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2.1 ELECTRIC FIELD

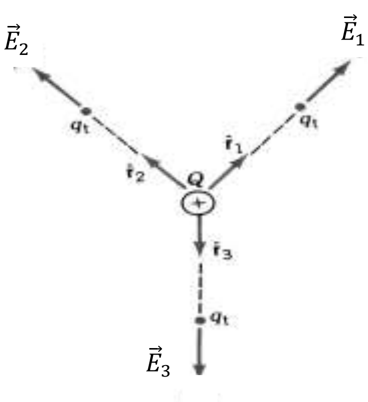
- If one would use a thermometer to measure the temperature at different locations of a big city, one would record a set of different values. This set of temperature values at *different locations* is labelled as a ***field*** of **temperature**. Similarly, a *distribution* of air pressure values at *different locations* is a **pressure *field*** . Those are two examples of ***scalar fields*** (because of the *scalar nature* of ***field parameter***). The main objective of field's models is the quantitative comparison of the values for a physical parameter at different space locations. If the considered parameter is a vector, one deals with a ***vector field***. The ***electric field*** is a ***vector field***.   How does one define the ***vector of electric field***?

-Initially, the electric field appeared as an useful intermediary concept to answer questions:
What way interact two electric charges?..or.. How does a charge "knows" the presence of the other charge? However, with the development of electromagnetic theory, the scientists realized that the **electric field** is one **of the basic constituents (*matter and fields*) of the nature**.

- One has adopted the following model for the **interaction** between two electric charges:
Once an electric charge **Q** "appears" at a given point of space, *it creates its electric field at all space locations around it*. Then, if another electric charge **q** "shows up" at a point P of this space, it gets ***straight away*** *the* ***action*** *of the* ***local electric field*** (created by the charge Q) ***at point P***.

- One *defines the electric field* ***vector*** at point P of space from relation $$\vec{E}_P = \frac{\vec{F}_P}{q_t} \qquad (1)$$

At this expression, $\vec{F}_P$ is the ***electric force*** exerted on a small(<< 1C) **test charge** "q_t" placed at point "P". From expression (1) comes out that the **SI unit** of ***electric field*** is [***N/C***] and its magnitude is equal to numerical value of electric force applied on the charge "+1C" placed at point " P ".



-By using the Coulomb law, one can get the electric field created by a *point charge* Q at the space location $\vec{r} = r * \hat{r}$ (*origin of r-vector is at Q charge*) as

$$\vec{E}(r) = \frac{\vec{F}(r)}{q_t} = k\frac{Qq_t}{r^2}\hat{r}/q_t = k\frac{Q}{r^2}\hat{r} \qquad (2)$$

So, the magnitude of electric field at distance " *r* " is $E(r) = kQ/r^2$

Note*: Expression (2) is valid for "P" points outside the volume of a charged object.*

Fig 1. Electric field of a point charge Q at different locations of surrounding space.

Note: One can find ***the electric field vector due to point charges at a given point P of space by referring to the Coulomb force exerted on the unit positive charge (+1C) placed at point P; but the unit is N/C.***

- Actually, the definition (1) disregards the sources of electric field at P point and defines the ***electric field as a propriety of space*** *at this location,* ***no matter what way it is built***. So, one can calculate ***the electric force*** exerted on a charge " *q* " placed at " P " point of space ***straight away from local electric field*** $\vec{E}_P$ as

$$\vec{F} = q * \vec{E}_P \qquad (3)$$

If ***q > 0****, the force has the* ***same direction*** *as the* ***local field*** *and if* ***q < 0*** *it has the* ***opposite direction.***

-If the electric field is due to a set of point charges $q_1, q_2,..q_N$, by using the <u>principle of superposition</u> for Coulomb's forces, one get that the principle of linear superposition is valid for electric fields, too.

$$\vec{E}_{net} = \frac{\vec{F}_{net}}{q_t} = \frac{\vec{F}_1}{q_t} + \frac{\vec{F}_2}{q_t} + ... \frac{\vec{F}_N}{q_t} = \vec{E}_1 + \vec{E}_2 + ... \vec{E}_N \qquad (4)$$

- Note that *one defines similarly the **gravitational field** due to a mass "M" by using a **test mass*** " $\boldsymbol{m_t}$ " *and the gravitation law* $\vec{F}_G = -G\frac{Mm_t}{r^2} * \hat{r}$. *It comes out that the gravitational field is* $\vec{E}_G = -G\frac{M}{r^2} * \hat{r}$. *One says that a mass " M " creates a gravitational field* $\vec{E}_G$ *in the space around it. Then, if another mass* " $\boldsymbol{m}$ "appears *in this space, it will undergo the force* $\vec{F}_G = m * \vec{E}_G$ *due to local gravitational field. In the case of **gravitational fields,** the direction of exerted force is always the same as that of the local field.*

2.2 ELECTRIC FIELD LINES

-Figure 2.a shows the map of *electric field* due to a positive charge +Q. The electric field due to a set of two point charges +Q and -Q (see fig.2b) is found by applying relation (4). One may figure out that by drawings all electric field vectors at any point of space, the picture becomes very complicated. To avoid this problem, one uses the ***field lines;*** they help to ***visualize*** the ***basic information about the field pattern***.

