1] AREA UNDER THE KINEMATIC'S GRAPHS

- We saw how to find ***instantaneous velocity*** from the graph $x = x(t)$ and ***instantaneous acceleration*** from the graph $v = v(t)$ by using the graph slope *(or derivatives dx/dt; dv/dt)* at the point of interest. The following " inverse " situations are met in practice, too:
  a) Given the graph $v = v(t)$, find the location of particle "***x at a given moment t***".
  b) Given the graph $a = a(t)$, find the velocity of particle "***v at a given moment t***".

**A] Finding the location of particle from the velocity graph $v = v(t)$**
 - The expression $v = dx/dt$ *(1)   is a pure mathematical definition.* In physics, when referring to s*mall but **finite** and* **measurable** intervals (Δx or Δt), one writes expression (1) as $v = \Delta x/\Delta t$ (2)
   So, for small intervals (Δx or Δt) in real measurements, one gets $\Delta x = v^* \Delta t$ (3)
   Then, for a motion at constant velocity (graph in fig. 1) *one may calculate the **displacement** Δx from an initial location X_I (at time t_I) as* $\Delta x = X_F - X_I = v^* \Delta t = v^*(t_F - t_I)$ (4)
   As *$v^*(t_F-t_I)$ is the area of rectangle (fig.1), the displacement is equal to the area under the velocity graph for the considered time interval.* Finally, one gets $X_F = X_I + \Delta x = X_I + v^*(t_F-t_I)$ (5)
   Note that one must know X_I so that one can find X_F by expression (5).

Figure 1   Figure 2
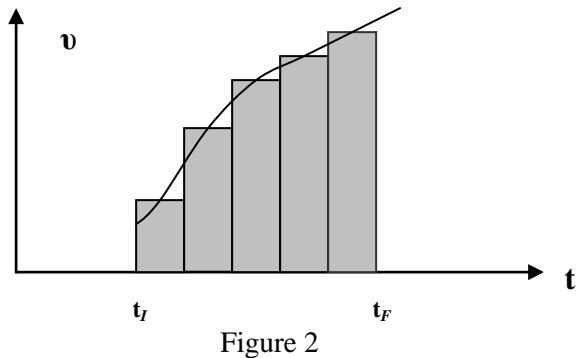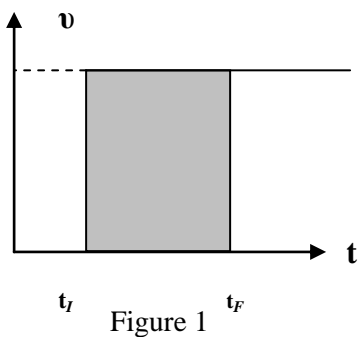
- The result "***displacement** ($\Delta x = X_F - X_I$) **equal to area under velocity graph**" is valid in all cases* but the way ones calculates the area is modified a little bit if the velocity is not constant (fig.2). One has to calculate the area by using a set of narrow rectangles covering the better way possible the area under the graph. One may figure out that this method is more precise[1] when using smaller time intervals Δt.

- The figures 3,4 show two possible velocity graphs. As the area under graph in fig.3 is a positive value, it comes out that $\Delta X = X_F - X_I > 0$ and $X_F > X_I$. This means that the particle is moving along ***positive direction*** of ***Ox***. In figure 4, the area under graph is negative, so one has $\Delta X = X_F - X_I < 0$ and $X_F < X_I$. This means that the particle is moving along ***negative direction*** of ***Ox***.

Figure 3   Figure 4

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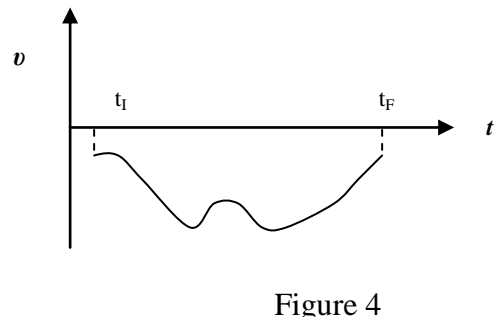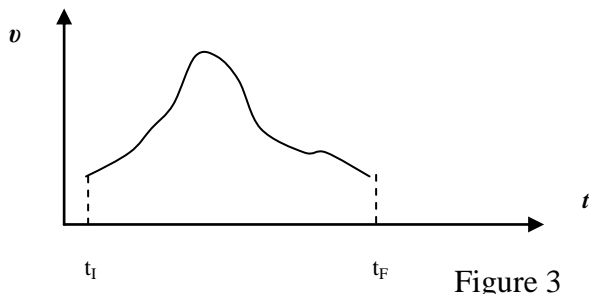
[1] The precise calculation can be done by using the integral calculation.

B] Finding the velocity of particle from the acceleration graph a=a(t)

- Similarly, if one has a graph $a = a(t)$, one may calculate the **change of** particle **velocity** by the area under the graph. In the case of motion at **constant acceleration** (figure 5.a,b), one may easily calculate the velocity at any moment " v_F "(F-final) if one knows it at an initial " v_I " moment of time.

