0,** it will start moving right side. Its kinetic energy will increase; at location "x"; K(x) = ME - U(x) = 100 - U(x) Beyond the location x_8, the particle will follow moving uniformly with constant K= 100 - 75= 25J. If the particle was placed(at rest) initially at x_2, then ME = U(x_2) = 75J and K(x_2) = 0. In this case, it would stop at x_8 because K(x_8) = 0 and f = 0. Note that x_8 is not a turning point but it is a point of ***neutral equilibrium***. A particle left initially at rest at a position of *neutral equilibrium* (x > x_8) *will remain there* (Ex. a book on the desk) all time.

If the mechanical energy is **ME = 50J,** there are ***two turning points*** (x_3 and x_7) while x_5 will be a location of ***unstable equilibrium*** because *any slight shift* either side will make internal force f # 0 with such a sign that it would move the particle away from equilibrium position. If initial **ME** < **50J,** the particle will be trapped in a space region between x_3 *and* x_5 or between x_5 *and* x_7. If the particle is initially **at rest** at x_6 and then given a light shift ether side, the internal force pushes the particle back to "x_6" point. This is a location of **stable equilibrium**.

-So, if one knows the potential function U(x) and ME value for an ***isolated system***, one can find out:
a) the value of ***kinetic energy K(x) at any location x*** ; b) the ***turning points***;
c) the configurations of ***stable equilibrium***; d) the configurations of ***unstable equilibrium*** ;
e) the configurations of **neutral equilibrium**.