ELECTROMAGNETIC INDUCTION

10.1 Experimental Observations

- A conductor rod that slides at constant velocity $\vec{v}$ over two conducting rails connected to a galvanometer is part of a closed <u>*passive circuit*</u> (***without efm***); *there is no current in such a circuit*. But, if the rod *moves inside a magnetic field*, the galvanometer shows the presence of current in the circuit (fig.1). So, "<u>*a current appears in a conductor if it is moving inside a magnetic field*</u> ". One can explain this current by the motion of charges (*free electrons*) inside the conductor (at velocity $\vec{v}$) with respect to magnetic field $\vec{B}$(see fig.1).

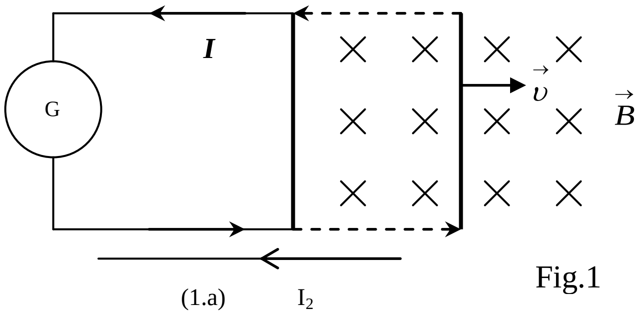


Fig.1

The magnetic force $\vec{F}_B = q^* \vec{\upsilon} \times \vec{B}$ (1) gives to these free electrons ($q = -e$) inside rod a *drift motion* directed "*downside* ". So, a current (*virtual q>0*) directed "*upside"* passes through circuit <u>***as long as the rod moves***</u>.

- Consider now that the rod is at rest and a current I_2 pass through another wire located close by as shown in figure 1.a. We know that this current builds a magnetic field around it and this field will penetrate in the area limited by the closed part of circuit. The observations show that, from the moment (t = 0) when the current starts to flow through the second wire and <u>*during a very **short** interval*</u> of time "***Δt "*** the galvanometer needle is not at zero; i.e. "<u>*there is a current in the circuit*</u>". Then, for t > ***Δt,*** even though I_2 is not zero, there is no more current in circuit. This effect was observed for the first time by Joseph Henry.

-It is clear that, in the two cases, the appearance of an electric current in circuit is due to "***some kind of action from the magnetic field "***. Before going into a better understanding of these phenomena, keep in mind that a current means *charges in motion* and this requires the ***presence*** of ***an electric field.*** So, one might figure out that, in a way, the <u>*action of magnetic field consists to the creation of an electric field $\vec{E}$ inside the conductors of circuit; one says that this is a current due to **electro-magnetic induction**.*</u>

- Also, one may say that, as a current passes through circuit, there is an ***emf in action*** inside the electrical circuit. It remains to define the ***magnitude*** and the ***direction*** of this induced ***emf***.

a) Consistent observations showed that the ***magnitude*** of the ***inducted emf*** is related to the <u>***change***</u> of <u>***magnetic flux***</u> ***"$\Delta\Phi_B$ "*** <u>***inside the closed circuit***</u>. The ***flux*** Φ_B of a magnetic field through a loop, (or a closed circuit) is defined as

$$\Phi_B = \vec{B} * \hat{n}A = \vec{B} * \vec{A} = BA\cos\theta \qquad (2)$$

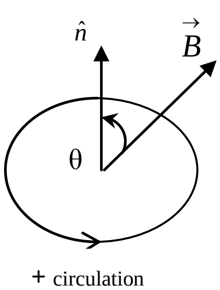
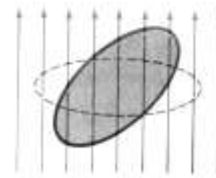


Fig.2

$\vec{A} = \hat{n}A$ is the ***area vector*** of loop. ***A*** is the area of loop; $\hat{n}$ is a unit vector with tail at loop center and directed perpendicular to loop plane such that ***angle to*** $\vec{B}$ ***at t = 0 is $\theta < 90^o$***. Then, after aligning the thumb on $\hat{n}$-direction, the right hand rule defines <u>***the positive sense of circulation***</u> in loop following the direction of curled fingers. *$\vec{A}$ is introduced only for flux calculation purposes; it has not any physical meaning.*

Note that for *an uniform field* $\vec{B}$, one can change the flux Φ_B by changing the loop area (fig 3a) or by changing the angle θ (fig3.b). The rotation of loop (*change of θ*) is the situation generally met in practice (AC generators, electric motors,..).