The physic's definition for acceleration is $\qquad\qquad\qquad\qquad\qquad$ $a = \Delta v / \Delta t$ $\qquad\qquad$ (6)

For constant acceleration one may write $\qquad\qquad\qquad\qquad$ $a = (v_F - v_I)/(t_F - t_I)$ $\quad$ (7)

So, $\qquad\qquad$ $v_F - v_I = a*(t_F - t_I)$ $\qquad\qquad\qquad$ *and* $\qquad\qquad$ $v_F = v_I + a * (t_F - t_I)$ $\qquad$ (8)

As time change is positive, **a-sign** decides if the velocity increases (5.a) or decreases (5.b) with time.
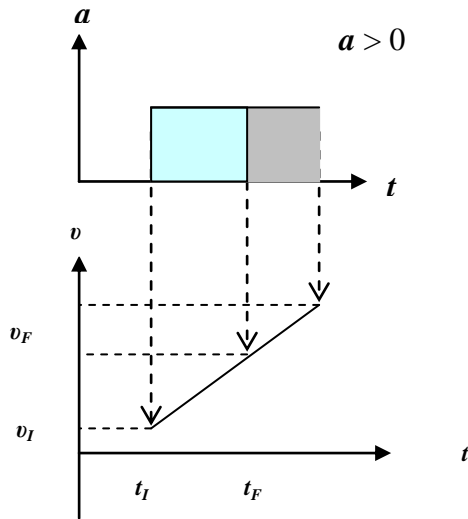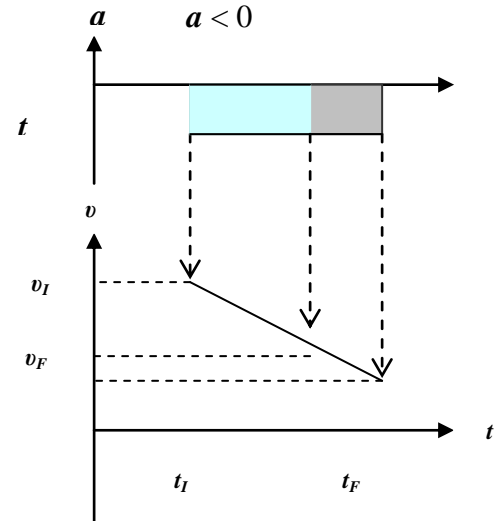

$\qquad$ Figure 5.a (a-c^{te}, positive, v-increases linearly) $\qquad\qquad$ 5.b (a-c^{te}, negative, v-decreases linearly)

Even when "a" is not constant, the area under the a(t) graph gives the *change of velocity* but in these cases the velocity does not change linearly with time (Ex. the "harmonic oscillations, see NYC course).

2] BASIC EQUATIONS OF THE MOTION WITH *CONSTANT ACCELERATION*

- Finding the velocity by the slope of graph $x = x(t)$ or the displacement by the area under graph $v = v(t)$ is useful, but when the graphs are not straight lines and one looks for instantaneous velocity of many points, the procedure becomes cumbersome and complicated. In these situations, the analytical methods provide the result in the form of equations and their solutions as functions. By using a function one can find quickly the numerical value of kinematic parameters at any moment of time.

- Let's consider a 1D motion at **constant acceleration "a"**. We know that in this case the change of velocity is given by expression (6). To simplify expressions, one takes time $t_I \equiv 0$ at starting moment and uses the notations:

$t_F = t; \ x_I \equiv x_o ; \ x_F \equiv x; \ v_I \equiv v_o \text{ and } v_F \equiv v$ $\qquad$ (9)

With these notations the expression (6) can be written as

$$v = v_0 + a*t \qquad (10)$$

The inclined line in fig.6 is described by expression (10).
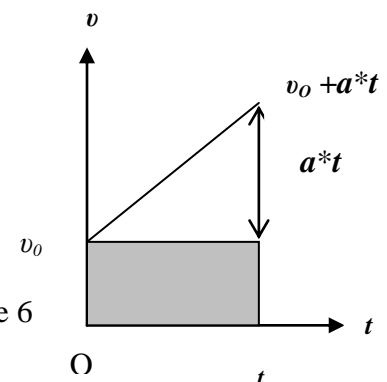Its slope is equal to "a".

Figure 6

- Then, one can find the displacement as the area under the velocity graph in figure 6.

