

LECTURE 3

KINEMATICS

Kinematics uses a set of parameters (**time, position, displacement, velocity and acceleration**) to describe the evolution of a movement in time.

1] THREE MAIN TYPES OF MOTION

- One may distinguish three basic ways of objects' motion: **translation, rotation and vibration**. During a translation and rotation there is *no deformation*; the *shape* of object and its *dimensions remain the same*. In a vibration motion, the object shows changes of one dimension or of its whole shape.

-**Translation**: All points of the object move in the same way and their *final position can be found by using the same displacement vector*.

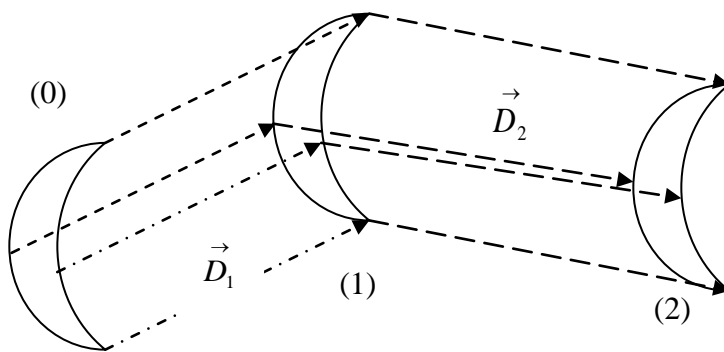


Figure 1 Translation

The same displacement vector \vec{D}_1 shifts each object point from position (0) to position (1). The same displacement vector \vec{D}_2 shifts each object point from position (1) to position (2).

-**Rotation**: The object changes orientation in space. Each object point has a **different displacement vector**.

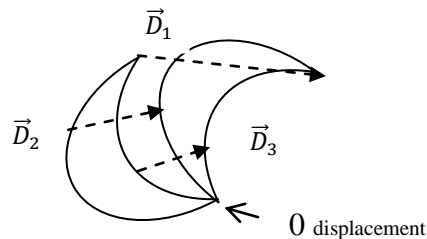


Figure 2 Rotation

Each point of object has a specific displacement vector.

-**Vibration**: There is oscillations *in time of one, two or the three dimensions* of the object.

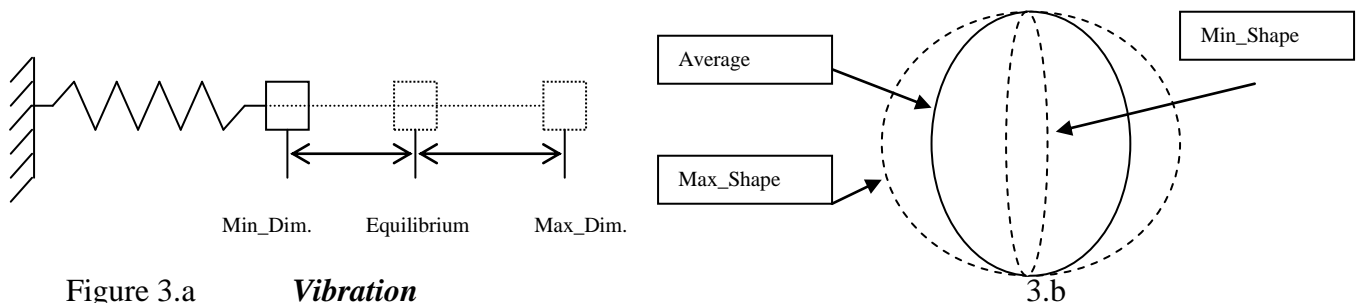


Figure 3.a **Vibration**

3.b

In fig.3.a, the object of study (*spring & block*) changes only one dimension in periodic way. In fig.3.b, the object (an elastic ball) changes the shape (three dimensions).