Fig3.a  Fig3.b 

The *unit of* ***magnetic flux*** in SI system is the Weber (Wb). From the expression (2), one may find that

$$1Wb = 1T * 1m^2 \tag{3}$$

During this introduction for flux concept, we considered a loop inside an uniform magnetic field. In a general situation, the magnetic field vector $\vec{B}$ may change from one location to another and the flux is defined as the sum of all local contributions. This brings to the general formulae for the magnetic flux

$$\Phi_B = \iint_{Loop\_area} \vec{B}\, d\vec{A} \tag{4}$$

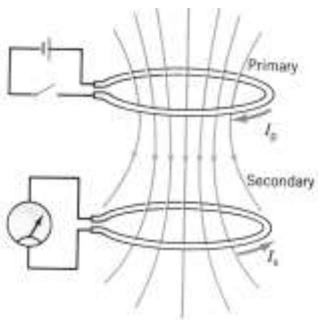


Fig.4

One may change the flux $\boldsymbol{\Phi_B}$ through a fixed shape loop by changing the magnetic field $\vec{B}$. In fig.4, the magnetic field built by the current in the *primary circuit* passes through the *secondary circuit* area.

One can change the magnitude of $\vec{B}$ inside the area of secondary circuit and its flux $\boldsymbol{\Phi_B}$ by changing the current ***Ip*** in primary circuit.

- Michael Faraday discovered that the ***magnitude*** of ***emf*** inducted in a conducting loop is ***proportional to the change rate of the magnetic flux*** through the loop

$$\varepsilon \sim \frac{d\Phi_B}{dt} \tag{5}$$

As $$\frac{d\Phi_B}{dt} = \frac{d(BA\cos\theta)}{dt} = \frac{dB}{dt}A\cos\theta + B\frac{dA}{dt}\cos\theta - BA\sin\theta\frac{d\theta}{dt} \tag{6}$$

the ***Faraday law*** shows that there are three ways to induct an E&M ***emf*** (B *change*, A *change*, θ *change*).
Important: *In practice, one deals mainly with the change of the first and the third factors " B ; θ ".*

b) The ***direction*** of inducted ***emf*** in a circuit is defined by ***Lenz law***. This is a particular expression of action - reaction behaviour in nature. This law says: **While** an **exterior action** is **changing** the **flux** $\boldsymbol{\Phi_B}$**,** the **inducted** ***emf*** and the **induced current** in conducting loop have such a **direction** that produces an additional **magnetic field** which **tents** to ***keep the previous* value of *magnetic flux*** through the circuit**.**

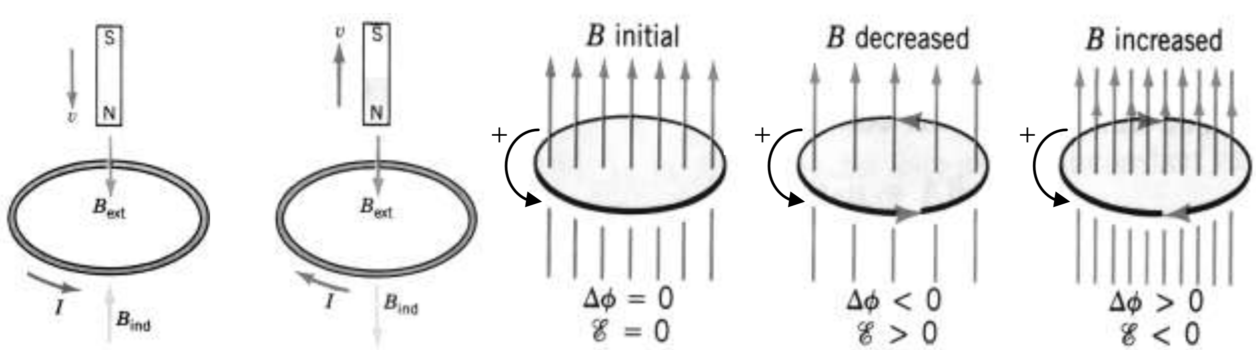


Fig5.a (b) (c) t = 0; $\Phi > 0$ (d) t = dt (e) t = dt

In fig5.a the external action is *increasing* the flux; the inducted current "***I*** " has such a direction that the related field B_{ind} (directed ***upside***) works <u>against the exterior action</u> (*by decreasing the net field*). In fig5.b the external action is *decreasing* the flux; the inducted current "***I*** " has such a direction that the related field B_{ind} (directed ***downside***) works against the exterior action (*by increasing the net field*).

