14.1 R,L,C IN SERIES IN AN AC CIRCUIT.

The scheme in figure 1 shows a resistor R, a capacitor C and an inductor L ***in series*** connected to the terminals of an ***AC source***. As the current is the same for all elements in series, one starts by writing

$$i_R = i_C = i_L = i = i_0 \sin \omega t \qquad (1)$$

As the voltage and current have a phase shift in AC circuits, in general, $\upsilon = \upsilon_0 \sin(\omega t + \phi)$ (2)

The values of υ_0 and ω are fixed by the source while the ***amplitude of current*** "i_o" and the ***phase shift "φ"*** between the *voltage* and the *current* depend on the numerical values of R, L, C in a circuit. Let's refer to an instant "*t*" when the " *current* ***phasor*** " has the direction shown in figure 2 and let's label as υ, υ_R, υ_C and υ_L the voltage at the source, resistor capacitor and inductor. Next, by applying the second rule of Kirchhoff along the ***current direction,*** one get :

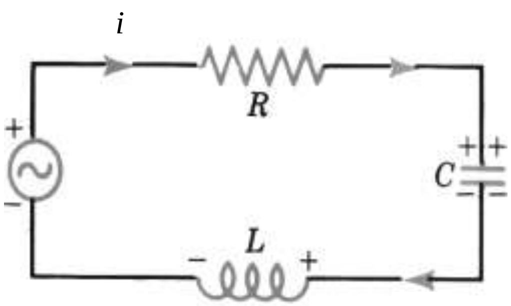

Figure 1

$$\upsilon - \upsilon_R - \upsilon_C - \upsilon_L = 0 \Rightarrow\Rightarrow\Rightarrow \upsilon = \upsilon_R + \upsilon_C + \upsilon_L \qquad (3)$$

As the phase shift between the current and the voltage is different for each element, in general, there is phase shift between terms in expression (3) and one must consider it as a phasor relationship.

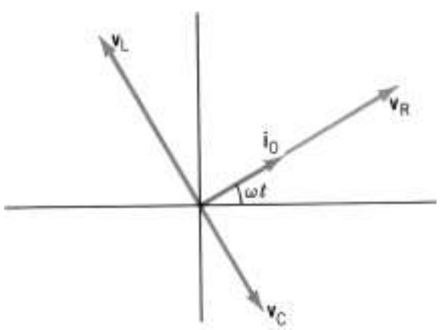

Figure 2

- So, one draws the phasors diagram shown in fig.2 as follows:

1. Start by drawing the current phasor at the moment "t" ($\varphi_i = \omega t$).
2. The phasor for **R-voltage** has zero phase shift versus current. So, this phasor is drawn along the same direction as *i-phasor*.
3. The phasor for **C- voltage lags** by **π/2** vs. *i-phasor*.
4. The phasor for **L-voltage** is **ahead** by **π/2** vs. *i-phasor*.
Then, the *phasor for applied voltage* by source will be the sum of three *potential drops' phasors* (see fig.3)

$$\upsilon_S^{ph} = \upsilon_R^{ph} + \upsilon_C^{ph} + \upsilon_L^{ph} \qquad (4)$$

Note that the expression (4) is a "*vector sum of vectors"* with magnitude equal to the maximum value of corresponding voltage ($\upsilon_{0R}, \upsilon_{0C}, \upsilon_{0L}$). As the C and L phasors align in opposite direction, their "*vector"* sum will be a *phasor with magnitude*[1] $|v_{0L} - v_{0C}|$. So, the sum of three phasors is reduced into the sum of two phasors (figure 3). Now one can express the *amplitude of applied voltage* through the sum of two other phasors' amplitudes as

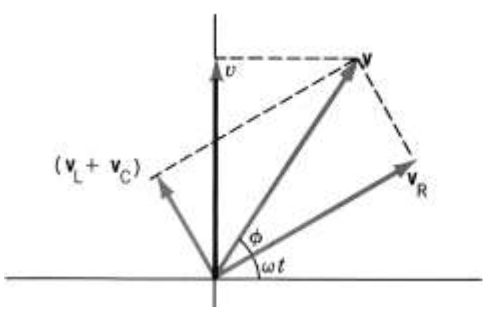

Figure 3

$$\upsilon_0^2 = \upsilon_{0R}^2 + (\upsilon_{0L} - \upsilon_{0C})^2 \qquad (5)$$

Using the relations between the ***maxima values*** of *instantaneous voltage and instantaneous current* through R, C and L, relation 5 transforms to

$$\upsilon_0^2 = (i_0 Z)^2 = i_0^2 [R^2 + (X_L - X_C)^2] \qquad (6)$$

The quantity

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad (7)$$

is called the ***impedance*** of R, C, L in **series circuit**. This quantity

[1] To avoid ambiguities, we have assumed that $\upsilon_{0L} > \upsilon_{0C}$.

determines the amplitude of current in an AC circuit; its unit is ohm. Then, the relation (6) can be transformed to Ohm's law relating ***the amplitudes*** of ***applied voltage*** and the ***current*** in the circuit

$$\upsilon_0 = i_0 * Z \qquad (8)$$

By multiplying by 0.707 both sides, one gets the Ohm's law for ***rms*** values $\quad V = I * Z \qquad (9)$