This area is constituted by two parts: the rectangular area $\qquad$ $A_1 = v_{O*}t$ $\qquad$ (11)

and the triangle with area $\qquad$ $A_2 = t*at/2 = a*t^2/2$ $\qquad$ (12)

So, the total area under the velocity graph is $\qquad$ $A_{tot} = A_1 + A_2 = v_{O*}t + a*t^2/2$ $\quad$ (13)

This way, one finds out that $\quad \Delta x \equiv A_{tot} = x - x_0 = v_0*t + a\dfrac{t^2}{2}\quad$ and $\quad x = x_0 + v_0*t + a\dfrac{t^2}{2}$ $\qquad$ (14)

Note: The graph x = x(t) of expression (14) is a parabola which, at t = 0s, crosses *x-axe* at x_0 and has a slope (at t=0 sec) equal to v_0. *For different values of x_0 and v_0 this parabola has different orientations in x-t plane. A motion at constant accelerations is presented always by a parabola in x-t plane.*

- The expression (14) can be modified so that it relates the velocity and position without the variable "time". One may start by isolating "**t**" at expression (10) $\qquad\qquad t = \dfrac{v - v_0}{a}$ $\qquad$ (15)

Next, by substituting it at expression (14) one gets:

$$x = x_0 + v_0*\left(\frac{v-v_0}{a}\right) + \frac{a}{2}\left(\frac{v-v_0}{a}\right)^2 \Rightarrow 2a(x - x_0) = 2v_0*(v-v_0) + (v-v_0)^2 = 2v_0v - 2v_0^2 + v^2 - 2vv_0 + v_0^2$$

$2a(x - x_0) = v^2 - v_0^2$ $\qquad$ and finally $\qquad\qquad v^2 = v_0^2 + 2a(x - x_0)$ $\qquad$ (16)

-The three equations $\qquad\qquad\qquad x = x_0 + v_0*t + a\dfrac{t^2}{2}$

$$v = v_0 + a*t \qquad\qquad\qquad (17)$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

are the ***three basic kinematics equations*** for the 1D motion at ***constant acceleration***.

Note: *In some textbooks the first expression is written as $\Delta x = x - x_0 = +v_0*t + a\dfrac{t^2}{2}$ and calculations are referred to displacement "Δx". As this requests strong attention to the sign of Δx, it is better to use these expressions in the form given at (17) and derive the displacement Δx if the problem asks for it.*

-By transforming eq. (16) as follows $v^2 - v_0^2 = 2*\dfrac{\Delta v}{\Delta t}(x - x_0) \rightarrow (v - v_0)(v + v_0) = 2*\dfrac{v - v_0}{t - 0}(x - x_0)$

one gets $\qquad\qquad\qquad\qquad\qquad x - x_0 = \dfrac{v + v_0}{2}t$ $\qquad\qquad\qquad$ (18)

This expression is useful if one knows that the ***acceleration is constant***, but does not know "***a-value***".
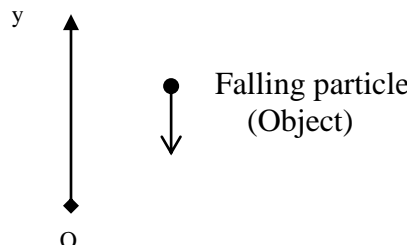
3] FREE FALL

- One say that an object is in a ***free-fall*** *motion if the* <u>**gravity is the only force**</u> *acting* on it.
 A common[2] situation of free fall motion is that of an object falling vertically from a certain height.
The acceleration of ***free-fall motion*** near to earth surface ***is due to earth gravity*** and it is $\approx 9.8\text{m/s}^2$.

- In the case of free-fall "*earth objects*" one has to deal with ***motion along a fixed direction*** (translation)
in space and one can ***model it as a point*** in 1D Kinematics. Furthermore, as the acceleration is constant
$(9.8\text{m/s}^2 \equiv g)$, one may apply the kinematics equations for the motion at constant acceleration.

- When studying the two dimensional kinematics, in general, one uses the ***Ox*** axe for horizontal motion
and ***Oy*** axe is directed ***vertically up***. Based on this choice of Oy axe, the *free fall acceleration* will be

$$a_y = \text{- } g \qquad (19)$$

- So, for a free-fall motion of objects near earth, the kinematics equations take the form

y

Falling particle
(Object)

$$\upsilon = \upsilon_0 - g * t \qquad (20)$$

$$y = y_0 + \upsilon_0 * t - g\frac{t^2}{2} \qquad (21)$$

$$\upsilon^2 = \upsilon_0^2 - 2g(y - y_0) \qquad (22)$$

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- The value $g = 9.8\text{m/s}^2$ is a good approximation for the magnitude of acceleration of objects in free fall
(no other forces but the earth gravitation) <u>***near to earth surface***</u>. In fact, the magnitude of acceleration
due to earth gravity depends on the altitude and location of object. Note that the free-fall motion is ***not a
good model*** for any dropped objects. For example, in the case of skydivers, the action of air friction
force, which is directed against the direction of motion, does not allow the gravitation to increase the
speed at constant acceleration. One say that the presence of ***air friction force*** destroys the condition of a
free-fall motion.

Use the following link to generate 1D kinematics graphs:
https://phet.colorado.edu/sims/cheerpj/moving-man/latest/moving-man.html?simulation=moving-man

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[2] Another situation is that of a satellite moving around the earth. We will study this motion later on.