2] ONE DIMENSIONAL KINEMATICS

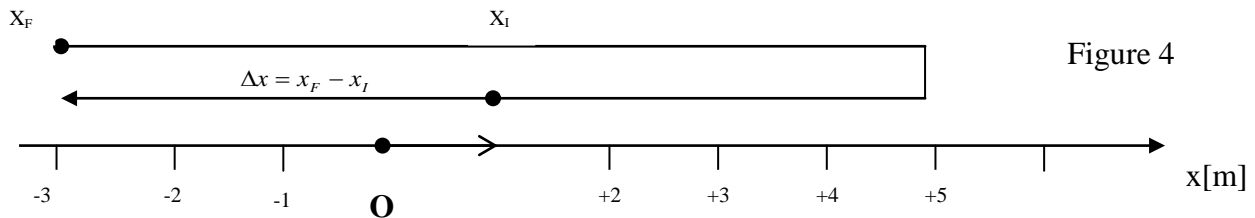
- Mechanics uses a specific **physical model** for the study of each type of motion. This chapter deals with the translation of an object along a fixed direction in space (**1-D translation**). In the next chapter, one will present the main steps for the study of the translation motion on a plane (**2-D translation**¹). As mentioned above, **in a translation**, all object points are shifted by *the same displacement vector*. This simplifies greatly our work; "One may study the movement of just one object point and the results apply over the object as a whole". So, one uses **a point particle motion as a model for the object translation**.

2.a DISPLACEMENT VERSUS THE TRAVELLED DISTANCE

-Consider an object (*plane, car..*) moving **along a straight line**. One starts by modeling it as a particle in motion along a straight line. To define the **position** of this particle, one draws one axe and chooses :
a) its *origin* O; **b)** its *positive direction*; **c)** Length unit "**m**"; **d)** Time unit "**sec**"; **e)** $t=0$ at initial location. Figure 4 presents 1-D motion of a point particle that starts the motion at $t_0 = 0\text{sec}$ from $X_{I(\text{initial})} = +1\text{m}$, moves **4m** to the right, turns back at $X = +5\text{m}$ and stops at $X_{F(\text{final})} = -3\text{m}$ after $\Delta t = t_F - t_0 = 2\text{s}$.

-One defines the particle **displacement** as $\Delta x = X_F - X_I$ where X_F is the final location and X_I is the initial location. The displacement " Δx " may be **positive or negative**. If $\Delta x > 0$ then, **at the end of Δt** , the particle is **shifted on positive direction** and **if $\Delta x < 0$ it is shifted on negative direction of Ox axe**.

Remember: The **displacement** is not the same as the **travelled distance** (always positive). In the case of Fig. 4, the **travelled distance** is $S = 4 + 8 = 12\text{m}$ while the **displacement** is $\Delta x = (-3) - (+1) = -4\text{m}$.



2.b VELOCITY VERSUS SPEED

How fast is moving the particle?

- In *everyday vocabulary* one uses **the speed (positive scalar)** to answer this question in case of cars, planes etc. If one does not need any information about the way the particle is moving in particular portions of its path, one refers to the

$$\text{average speed along the path} = sp_{av} = \text{travelled distance} / \text{time interval} = S / \Delta t \quad (1)$$

The travelled distance " S " and the average speed " sp_{Av} " along the path are positive (always) scalars.

In the upper mentioned example, one would find: **average speed, $sp_{av} = 12\text{m} / 2\text{sec} = 6\text{m/sec}$**

¹ The movement in space, i.e. 3-D motion is not included in programme.

-In physics, one uses mainly **the velocity** to describe the way a particle is moving. One starts by defining

the average velocity as
$$\text{average_velocity} = v_{Av} = \frac{\Delta x}{\Delta t} \quad (2)$$

Note that the velocity may be **negative or positive**; in our example $v_{av} = -4/2 = -2\text{m/s}$ which is different from average speed (6m/s).