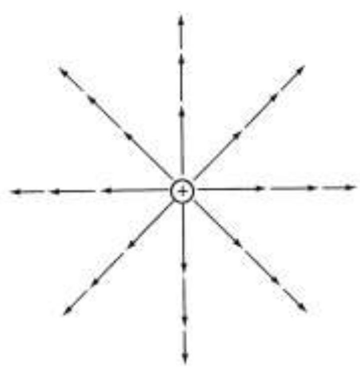


Fig. 2a Electric field map (+Q)

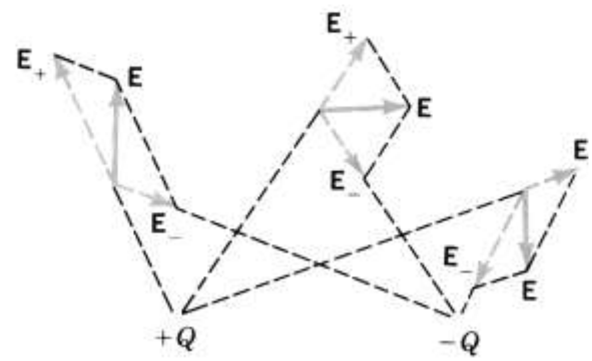


Fig.2b Electric field map (+Q and -Q)

-Here are the rules that *relate* the electric **field lines** and the electric **field vectors**:

a) The electric *<u>field lines</u>* have a *direction*; they **emerge from (+)** charges and **enter to (-) charges**.
b) At any point of space, the direction of the **<u>electric field vector</u>** fits to the tangent to the **<u>field line</u>** passing by this point.
c) The **field lines never cross each other**. Otherwise, it would be more than one direction for the vector of electric field at a given point!!
d) The **density**[1] of field lines around a point in space is **proportional to the magnitude of electric field** at this point. Thus, a high density of field lines tells that the magnitude of electric field is large and a low density of field lines shows an area with small magnitude of electric field.
e) While the **electric field is a physical reality**, the field lines are not. They are just an intermediary tool used to give a visual and general information about the field.

[1] The number of lines that traverse a unit area perpendicular to local lines' direction

Figure 3 shows the " map " of electric field lines related to a set of two electric charges (+,-) and (+,+) .

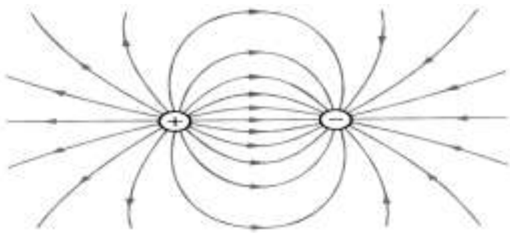


Fig.3a The field lines for (+Q and –Q) known as "electric dipole"

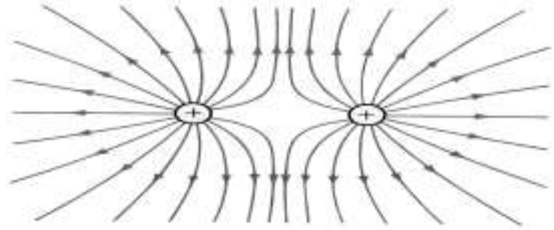


Fig.3b The field lines for a set of (+Q and +Q) charges

-Figure 4 shows the central part of an *infinitely large plane* with *uniform distribution of charges* on it. The electric field vector at any point of space in either side of this plane is perpendicular to the plane (*because the components parallel to plane cancel out each other*). A detailed calculation shows that the *magnitude* of its electric field is the same in all space locations "*close to this plane"*.

If **the vector $\vec{E}$ is the same at every point of space region, one calls it a *<u>uniform electric field</u>***. One may see that there is a *uniform electric field in the space close to a uniformly charged plane on one side of it; there is another (opposite direction) uniform field on the other side of plane.*

