Physics NYA
Measurements and Relationships

PART A

Objectives: a) To introduce a convenient unit of measurement
b) To identify the type of uncertainty and calculate it
c) To verify experimentally:
    c-1) the type of relationship between the circumference of a circle and its diameter.
    c-2) the type of relationship between the area of a circle and its diameter.
    c-3) π - value

Materials: A large sheet of white paper, 5 circular objects with different diameters, string, paper tape.

Procedures:

1. Introduce a *new* (**not SI unit**) convenient *length measuring **unit*** (*call it **1div***) such that, for all the measurements performed during this lab, the **numerical results** are easy to manipulate:
  a) say, between 0 and 100 (not **very big** numbers)   and   b) **not equal** for different objects.
Draw several such *div units* on a *paper tape* and use it *as a ruler for length measurements*.

2. Use the *scaled paper tape* to measure the diameter "***D***" of five *round objects with* significantly different diameters, yet not so large that they cannot be traced onto the large sheet of paper (see below). Apply the "*half minimum unit rule*" and take the ***absolute uncertainty*** "*ΔD = 0.5div* " for any diameter and length measurement procedure. Record diameter values as D_{best} [div] in table .

3. For each of the circular objects, measure the circumference "***C*** " and record it as C_{best} [div] together with its ***absolute uncertainty*** "*ΔC= 0.5div* " in data table. For this measurement, fit a piece of string around the object circumference, cut it and measure its length by using the *scaled paper tape*.
Plot a ***graph*** C = C(D) of the ***circumference*** versus ***diameter*** and label it as graph No.1.
Put on horizontal axe the *D-values* , on vertical axe the *C-values* and select the same length for unit div on both axes. For each circle, use the D_{best}, C_{best} in table to get a point on the graph area. Use the length " ***0.5div***" to draw the "*uncertainty bars*" (known as "*error bars*" see fig.1)). Use the method shown in "Brief survey on uncertainties" to ***determine the slope*** of this graph. Find what is the *significance* of the *slope* value. Write the result by presenting the absolute and relative uncertainty for slope values. (i.e. $Sl = Sl_{best} +/- \Delta Sl$; $\varepsilon_{Sl}\% = (\Delta Sl / Sl_{best})*100\%$)
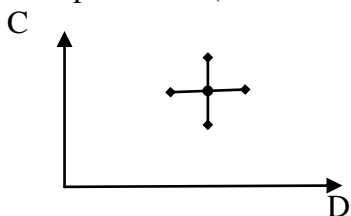

Fig.1 Presentation of an experimental result
and associated uncertainty bars in a graph.

4. Measure the ***area*** of each circle by counting the *number* of squares that make up the area. Count first the *minimum area value* A_{MIN} (number of full squares inside) and then *maximum area value* A_{MAX} (by adding squares half-in/half-out). Introduce those results in table as numerical values with one decimal (ex: 15.0) ***div²*** . Next, use max-min method to find the best estimation $A_{best} = (A_{MAX} + A_{MIN}) / 2$, the ***absolute uncertainty*** $\Delta A = (A_{MAX} - A_{MIN}) / 2$ and relative uncertainty $\varepsilon_A\% = (\Delta A/ A_{best})*100\%$.
The ***diameter*** has (***div***) units and the ***area*** has "***div²***" units. Plot the graph (No.2) A = A(D) of *area A versus diameter D for the set of 5 objects*. You don´t need to draw the "uncertainty bars"

in this graph; only pass a fitting line through best estimation points and get out its expression.
What type of mathematical relationship represents the graph line ? Does it fit to what you expect?

5. Use the data in table to plot the graph No.3 $A=A(D^2)$ of the ***area A*** versus **D^2**. *Does the line you see on graph fit to a straight line* ?   Explain why it should be a straight line.
Use the data in the table to draw the "*uncertainty bars*" for ΔA and $\Delta(D^2)$. Calculate $\Delta(D^2)$ as $\Delta(D^2) = 2D_{best}* \Delta D$ (use max-min method to derive this expression). Use the method shown in "Brief survey on uncertainties" to ***determine the slope*** of this graph from fitting line. Next, identify the relation between slope and π; derive an *estimation for π-* value. Write the result of measurement by presenting the best estimation π_{Best}, absolute uncertainty $\Delta\pi$ and relative uncertainty ε_π%.
Verify if the known value of **pi=3.14** falls inside the ***uncertainty interval*** and state clearly if your measurement is **accurate** or not. Compare its ***precision*** of **pi-value** (refer to ε_π%) to precision of the result for derived from graph No.1 and state which result is more precise.

