K.Angoni, Physics Department, Vanier College

Measurement of the ratio "e/m" (*charge to mass*) for the Electron

Objective: To measure the ratio q/m of the charge of the electron (q_{el}=-e) to its mass (m_{el}).

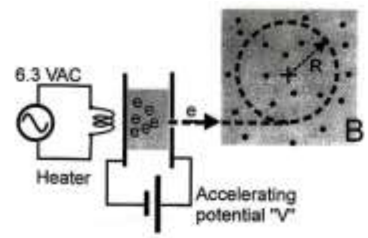
Introduction

In this experiment, one uses a hot filament (heater) to generate free electrons. Next, one accelerates them in an "*electron gun device*" through a potential difference V. By applying the principle of energy conservation for an electron (*moving from "-" to "+" plate*) inside electric field, ΔE=ΔK+ΔU=0 one gets ΔK = -ΔU = -Δ(q*V)= -q*ΔV = - q_{el} *[V - 0] = -(- e)*V = e*V . Finally, by assuming that the electron starts its "*directed motion*" from rest one gets :

$$\tfrac{1}{2} m v^2 = e V \qquad (1)$$

Next, the beam of electrons enters inside an uniform magnetic field $\vec{B}$ perpendicular to their velocity $\vec{v}$. In these circumstances, the magnetic force on each electron has the magnitude

$$F_B = e\boldsymbol{v}\boldsymbol{B} \qquad (2)$$



As the net (where F_{net}=F_B) force is perpendicular to its velocity, the electron moves on a circle (with radius R), at constant speed v and an acceleration $\boldsymbol{a_c = v^2/R}$. Since the net force is centripetal, from Newton's second law

$$F_{Net} = F_B = m\, v^2/R \qquad (3)$$

Substituting $\boldsymbol{F_B}$ from (2) to (3) one gets $\quad evB = m\, v^2/R$

and $\quad (e/m) = v \,/\, B*R \qquad (4)$

After getting the square of both sides at (4) $(e/m)^2 = v^2 / B^2*R^2$ one substitutes $v^2 = 2(e/m)V$ from (1) and gets $(e/m)^2 = 2(e/m)V / B^2*R^2$ and

$$e / m = 2 V / R^2B^2 \qquad (5)$$

The expression (5) can be rewritten as $\quad \boldsymbol{V = [(B^2/2)*(e/m)]*R^2} \qquad (6)$

If one draws the graph of ***accelerating voltage V** vs **R²*** of circle it should be a *straight line*

passing by origin with a slope $\quad Slope = (B^2/2)*(e/m) \qquad (7)$

Knowing the slope, one can calculate $\quad \boldsymbol{e/m = Slope * (2/B^2)} \qquad (8)$

The magnetic field used in this experiment is created by a set of Helmholtz coils. By using the geometry of the circular coils with radius R and N turns, one may find that the strength of magnetic field *in the central area of two coils set* is given by expression $B=(4/3)^{3/2}\mu_o NI/R$. In case of our Helmotz sets (*given number of turns N and radius R*) this expression is simplified to :

$\boldsymbol{B[T] = 7.80 \times 10^{-4} * I[A]}$ (for old version) and $\boldsymbol{B[T] = 7.40 \times 10^{-4} * I[A]}$ for new version apparatus (9)

By substituting expression (9) into relation (8), one gets:

$$\boldsymbol{e/m\,[C/kg] = Slope * 2 / (7.8\times10^{-4} * I)^2} \quad \textit{and} \quad \boldsymbol{e/m\,[C/kg] = Slope * 2 / (7.4\times10^{-4} * I)^2} \qquad (10)$$