- Knowing υ_0 of AC source , one may use (8) to calculate the ***amplitude of current*** " $\boldsymbol{i_0}$ " in circuit. Also, from phasor diagram in fig.3, one can find the *phase shift* between *source voltage* and *current* in circuit

$$tan\emptyset = \frac{|v_L^{ph} + v_C^{ph}|}{|v_R^{ph}|} = \frac{v_{0L} - v_{0C}}{v_{0R}} = \frac{i_0(X_L - X_C)}{i_0 R} = \frac{X_L - X_C}{R}$$

So, $$\tan\phi = \frac{X_L - X_C}{R} \qquad (10)$$

If $\mathbf{X_L = X_C}$ the phase shift **φ = 0,** i.e**.** the current in circuit and the driving potential are all time in phase.
If $\mathbf{X_L > X_C}$ the phase shift **φ > 0** which means that the driving ***potential is ahead of current***.
If $\mathbf{X_L < X_C}$ the phase shift **φ < 0** which means that the ***current is ahead of driving potential***.

14.2 RESONANCE IN A CIRCUIT WITH RLC *IN SERIES*.

- The ***impedance "Z*** " defines the characteristics of current passing through the circuit when an AC voltage applies on it. One studies the "*average behaviour* " of circuit through ***rms*** values for current, voltage and power. If the ***rms voltage*** of source is ***V***, the ***rms current I*** in a R,L,C <u>*in series*</u> circuit is

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} \qquad (11)$$

The *rms* **I** value ***depends on the frequency*** of applied AC voltage because X_L and X_C depend on ω.
As seen from (11), there is a maximum ***rms current*** in circuit when $X_L - X_C = 0$; $\quad I_{\max} = \frac{V}{R} \qquad (12)$

As $X_L = X_C \rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0^2 = \frac{1}{LC} \Rightarrow\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$, the ***frequency of applied voltage***

which produces ***I_{max} (i.e. a resonance)*** in circuit is $\quad \omega = \omega_0 = \frac{1}{\sqrt{LC}} \qquad (13)$

Remember that $\omega_0 = \frac{1}{\sqrt{LC}}$ is the *natural circular frequency* of a C, L circuit without damping(R = 0).

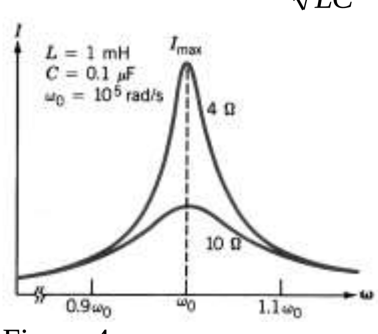

Figure 4

The graphs in fig.4 show the evolution of ***rms current*** in circuit when the *frequency of applied AC voltage is close to the resonance frequency of circuit* $\boldsymbol{\omega_0}$. Note that the relation (11) shows that ***at resonance***, an AC circuit behaves as if containing only a resistor. At ***resonance*** φ = 0 and the ***voltage*** and ***current*** are ***in phase*** which means that the *phasors* of source voltage and current are aligned following the same direction at any moment of time.
As shown in fig.4, for the same C, L values, the ***resonance curve*** of a L, C, R circuit becomes ***sharper*** (fig.4) for ***smaller values of R***.

14.3 POWER IN AC CIRCUITS

- The instantaneous power delivered by an AC source in a circuit is a function of time

$$p = i * \upsilon = i_0 \sin(\omega t) * \upsilon_0 \sin(\omega t + \phi) = (i_0 \upsilon_0) \sin(\omega t)[\sin(\omega t)\cos\phi + \cos(\omega t)\sin\phi)] =$$

$$p = i_0 \upsilon_0 [\sin^2(\omega t)\cos\phi + \sin(\omega t)\cos(\omega t)\sin\phi] = i_0 \upsilon_0 \{\sin^2(\omega t)\cos\phi + \frac{[2\sin(\omega t)\cos(\omega t)]}{2}\sin\phi]$$

$$p = i_0 \upsilon_0 [\sin^2(\omega t)\cos\phi + 0.5 * \sin(2\omega t)\sin\phi] \qquad (14)$$

For practical calculations, one refers to average value of power and this average is referred to one period of oscillation. The ***average power delivered by the AC source in circuit during one period*** is

$$p_{av} = \frac{1}{T}\int_0^T p dt = i_0 \upsilon_0 \cos\phi \frac{1}{T}\int_0^T \sin^2(\omega t) dt + i_0 \upsilon_0 0.5 \sin\phi \frac{1}{T}\int_0^T \sin(2\omega t) dt \qquad (15)$$

$$\int_0^T sin^2(\omega t) dt = \left(\frac{t}{2} - \frac{\sin(2\omega t)}{4\omega}\right)_0^T = \left(\frac{T}{2} - \frac{\sin(2\omega T)}{4\omega}\right) - 0 = \left(\frac{T}{2} - \frac{\sin(4\pi)}{4\omega}\right) = \frac{T}{2}$$

$$\int_0^T \sin(2\omega t)\, dt = -\frac{1}{\omega}(cos\omega t)_0^T = -\frac{1}{\omega}\left(\cos\frac{2\pi}{T}T - cos0\right) = -\frac{1}{\omega}(cos2\pi - cos0) = -\frac{1}{\omega}(1-1) = 0$$