Important Note: *How to get a **right length** for div unit used in our experiments?*
You will measure the diameters, the circumferences and the areas of circles. One may choose easily a "*good length* for div unit" that fits to request "a" at point #1 for measuring the diameters and circumferences length but it may not fit to request "b" when measuring the circle areas. So, while the presentation of results follows the steps 2-3-4, during measurements, you will proceed as follows:
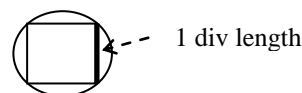
-Start by cutting the string lengths that fit to the diameters and circumferences of circles (cut *a string length that fits to diameter of first circle, tape it on a paper and write aside D_1; follow the same way for the string that fits to its circumference C_1 ; do the same for other circles*).
 At this step, you do not have to measure the length of those strings.

-Draw the **smallest circle** on a large sheet of paper.
 Next, draw carefully a square inside it.   Fig.2
Take the ***side of this square as 1div*** (as shown in figure 2) and use its length to draw several lines at distance 1div from each other *on the paper tape*. This way you get a *scaled paper tape* (fig.3).
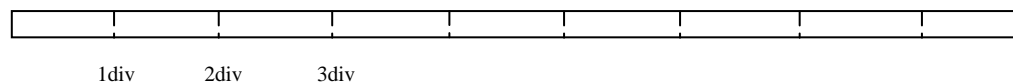

Fig.3   1div   2div   3div

Use this as a ruler and measure the length of pieces of string that correspond to diameters and circumference of circles. Record the found values as D_{best} and C_{best} in the table with 0.5div uncertainty (ex. 5.0 or 5.5   and *not 5.2 or 5.7*) .

- Follow by drawing vertical and horizontal ***equidistant*** parallel lines at distance 1div, as shown in figure 4 and after that draw the other 4 circles with increasing diameter to the right (see fig.4).
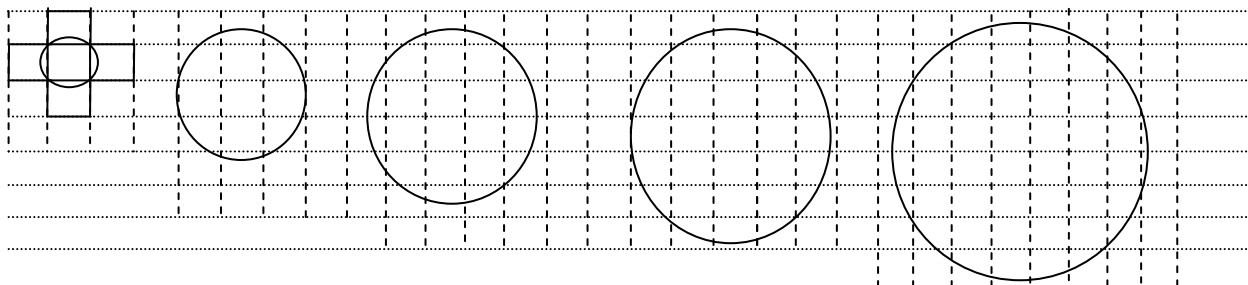

Fig.4 Measuring the area of circles on large paper (ex.: For the first circle $A_{min}=1div^2$ and $A_{max}=5div^2$)

Bold the extreme sides of squares inside (*dashed line*) and outside the circles (*full line*, see fig.5) . Count the number of full squares fitted from inside the circle A_{min} and the number of full squares fitted from outside the circle A_{max} (in the case of fig.5 $A_{min} = 8.0 div^2$ and $A_{min} = 26.0 div^2$).
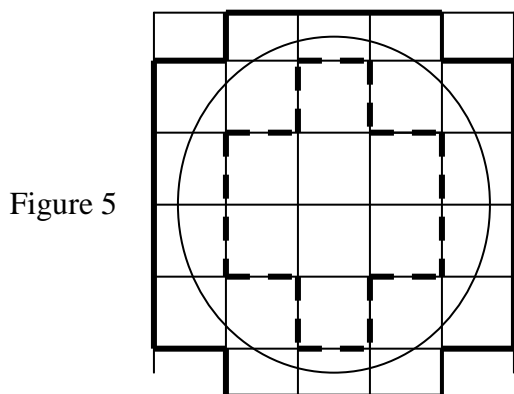
Figure 5