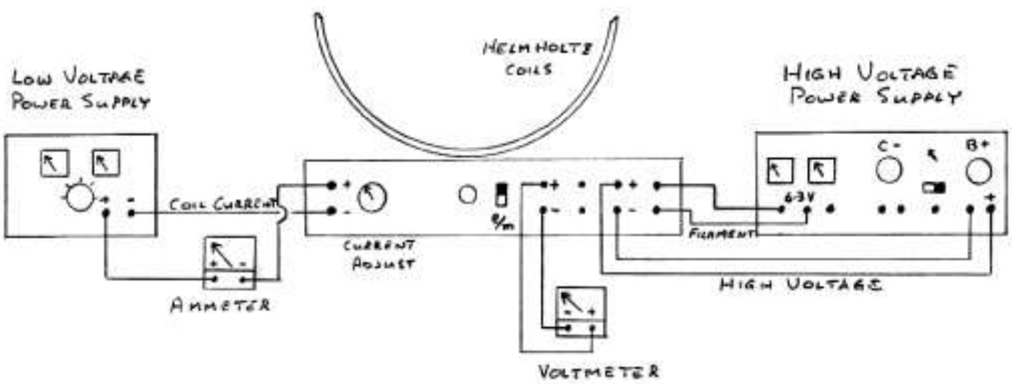
where I is the current passing through the coils.

General Adjustments

1) In this experiment, one must take into account the effect of the magnetic field of earth. The vertical component is not a problem because the effect cancels out over the orbit of the electrons. To cancel the effect of the horizontal component, ***the plane of the Helmholtz coils must be aligned parallel to it.*** You will *u*se a magnetic compass to align the plane of coils with the horizontal component of the earth's field. This must be done with the current in the coils turned off. Also, move the apparatus to the edge of the workbench and move other instrument at least a foot away from the coils.

2) Connect the apparatus to the low voltage power supply, the high voltage power supply (HVPS), a voltmeter and ammeter as shown in the diagram below. Make sure both power supplies are turned off and the control knobs are turned to zero. Use the highest scale on both meters until you know the size of the voltage and current you will be measuring.
3) Put the DC ON switch of the HVPS to "STANDBY" and turn the B+ knob to zero. Switch the AC power on and wait a minute or two for the filament to start glowing.

4) Turn the current adjust switch for the Helmholtz coils to zero and set the selection switch to e/m. Turn the control knob on the low voltage power supply to zero, and then turn the supply on.



MAKE SURE THE FILAMENT OF THE ELECTRON GUN IS CONNECTED TO THE **6.3 VAC** TERMINALS OF THE HIGH VOLTAGE POWER SUPPLY. **HAVE YOUR TEACHER CHECK ALL CONNECTIONS BEFORE TURNING ON EITHER POWER SUPPLY.**

5) Switch the HVPS from "Standby" to "DC ON" and gradually increase the B+ control until the voltmeter reads 150 V. A green beam should appear in the glass envelope as the electron beam collides with the mercury vapour inside. Adjust the focus control to obtain as sharply defined a beam as possible.

NOTE: At this point, we will have to turn off the lights in the lab to make the beam more visible. You can also use the cloth hood supplied to shield out excess light. If the High Voltage Power Supply starts blinking, switch it to Standby and call over the teacher.

6) Turn the current adjust on the apparatus about half way up. Switch the low voltage power supply(EPSCO) to 0 - 16 V , turn it on and raise the current through ammeter until the electron beam is curved into a circle filling about 2/3 of the glass envelope. Check to see that the electron beam makes a closed circle bringing it right back between the two struts which take current to the filament. If it does not, **carefully and gently** rotate the glass envelope until the beam does form a closed circle. Note the shape of the path of the beam when it does not close.

Part A: measuring e/m value

1) Set the current in the coils to a ***fixed value (I=0.8 to 2A)which means keeping B magnitude constant. For this*** adjust both the voltage control on the low voltage power supply and the current adjust knob on the e/m apparatus. The beam should form a fairly small circle. Watch the ammeter carefully when you do this to make sure it does not go off scale. ***Record the value of the current and keep it constant during measurements. Record the corresponding high voltage value.***
In this experiment, you will increase gradually the accelerating ***voltage*** (read on the voltmeter) ***and for each value you have to measure the radius of circular electron orbit***.