Remember: *Speed and velocity are different parameters but they have the same units.*

Note: *The displacement and the velocity are both positive when the particle moves along +Ox direction. When the particle moves along opposite(-Ox) direction, they are both negative. So, the sign of velocity and displacement indicates the direction of motion.*

- One uses often the graph $x = x(t)$ to present the *history of a particle motion*. The figures 5, 6 present two such graphs. The **average velocity** is calculated easily from the slope at these graphs. In the graph of figure 5, the slope ($\Delta x/\Delta t$) does not depend on the moment one starts counting the interval of time or the magnitude of Δt . In this case the motion follows *all time at the same velocity*; it is an **uniform motion**. In the graph of fig.6, the **average velocity** depends on the initial moment and on the magnitude of the time interval Δt . So, v_{Av} does not offer enough information for all situations.

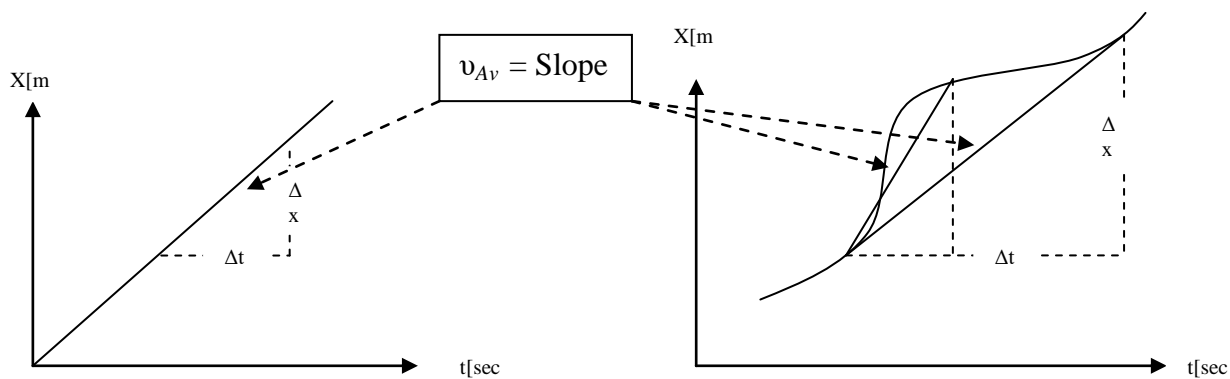


Figure 5

Figure 6

2.c INSTANTANEOUS VELOCITY

- A simple observation of figure 7 shows that, for right information about the *particle velocity* at point P one must refer to the *shortest time interval* Δt counted from the moment $t = t_p$.

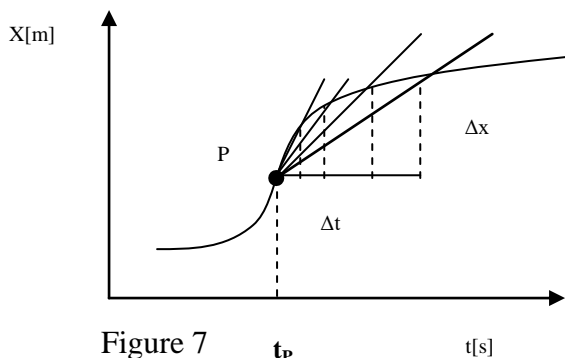


Figure 7

One may figure out that the best estimation of velocity close to point P is the limiting value of average velocity when the time interval Δt goes versus zero. In fact, this is the definition of **instantaneous velocity** at point P

$$v^P = \lim_{\Delta t \rightarrow 0} \left. \frac{\Delta x}{\Delta t} \right|_{t_p} = \left. \frac{dx}{dt} \right|_P \quad (3)$$

Note_1: As this derivative is equal to the slope of **tangent of curve at point P**, one may find the velocity from the function $x = x(t)$.

Note_2: From this definition it comes out that the **instantaneous speed is equal to absolute value of instantaneous velocity**.