Note_1: *MEASURE the area by counting the number of squares with area 1div².*(*8div² in fig.5*) *We all know that you can determine the circumference or the area by calculations using a well-known formula. This is NOT what you are asked to do in this lab. Here you must MEASURE the circumference by using the measuring unit "div" you created.*

Analysis of results

1. Write the theoretical expression relating the **circumference** of a circle to its **diameter**.
2. From your graph No.1, describe the *type of relationship* between the circumference and its diameter. Does this relationship agree with the theoretical expression ?
3. State the theoretical expression relating the **area** of a circle to its **diameter**.
4. From your graph No2, describe the *type of relationship* between the circle **area** and its **diameter**. Does the relationship you discovered in "4" agree with the theoretical expression ?
5. From the slope of graph No.1 and graph No.3, estimate π_{best}, $\Delta\pi$ and $\varepsilon_\pi\%$. Are those estimations **accurate** and which of them is **more precise**?

Table No1   Measured Data for Diameter, Circumference and Area

No	D_{Best}	ΔD	$\varepsilon\%$	C_{Best}	ΔC	$\varepsilon\%$	A_{MAX}	A_{MIN}	A_{Best}	ΔA	$\varepsilon\%$	D^2_{Best}	ΔD^2	$\varepsilon\%$
1														
2														
3														
4														
5														

Checking for "*Sources of Error*" and distinguishing between *errors* and *uncertainties*

a) If you find *major discrepancies* between your experimental results and expected ones(ex. the expected value falls out and far from the interval of uncertainty), you should start by checking for *mistakes* and making sure that the result of each step does make sense. If no mistake appears, one should say that:
   a-1) the theoretical prediction contains **errors**   or   a-2) the measurements are **inaccurate**.
After this, one has to verify the theoretical calculations and the calibration of measurement devices.

b) If you find *minor discrepancies* between your experimental results and expected ones(ex. the expected value falls out but very close to interval of uncertainty), the chances are that you have underestimated the uncertainties. In this case, you have to check if you should *increase* the measurement uncertainties (the uncertainty interval of result will get larger).

c) If the theoretically expected value falls inside the uncertainty interval your result is **accurate**. But, it may happen that the precision is very low (relative uncertainty of the result is very high, ex. $\varepsilon\% = 60\%$). This means that you have *low quality of measurements and __you are dealing with large experimental uncertainties__*.

In this case, you may increase the precision of result (i.e. decrease its ε%) by improving the measurement procedure (decreasing uncertainty of direct measured parameters) or by changing the method.

Fig.6 Select five object with gradual increase of diameter. Draw a line somewhere around the middle of page. Align the circumference of objects along this line and draw five circles. Cut pieces of string with length equal to diameter and circumference of each circle. Label them as D_1, C_1; D_2,C_2;.....D_5,C_5.
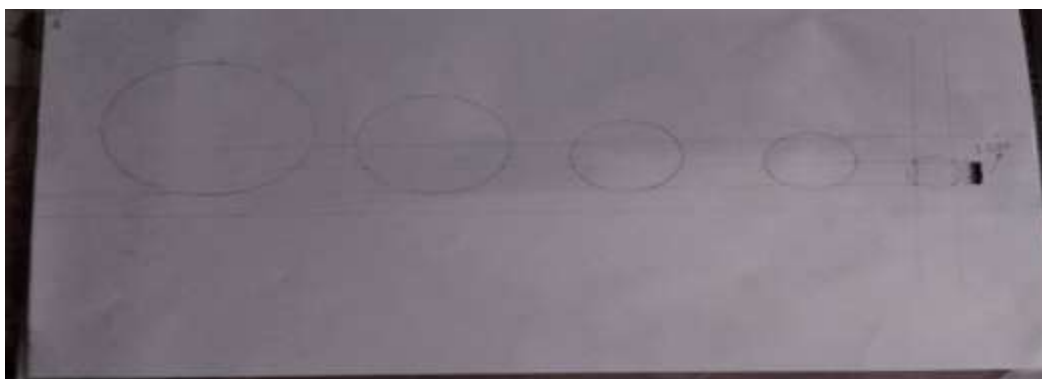