Measuring the radius:
The radius of the circular beam is measured using the illuminated scale behind the glass envelope. Since it is difficult to ensure that the zero on the scale is at the center of the circle, read the position of the left side of the circle, r_L, and the position of the right side of the circle, r_R, and calculate the average radius:

$$R = (r_L + r_R) / 2 \qquad (11)$$

The scale has an anti-parallax mirror behind it. When making the measurements, position your head so that the image of the beam in the mirror is aligned with the beam itself and the scale goes across the middle of the circle as shown below:

When you are properly aligned, the markings in the part of the scale you are using appear sharp and somewhat darker than the rest of the scale. If you are not properly aligned, they will be bright and fuzzy or doubled. <u>Record 3-5 measurements of r-value for each voltage</u>.

2) Keeping ***I constant***, increase the voltage in ***five to six steps*** from 150 V to 200 V. **Do not exceed 300 V!** Increasing the voltage will increase R-value. At each value of V, masure three times the radius "R" of the electron path, record the V value and make sure that I is still at its fixed initial value. Record all values. Calculate the best estimation for radius R and its uncertainty.

Table #1 I=.....

V_{Source}	V_{Device}	R_1	R_2	R_3	R_{Best}[**cm**]	ΔR[**cm**]	R_{Best}[**m**]	ΔR[**m**]	$(R_{Best})^2$[**m²**]	$\Delta(R^2)=2*R_{Best}*\Delta R$ [m²]

Based on recorded data, draw the graph#1 **$V = V(R^2)$** . Fit this graph by a straight line passing by the origin and get out its slope(see expr.6,7).Use this slope and the expression (10) to calculate the ratio e/m for electron. Use the uncertainties of measurements for V and R, to find uncertainty of e/m value. Compare the result to the officially accepted value for e/m.

Part B: Verifying the expression for the radius of circular orbit (keep voltage V constant)

At first, one uses the expression (4) to get the relation $R = (m/e)^* \upsilon^*(1/B)$ (12)

So, the graph R=R(1/B) is a straight line passing by origin; its slope is $Slope = (m/e)^* \upsilon$ (13)

1) Reduce the accelerating voltage and keep it to a _fixed value_ (150 to 200 V). Then vary the current over a wide range (1 to 3*A*) to obtain ***five to six I-readings***. At each **value of I**, make sure that the voltage is at the _chosen fixed value_ and **measure the radius***(3-5 times)* of the circular beam. Record I, R values in table#2. Use expr.(9) to calculate corresponding B-values for each I-value. Then, draw the graph#2 R **= R(1/B),** fit it by a straight line passing by origin and get ***its slope***. From expression (1) one can get the speed of electrons as

$$\upsilon = [(e/m)^*2V]^{1/2} \qquad (14)$$

From expression (13) one get it as $\upsilon = \text{Slope} ^*(e/m)$ (15)

By equalizing the right side of expression (14) to that of (13) one get

$$\text{Slope} ^*(e/m)= (e/m)^{1/2} {}^*(2V)^{1/2} \quad \text{or} \quad (e/m)^{1/2} = (1/\text{Slope})^*(2V)^{1/2}$$

Finally, one can get out the expression $(e/m) = 2V / (\text{Slope})^2$ (16)

In this experiment, one starts by drawing the graph R=R(1/B) and finding its slope. Next, one uses this slope at expression (16) to find the ratio (e/m) for electron.

Table #2 V_{Device} =.......... (V_{Source}=....)

I	B	1/B	R_1	R_2	R_3	R_{Best}[cm]	ΔR[cm]	R[m]	ΔR[m]

Important note: *Normally one should refer to the voltage provided by the source terminals but these are old devices and V_{Device} may be not an accurate value. So, one suggests to record both values of voltage at device and at source terminals. If they are different and V_{Device} gives inaccurate results, repeat the calculations with V_{Source} and refer to the voltage that provides more accurate results.*

Analysis:

- Calculate the ***best estimation, absolute uncertainty and precision*** for e/m from part A and part B.
- Do the uncertainty intervals overlap to each other?
- Calculate the ratio e/m by using the values of e and m for the electron in your text book.
  Use precision and accuracy to compare your result to this value.

Conclusion: Does your measured value of e/m agree with the text book value? What is the main source of random uncertainty? Is there evidence for a systematic error in your measurements?