In fig5.c the circuit is in a ***<u>stationary state</u>***(*constant magnetic field, a positive flux, no current*) and there is no flux change; so, $\varepsilon = 0$. In fig5.d an external action is decreasing B-magnitude ($\Delta\Phi_B < 0$). The ***inducted emf*** (and current) has a direction which tents to compensate the external action by creating a "B_{ind}" directed "up". In fig5.e the external action is increasing B field ($\Delta \Phi_B > 0$); the ***inducted emf*** and ***current*** follow direction "-" so that the inducted magnetic field B_{ind} is directed "down "(to decrease the net magnetic flux).

- Now, let's use the figures 5.c,d,e to include the Lenz law into expression (5). At t = 0, there is a field $\vec{B}$ through the circuit area. One starts by drawing $\vec{A}$-vector parallel to $\vec{B}$. So, ***at t = 0, Φ_B*** is ***positive*** and ***one fixes*** what is considered as ***positive sense*** (fig5.c) for the ***current and emf*** in this circuit in conformity to the direction of $\vec{A}$ - ***vector.*** Then, at a moment t = dt, the figures 5.d,e show that the inducted ***emf " ε "***, has the ***opposite of sign*** compared to that of flux change ***$\Delta\Phi_B$.*** So, one gets the Faraday-Lenz law

$$\varepsilon = -\frac{d\Phi_B}{dt} \tag{7}$$

In case of a ***coil*** with ***N turns*** the expression (7) becomes

$$\varepsilon = -\frac{d(N\Phi_B)}{dt} \tag{8}$$

<u>Precisions</u>

a) The ***source*** of the ***inducted emf*** is the magnetic field itself. This type of emf is ***distributed along*** the ***whole loop*** and its ***action*** is ***distributed*** around the ***whole loop***.

b) As mentioned previously, the field lines are used to visualise information about direction of vector $\vec{B}$ and its magnitude(their density at a given location is proportional to $\vec{B}$ magnitude) but, similar to $\vec{A}$ - ***vector,*** they do not have physical meaning. However the ***magnetic flux is a measurable basic physical parameter***.

c) The ***change*** of ***magnetic field*** $\vec{B}$ ***induces*** an ***electric field*** $\vec{E}$ which ***generates*** a current into the loop. This current induces " *another* " $\vec{B}$ - field in the surrounding space which brings a change of the total magnetic flux and so on…This short comment is sufficient to figure out that there is a close relationship between ***<u>the changes</u>*** of magnetic and electric fields. Actually, the electromagnetic induction acts in a much larger frame. It is at the *origin of all types of existing electromagnetic waves* (light, TV, radio, …).

10.2 GENERATORS OF ELECTRIC CURRENT

An *electric generator* is a **coil** with ***N turns***[1](or N loops) that rotates at *constant angular velocity "**ω**"* inside an *uniform magnetic field* (see Fig.6) and generates an *inducted emf* by changing the angle " $\theta = \omega * t$ ". One takes $t=0$ when the coil plan is perpendicular to $\vec{B}$ and assigns the area vector $\vec{A}$ parallel ($\theta=0$) to $\vec{B}$. So, at $t = 0$, the flux through the coil $\Phi_B = N(\vec{B} * \vec{A}) = NBA\cos\theta = NBA$ is *positive and maximum.*

[1] The magnetic flux through the coil is $\Phi_{coil} = N * \Phi_{one\text{-}loop}$