Fig.7 Remove all objects and pieces of string from the paper and erase the writings on it. Then, draw a square inside the smallest circle. Cut a small piece of string or thin wire with length equal to the side of the small square. Label it as 1Div. Next, draw parallel lines at distance 1Div from each other as shown in figure 4.
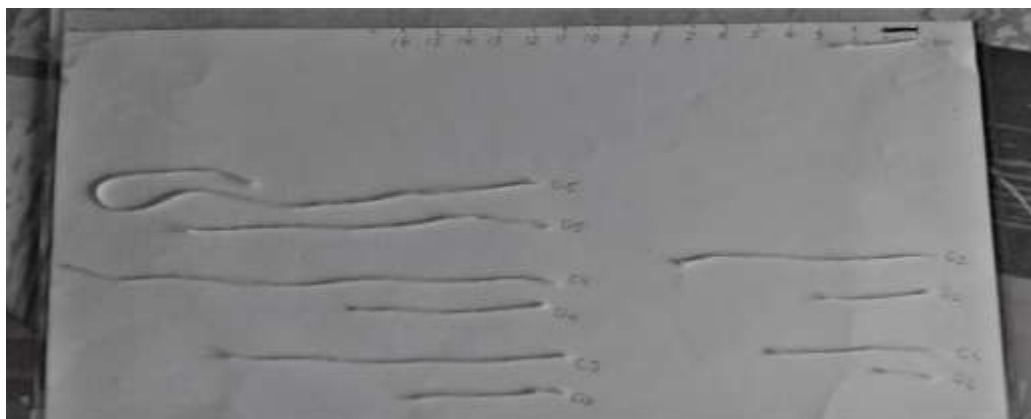

Fig.8 Place all strings on a white paper in order as shown in figure and label them D_1, C_1... D_5, C_5.

　Use the 1Div length of wire (black) to build a ruler with unit length equal to 1DIV at the side of paper.
　Measure the length of string pieces in DIV units by using this ruler.
　Example; the piece of string shown in figure has a length 2.5DIV.

PART B

Objective:

To measure and compare the reaction time of two persons.

Materials: A meter stick.

1. **Procedures:**
   One student keeps a meter stick vertically at the end so that the whole length is down while the other student put his open hand around the **_bottom_** i.e. at 0-*mark of the meter stick*. At a given moment, without notice, the first student leaves the meter and the second student closes his hand to catch it. One measures the *length L dropped down the closed hand*. One repeats 5 times the measurement, get the values **L₁, L₂, L₃, L₄, L₅** and record them in table No2. Next, one calculates the average **L_Av**, the *mean deviation* **ΔL** (i.e. a*bsolute uncertainty) and the two values
   $$L_{max} = L_{AV} + \Delta L \_and\_L_{min} = L_{Av} - \Delta L$$

   Then, by the relation[1] **L = g* t²/2** one calculates the *reaction time* as $t = \sqrt{\dfrac{2*L}{g}} = \sqrt{\dfrac{2*L[m]}{9.8[m/s^2]}}$

   One starts by finding best t-value as $t_B = \sqrt{\dfrac{2*L_B[m]}{9.8[m/s^2]}}$ and follows by calculating the

   maximum value $t_{max} = \sqrt{\dfrac{2*L_{max}[m]}{9.8[m/s^2]}}$,   the minimum value $t_{min} = \sqrt{\dfrac{2*L_{min}[m]}{9.8[m/s^2]}}$

   and $\Delta t = \dfrac{t_{max} - t_{min}}{2}$

2. Repeat the measurements for the other student.
3. Write reaction times as t=t_b±Δt; ε_t=(Δt/ t_b)*100%
4. Draw uncertainty intervals in comparison graph.
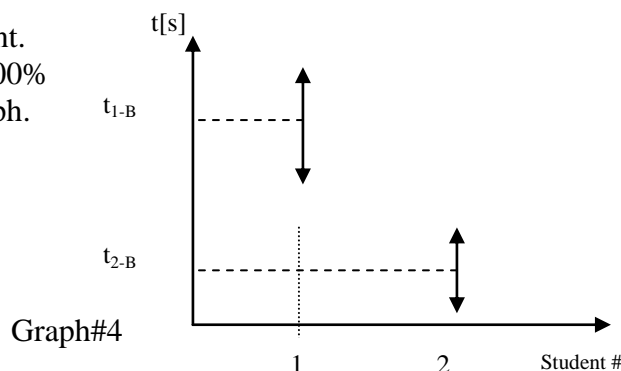5. Compare the results from the graphs and state
   who is reacting quicker.

Graph#4

Table No2

No	L	L_Best	ΔL	ε_L%		t_Best	Δt	ε_t%
Student 1	. . . . .	.	.	.				
Student 2	. . . . .	.	.	.				

Comments:
1. Explain how you decided that the two results are essentially different or not.
2. Where appears the precision of measurement on the graph?
3. How could you improve the precision of the measurements?

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[1] This relation we will find later during the course.