As the first integral is equal to "*T/2* "and the second integral is equal ***zero***, it comes out that the average power delivered by an AC source in circuit is

$$p_{av} = \frac{1}{2} i_0 \upsilon_0 \cos\phi \qquad (16)$$

This expression shows that the phase shift between the applied voltage and the current in circuit *affects* directly the *average power delivered* by the source in a circuit.

-If the circuit contains only a set of R, C, L *in series*, one may use the phasor diagram in figure 3 to get the cosine of phase shift between current and source voltage as follows

$$\cos\phi = \frac{\left|\upsilon_R^{ph}\right|}{\left|\upsilon_S^{ph}\right|} = \frac{\upsilon_{0R}}{\upsilon_0} \qquad (17\ )$$

and $$\upsilon_0 \cos\phi = \upsilon_{0R} = i_0 R \qquad (18)$$

Then by substituting this in expression (16) $p_{av} = \frac{1}{2} i_0 \upsilon_{0R} = \frac{1}{2} i_0^2 R = \frac{i_0}{\sqrt{2}} * \frac{i_0}{\sqrt{2}} R = I^2 R$ (19)

Expression (19) shows that the power supplied by AC source is dissipated as heat in resistor; $P_R = I^2 R$

- In general, disregarding the elements in a circuit, one may rewrite the expression (16) in the form

$$p_{av} = \frac{i_0}{\sqrt{2}}\frac{\upsilon_0}{\sqrt{2}}\cos\phi = IV\cos\phi = P \qquad (20)$$

At this expression the *average power supplied by the source is expressed through* ***rms*** *values of* ***<u>source voltage</u>*** *and* ***current*** in circuit**.** So, simply by measuring the ***rms*** values of ***current*** and **voltage** at ***source terminals,*** one may calculate the ***average power*** delivered by an AC source, even ***without knowing the elements of the circuit*** where is supplied this power.

- The value of " ***cosφ*** " factor in expression $P = IV\cos\phi$ depends only on the phase shift between the current in circuit and source voltage. It affects very much (*can change it from zero to maximum value)* the power released by source in a circuit. For this reason one has named " ***cosφ*** " as ***power factor***.

If ***cosφ = 0*** , ***φ =± π/2*** which means that *the* ***load is purely inductive or capacitive.*** In this situation, the energy delivered in circuit during a half-period is returned into the source during the other half of period when the current inverts the flowing direction. Then, *the average power* provided by the source in circuit during a period is ***zero***$(p_{av}=P=0)$.

If ***cosφ = 1*** one gets ***φ =0*** which means that *the* ***load is purely resistive****; all the energy sent into the circuit by the source is dissipated as heat at the resistor. It does not return any more into the source.*

Remember that, for known values of R, L, C *in series*, one may find the phase shift φ from the relation

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{\omega L - 1/\omega C}{R} \qquad (21)$$

- Let's consider another time the expression (19) and substitute there $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$

Then, we get

$$p_{a\upsilon} = I^2 R = \left(\frac{V}{Z}\right)^2 R = \frac{V^2 R}{R^2 + (X_L - X_C)^2} \qquad (22)$$

This expression shows that the ***average power delivered in circuit*** depends on the ***source frequency*** and it is a maximum $p_{av}^{max} = V^2/R$ (23) (***there is resonance***) when $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$ (24)

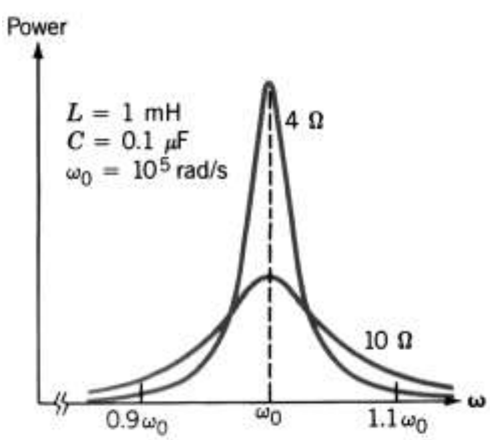

Figure 5

The resonance of delivered power in R , C, L circuits is used for the reception of radio and TV signals. When a package of electric signals with different frequencies is applied at R, L, C circuit input, only those with frequency inside the resonance region "*are allowed* " to build up a current with their frequency through circuit and this way deliver their power into circuit. One should use small values of R to get sharp resonance curves(fig.5). Such R, C, L in series circuits are able to provide very selective reception by *distinguishing* and *rejecting* even *signals* with frequency *close to* $\boldsymbol{\omega_0}$**.** This feature is widely used to build sharp ***filters for EM signals***.