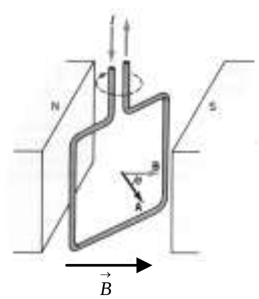


Fig.6

When the coil starts to rotate (*say CW as in fig*) the ***angle θ*** between vectors $\vec{A}$ and $\vec{B}$ ***changes*** and the magnetic ***flux changes,*** too. At time " *t* " the angle between them is " $\theta = \omega * t$ " and the flux is *decreased (see fig 6) to*

$$\Phi(t)_B = N(\vec{B} * \vec{A}) = NBAcos(\omega t) \tag{9}$$

The relation (7) tells that, during coil rotation, the induced ***emf*** in coil is

$$\varepsilon(t) = -\frac{d\Phi(t)_B}{dt} = NBA\omega sin(\omega t) \tag{10}$$

At angles $\theta < \pi/2$, ***emf*** *is positive* **and builds** an *induced* current which flows along ***the positive*** direction of ***circulation on the coil*** (as shown in figure).

From expression (10) one can see that the ***inducted emf*** is <u>maximum</u> $\varepsilon_0 = NAB\omega$ (11)

<u>*when θ = π/2*</u> i.e. when $\vec{A}$ is perpendicular to $\vec{B}$ which means the coil plane is parallel to field $\vec{B}$.
So, one can rewrite the expression (10) in form $\varepsilon = \varepsilon_0 \sin(\omega t)$ (12)

This expression tells that the induced *emf* and induced *current* in coil *change* their sign (i.e ***direction)*** with time in *harmonic way*. So, an <u>***alternating harmonic emf***</u> (fig.7) (and ***current)*** is induced in coil.
In a AC generator, the current goes out of coil (*into the external circuit*) via a set of two brushes in contact to the exterior of two sliding rings. Those rings are welded at coil ends and rotate together with the coil (fig.8).

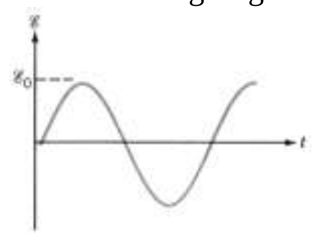


Fig.7

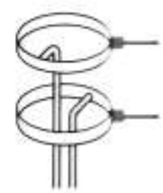


Fig.8

The ***commutator*** is a set of ***two half-rings*** welded to coil wires (from inside, Fig.9) and sliding over two *fixed brushes* (from outside) while rotating with the coil. At $t=0$, $\theta=0$ and $\varepsilon = 0$. For a *CW* rotation, when $\theta = 90^0$ ($\varepsilon = +\varepsilon_o$) the emf is positive and current is sent outside by *right side brush*, as shown in fig.9.

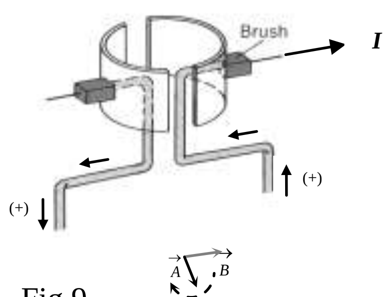


Fig.9

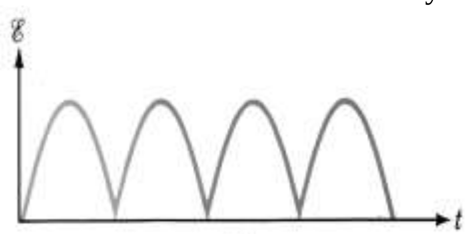


Fig.10

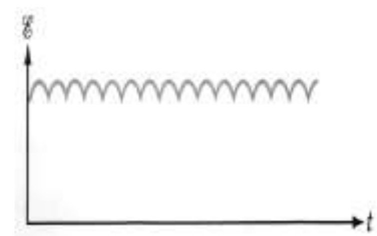


Fig.11

While the coil is rotated from $\theta = 90^0$ versus $\theta = 180^0$, the value of "**ε**" decreases but it remains positive. For $\theta = 180^0$, **ε** = 0 and for $\theta > 180^0$, **ε** < 0 (the direction of emf and current in coil becomes " - " versus the positive sense tied to area vector $\vec{A}$. This provides a current directed to the "*right side brush*", yet.
So, the ***brush to the right*** gets always the " ***positive terminal*** " of ***emf*** (generated by the coil and sends it out versus the external circuit. This device *transforms the alternating emf into a DC pulsed emf* (Fig.10).
Wheatstone improved the quality of pulsed DC current (see Fig.11.) by using a system of multiple coils and commutators oriented at different angles. This improvement brought to DC generators one uses today.
(https://phet.colorado.edu/sims/cheerpj/faraday/latest/faraday.html?simulation=faraday)