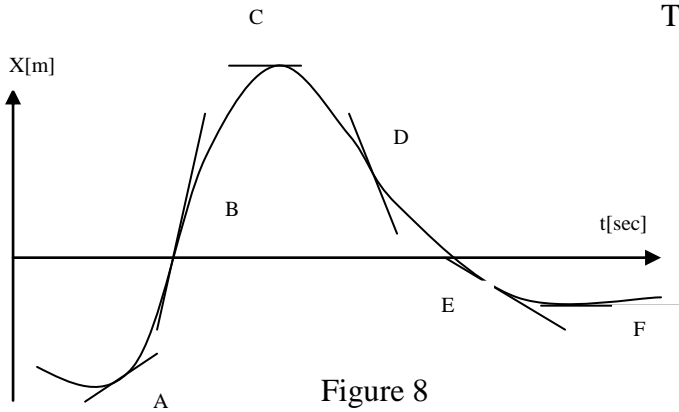


Figure 8

The graph in fig.8, presents a motion along Ox axe.

- A-point; $v > 0$; Motion in (+) Ox direction.
- B-point; **larger** $v > 0$; Faster motion along (+) Ox
- C-point; $v = 0$; Instantaneous rest, turn back point.
- D-point; $v < 0$; Motion in (-) Ox direction.
- E-point; **smaller** $v < 0$; Slower motion along (-) Ox
- F-point; $v = 0$ **for a while** ;
Long rest before turning back along (+)Ox direction.

2.d ACCELERATION

-In the popular vocabulary the word "**acceleration**" means *speed increase*. In physics, it means simply a **change of velocity** (*magnitude, direction or magnitude & direction simultaneously*). If one knows the velocity at each moment of time one may build the graph of instantaneous velocity versus time $v = v(t)$. If this graph is a *straight line*, the **acceleration** $a = \Delta v / \Delta t$ is *constant* all time. If the graph $v = v(t)$ is curved one has to introduce the **instantaneous acceleration**. The *average acceleration* is $a_{Av} = \Delta v / \Delta t$ is the

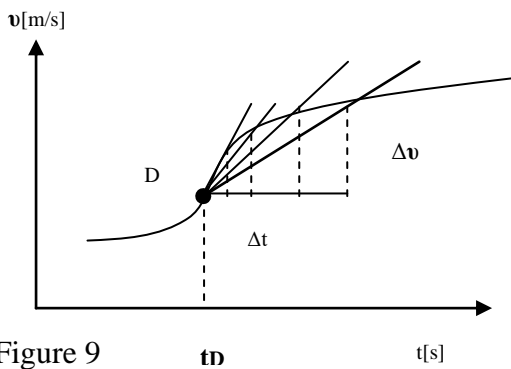


Figure 9

ratio (change of velocity / time interval) **and the instantaneous**

$$\text{acceleration at point D is } a_D = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} \Big|_{t_D} = \frac{dv}{dt} \Big|_{t_D} \quad (4)$$

The **acceleration** at any point on the graph $v = v(t)$ is equal to the **slope** of graph at that point.

The *acceleration may be positive or negative*. It's important to note that acceleration sign alone is not sufficient to find whether the particle is *speeding up* or *slowing down*. To get this information, one must compare the sign of "a" to sign of "v"

- Notes: a) If **a and v** have the same sign the particle is **speeding up** (or accelerating).
- b) If **a and v** have opposite sign the body is **slowing down** (or decelerating).

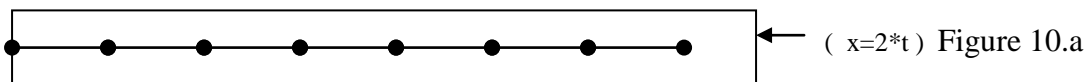
Remember: The direction of motion is shown by sign of displacement and the sign of velocity.

If $v > 0$ the motion is **along + Ox direction**; if $v < 0$ the particle moves in **opposite** direction to + Ox.

- **How to distinguish** an accelerated motion ($a \neq 0$) **from** a uniform motion (constant velocity, $a = 0$)?

