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THE ESSENTIAL INFORMATIONS CONTAINED INSIDE A POTENTIAL ENERGY FUNCTION

- Any **potential** energy function is related to a given **conservative force**. We considered the problem in 1D space and we showed that a simple mathematical relation holds on between them $F(x) = -\frac{dU}{dx}$ (1)

We underlined that only the change of potential energy $\Delta U = U(x_f) - U(x_i)$ has a physical meaning. The choice of space location where *U(x)* is zero does not affect the result and one chooses the location where *U(x) = 0* in the way that simplifies the math calculations for the solution of the considered problem.

After defining the potential ***function U(x)*** (*potential energy*) , one can write total mechanic energy of the system as function of "*x* ", i.e. ***E(x) = K(x) + U(x).*** Then, if the system is ***isolated*** , the principle of mechanic energy conservation tells that ME is a constant(i.e. " ***E(x) = E*** ") and it comes out that $K(x) = E - U(x)$ (2)

- The *potential function* ***U(x)*** contains some important information that are introduced by the following example. Assume that a 10 kg block of ice is sliding without friction on the sides of two icy hills which profile is shown in the Fig.1. The *system block - earth* has constant "***E"*** for any position of block on hills because Wext = $W_N = 0$.

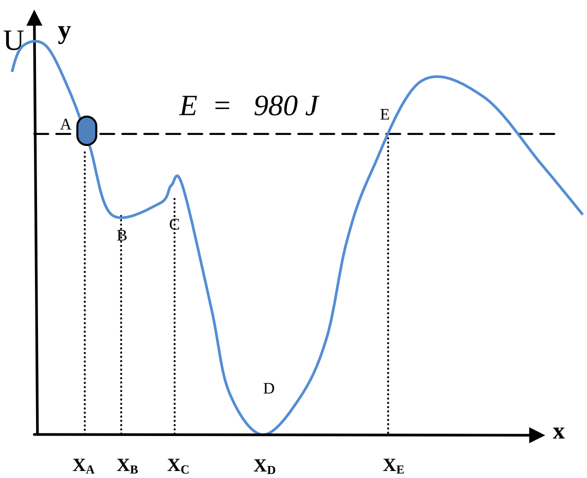

Fig.1

With Oy axe directed vertically up and origin at the lowest level(i.e. D- point), the *potential energy is* $\boldsymbol{U(y) = mgy}$; Note that, the shape of ***U***-function follows the y-values of hills; but the numerical values of " *U* " are in joules. If the block starts moving ***from rest*** at A-point $K(x_A) = 0$**.** If A is ***10m*** higher than D, $E(x_A) = 0 + U(x_A) = U(x_A) = 10 * 10 * 9.8 = 980J$ ***As "E"*** is ***constant,*** its value remains $980J$ ***all time***. Note that at other locations $K(x)\#0$ but $K(x) + U(x) = 980J$ which means that $U(x) < 980J$. One says that during its motion the block cannot pass over the "***potential barrier"*** of ***980J***. Also, this *potential barrier* fixes the space region [x_A , x_E)] inside which the block can move.

Once arrived at "x_E", the kinetic energy of block becomes zero (***K(x_E) = 0***), i.e. $\upsilon_E = 0$ and it returns back; the point $x = x_E$ is "***a turning point "*** ; the point x_A is a turning point, too. It comes out that the block would move all time between the two *turning points* (**x_A, x_E**). One says that the ***particle*** *(material point model for block)* ***is trapped inside a restricted space region*** *(due to potential barriers limiting it).*

- *At a* ***turning point*** the particle *changes the direction* of its *motion* (***velocity changes the sign)***. If the block is *moving from "D versus E"*, the derivative $\frac{dU}{dx}$ *is* > 0 but the force is negative (see eq.1) and it is slowing down the block motion. At turning point "E" the velocity becomes zero and acceleration imposed by the *internal force* inverts the direction of motion. Next, the same force speed ups the motion until the point " D " where the kinetic energy gets its ***maximum*** value (***K(x_D) = 980J)*** *while the particle moves at a* ***negative velocity***. *To the left* of the point " D ", $\frac{dU}{dx}$ *is* < 0 *i.e.* $F(x) > 0$ *and* *a > 0*. As the ***acceleration is positive*** but the ***velocity is negative,*** there is a *slowing down*. The slowing down motion follows till " C- point " where $\frac{dU}{dx} = 0$ and the force becomes *instantaneously zero* but block follows moving left side because its velocity is not zero (it has $K_{x_C}\#0$).

From C-point to B -point, $\frac{dU}{dx} > 0$ *i.e.* . $F(x) < 0$ and the block speeds up because *a < 0 & υ <0.*

Beyond B-point it slows down until the turning point " A" where it stops instantaneously and returns back.