10.3 MOTIONAL EMF

-Let's consider a conducting rod with length " *l* " travelling at constant ***velocity*** $\vec{\upsilon}$ *perpendicular* to an uniform magnetic field (Fig.12). Due to magnetic force action ($\vec{F}_B = -e\vec{\upsilon}\,x\,\vec{B}$), the free electrons *move* to the *lower end* and leave a net *positive charge* at the *upper end* of the rod. This distribution of charges builds up an *electrostatic field* $\vec{E}_0$ *directed downward*. The related electrostatic force acting upon the electron $\vec{F}_E = -e\vec{E}_0$ is directed upside i.e. in opposite sense of $\vec{F}_B$ ($\vec{F}_E \uparrow\downarrow \vec{F}_B$). The magnitude of electric force $\vec{F}_E$ increases with the increase of charge concentration at rod's ends. After a short interval of time it becomes equal to magnitude of $\vec{F}_B$ and electrons don't move any more versus the rod's lower end because the exerted net force becomes zero. This way a steady state equilibrium is set up. Starting from this moment

$$eE_0 = e\upsilon B\_i.e.\_E_0 = \upsilon B \qquad (13)$$

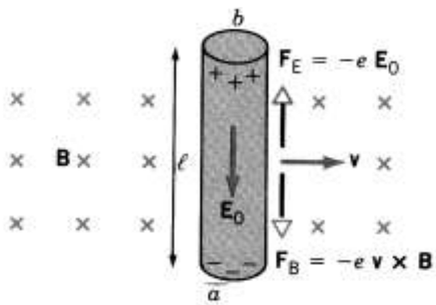


Fig.12

At steady state equilibrium there is *a potential difference* between rod's ends

$$V_b - V_a = E_0 l = B\upsilon l \qquad (14)$$

By referring to the terminology used for electric circuits, one should say that this ***potential difference*** is due to the *presence of an* ***emf*** inside the rod. This kind of ***emf*** is known as ***motional emf***. So, if a conductor with length " ***l*** " travels perpendicular to a magnetic field at speed " υ", there is a motional emf born inside it; this emf builds a potential difference between two rod extremes

$$\varepsilon_{motion} = V_b - V_a = B\upsilon l \qquad (15)$$

- What is the source of ***motional emf*** ? To answer this question one must look closer the *external work spent during the shift of electrons*. There are two external (*versus electric field*) sources that act on the rod; the *magnetic field and the external force that makes the rod* move to the right.

The ***magnetic force is always*** perpendicular ***to velocity which means that it is perpendicular to displacement at any moment.*** Hence, *at any moment, their dot product is zero; so a magnetic force cannot achieve work*. The external force achieves positive work(*why?*); so, it does provide work. From the energetic point of view, one may conclude that, the ***work spent by the external force*** goes to ***shift the rod right side (motion at constant velocity)*** and to build up the ***motional emf*** (15) inside the rod**.** Note that, although the *magnetic force does not work by itself,* it acts as a kind of ***intermediary agent*** which, by pushing electrons downside, conveys a part of external work versus "***electrical energy*** ".

-What happens while the rod is sliding at constant speed "***υ***"over a set of metal rails that form a closed circuit as in figure#1? By assuming that the resistance of the rod is negligible compared to the resistance "***R***" of the rails, one get a current " $\boldsymbol{I = \varepsilon / R}$ " flowing through circuit. This current will tent to deplete the electric charge at rod ends. As a results, the E- field in rod would decrease while the magnetic field would pump more electrons at lower terminal of the rod to keep the same amount of charges at terminals. The result would be a ***dynamic equilibrium*** with a current " ***I*** " through circuit (*rails & rod*) , a constant motional emf ($\boldsymbol{\varepsilon = B\upsilon l}$) at rod's ends and an amount of power "$\boldsymbol{P_{th} = I^2 R}$ " dissipated in circuit as heat.