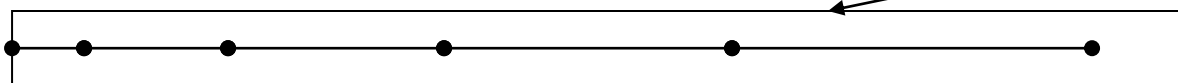
Fix an interval of time (say 1sec) and note the position of object for several consecutive time intervals. Then, if all **displacements** are **equal**, there is a motion at constant velocity (fig 10.a $v=2m/s$, $a=0m/s^2$).

If the consecutive **displacements change** (**increase** or **decrease**) there is an accelerated motion.



$\Delta x =$ 2 2 2 2 2 2 2

Acc. motion (Δx increased by 4)



$\Delta x =$ 2 6 10 14 18

Figure 10.b
($x=2*t^2$)

-The graphs in Figure 11 show how to follow the signs of $v(t)$ and $a(t)$ at different motion situations .

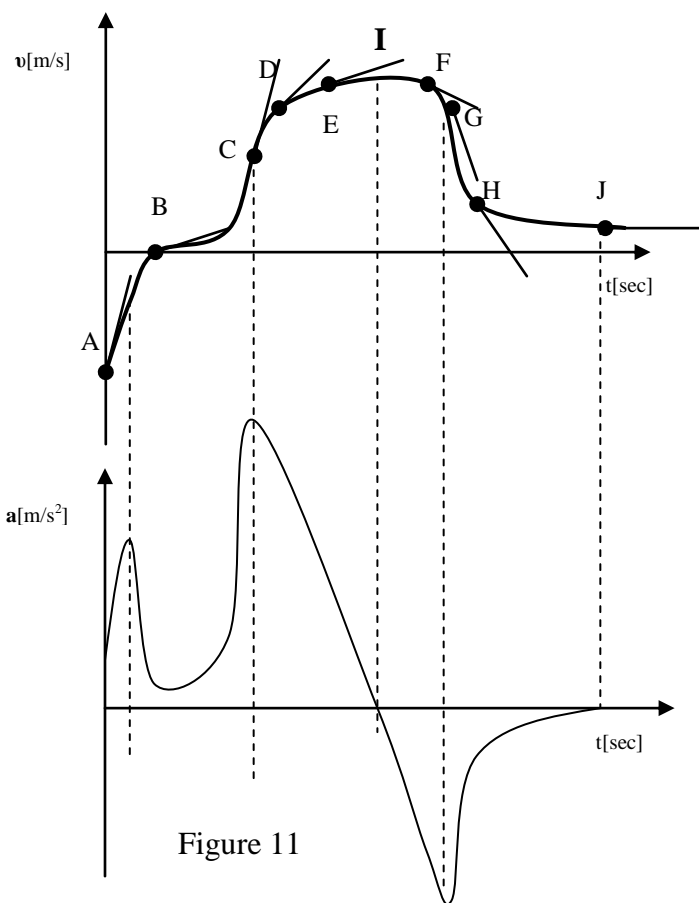


Figure 11

A-point; $v < 0$ means motion along $(-) \text{Ox}$,
As $a > 0$ (opposite sign) there is “slowing down”.

B-point; $v = 0$ but $a \neq 0$; this means instantaneous rest,
As $a > 0$ means $\Delta v > 0$; So, ready to speed up vs $(+\text{Ox})$.

Between B&C; $v > 0$ means motion **along** $+\text{Ox}$,
as $a > 0$ i.e. speeding up ; C-point max_acceler.

Between C&E; $v > 0$ motion along $+\text{Ox}$,
As $a > 0$ but decreasing means slight speeding up;

I-point = zero_acceleration

Between I &J; $v > 0$ means motion along $+\text{Ox}$,
As $a < 0$ but $v > 0$ the particle is slowing down.

major slowing down between G and H.

Beyond H-point, there is a slight slowing down until $a=0$
at point J (constant velocity)

Notes: In real life the velocity cannot change instantaneously and the acceleration ($a = \Delta v / \Delta t$) has always a finite value (no infinite).