- If the block is left **at rest** at "A", as $\frac{dU}{dx} < 0$ **and** $\boldsymbol{F > 0}$ (*directed along +x*), the internal force will make it slide down. If the block is left **at rest** at points "B or C or D" it will not move because $\frac{dU}{dx} = 0$ and consequently $\boldsymbol{F = 0}$.
The particle *(i.e. object) will be* ***at equilibrium*** *if it is* ***placed at rest*** at any location where $\frac{dU}{dx} = \mathbf{0}$. Now, assume that the particle is at rest at *equilibrium point* " D " and next, an external force (*say... exerted by your hand*) moves it slightly on the right. Once the external action is removed the particle will return to its equilibrium position (*D-point*). This happens because on the right side of D-point, $\frac{dU}{dx} > 0$ and **F < 0** (directed versus –x) will push it to D. If the external action shifts the particle left, once it is removed $\frac{dU}{dx} < 0$ and $\boldsymbol{F > 0}$ will drive it to D point, too. The same happens to B-point. If the particle is placed to an ***equilibrium point*** such that the ***internal forces*** around this location ***tent to keep the particle there,*** one says that this is a location of ***stable equilibrium of system***.
The configuration with ***lowest U-value*** is the ***most stable configuration*** of the system (D-point in fig.1).

-Assume now that the particle is placed at equilibrium at C-point and an external force moves it slightly on the right. The particle will slide down versus D-point and will not return to C. This happens because on the right of D-point $\frac{dU}{dx} < 0$ and **F > 0** (directed versus +x) will push it versus E-point. If the particle would be moved to the left of C-point it would slide down to B-point. One says that C-point is a point of ***unstable equilibrium of system.***
The *internal forces of the system* around this location *tent to remove* the particle *from* this *equilibrium* location.

- The shape of ***U-function*** graph is a characteristic of the system (block-earth, block-spring, ...) under study. If one knows it, one can use it very efficiently to get essential information on the motion of a particle in that space. Let's consider a particle located inside a space region where the profile of the potential function is that shown in fig.2.

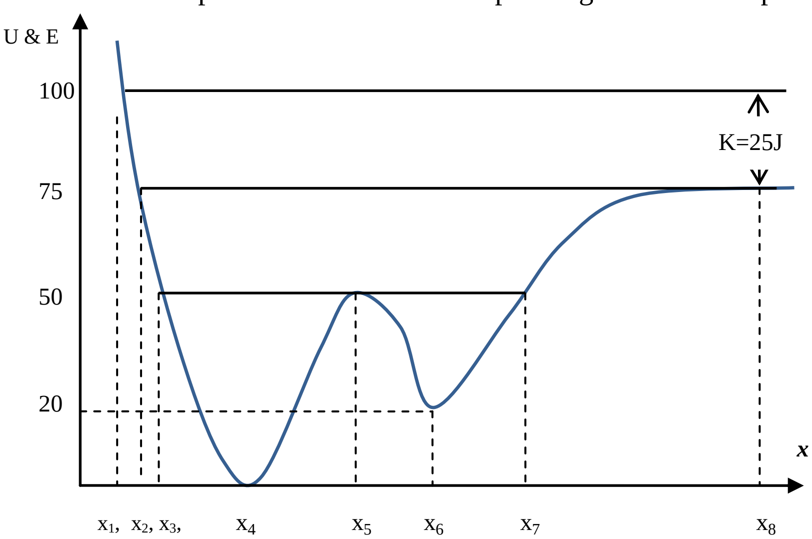

Fig.2

If the particle is placed *at rest* at x_1, it will have $U(x_1) = 100J,\ E = U(x_1) = 100J,\ K(x_1) = 0J$. Also x_1 is a **turning point**. As $\frac{dU}{dx} < 0$ and **F > 0,** it will start moving right side. Its kinetic energy will increase as $K(x) = E - U(x) = 100 - U(x)$ Beyond the location x_8, the particle will follow moving uniformly with constant K= 100 - 75= 25J. If the particle was placed(at rest) initially at x_2, then E = $U(x_2)$ = 75J and $K(x_2)$ = 0. In this case, it would stop at x_8 because $K(x_8) = 0$ and F = 0. Note that x_8 is not a turning point but it is a point of ***neutral equilibrium***. A particle left initially at rest at a position of *neutral equilibrium* ($x \geq x_8$) *will remain there* (ex: a book on the desk) all time.

If the mechanical energy is **E = 50J,** there are ***two turning points*** (x_3 and x_7) while x_5 will be a location of ***unstable equilibrium*** because *any slight shift* either side will make internal force F # 0 with such a sign that it would remove the particle from equilibrium position. If initial **E < 50J,** the particle will be trapped in a space region between x_3 *and* x_5 or between x_5 *and* x_7. If the particle is initially **at rest** at x_6 and then given a light shift ether side, the internal force pushes the particle back to " x_6" point. This is a location of **stable equilibrium**. Note that the **most stable equilibrium** corresponds to the ***lowest U-value of system***, which is at location $x = x_4$.

If one knows the potential ***function U(x)*** and ***E value*** for an ***isolated system***, one can find out: a) its ***kinetic energy K(x) at any location x*** ; b) the ***turning points***; c) the configurations of ***stable equilibrium***; d) the configurations of ***unstable equilibrium*** ; e) the configurations of **neutral equilibrium**.