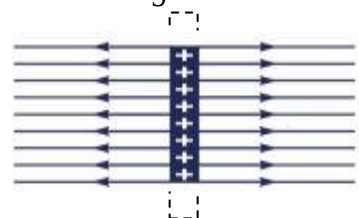


Fig.4

2.3 CONDUCTOR INSIDE AN ELECTRIC FIELD

-When a **neutral** homogeneous **conductor** is placed inside an electric field $\vec{E}_{ext}$, its "*free"* electrons move on opposite sense of $\vec{E}_{ext}$ (Fig.5a) and leave an unbalanced positive charge on the other side. This charge redistribution inside the conductor creates an internal field $\vec{E}_{int}$ directed in opposite sense to $\vec{E}_{ext}$.
So, the resulting field *inside the conductor* is $\vec{E}_{net} = \vec{E}_{ext} + \vec{E}_{int}$. As long as $\vec{E}_{net} \neq 0$ the "*free"* electrons continue to move in a way that increases the magnitude of $\vec{E}_{int}$ and consequently decreases the magnitude of $E_{net} = E_{ext} - E_{int}$; when $E_{net} = 0$ they stop moving and all conductor charges are at rest.
So, if all charges are at rest, the net macroscopic field inside a homogeneous conductor is zero.
Note that even though *the electric field may be different from zero* in *microscopic level (close to the ions of crystalline structure), the* ***field average (i.e. macroscopic field)*** *is zero* ***<u>everywhere inside</u>*** *the* **conductor**.

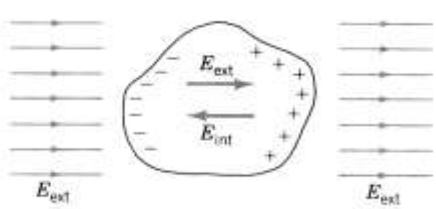


Fig.5a Two fields inside conductor

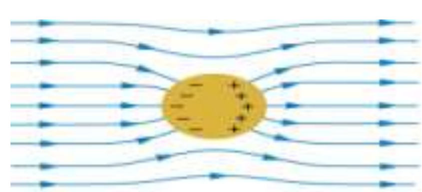
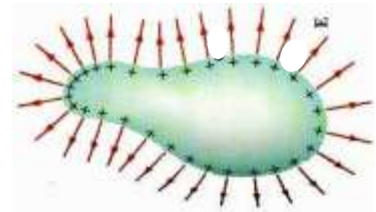


Fig.5b Deformation of external field lines close to conductor

-What happens with the *electric field* **outside the conductor close** to its **surface**? No matter what is the direction of external electric field, *at each outside location close to the surface,* one may decompose it into two components; one parallel to the surface $\vec{E}_{ext}\|$ and one perpendicular to the surface $\vec{E}_{ext}\perp$. The shift of free electrons to the conductor surface builds up an additional field that *reduces to* ***zero*** the ***net parallel field(*** $\vec{E}_{ext}\| = 0$); <u>otherwise the electrons would continue to move on conductor's surface</u>. Note that "*normally"* they cannot move along the vertical *<u>out of conductor without a "considerable force"</u>*.

The redistribution of free electrons affects but ***does not reduce to zero*** $\vec{E}_{ext}\perp$ ***out*** of conductor surface. So, in ***electrostatic***, the **electric field** at any nearby point **outside conductor** is **perpendicular** to surface of conductor **(fig.5b**). Meanwhile, the **net electric field everywhere inside the conductor, even close to its surface, is zero**; otherwise the free electrons would move and the condition of static electric charges would not be valid.



- Even without external fields, if one transfer electrons on a conductor, by contact method, they will push each other and, at electrostatic condition (*all charges at rest*) they will get distributed on object surface. A similar logic to that presented in previous paragraph brings to conclusion that, once all charges are at rest, the ***internal*** field is **zero and** it is perpendicular close to object surface ***outside*** because only this direction leaves all charges at rest on surface.

Fig.6

In fig.6, one has removed electrons; so, "+" charged remain distributed uniformly on conductor surface.

- At this point, one may figure out that, if one removes the material inside the conductor and get a ***conducting shell***, the electric field will be zero everywhere in the empty space inside no matter what is the electric field outside the conducting shell. Actually, this is the situation inside a " **Faraday cage "**.

2.4 MOTION OF CHARGES INSIDE UNIFORM FIELDS

-If an elementary charged particle *"q"* (*electron, proton, ion or many of them in a charged particle*) is inside a *static uniform electric field with magnitude " E "*, it is subjected to the electric force with magnitude

$$F_{el} = qE \qquad (5)$$

Due to their small mass ($G\sim10^{-11} Nm^2/kg^2$, $m_{el}\sim10^{-31}kg$, $m_p\sim10^{-27}$ kg), the gravitation force on *elementary particles* is much smaller than electric force. So, the net force exerted on a charged *microscopic particle* is almost equal to the electric force on it. In these circumstances, the second law of Newton gives

$$\vec{F}_{net} \cong \vec{F}_{el} = q\vec{E} = m\vec{a} \qquad (6)$$

and the particle moves at a constant acceleration ($\vec{E}$ *is constant through a uniform field and* $\vec{a}\uparrow\uparrow\vec{E}$ *or* $\vec{a}\uparrow\downarrow\vec{E}$)

$$\vec{a} = \frac{q\vec{E}}{m} \qquad (7)$$

2.5 CONTINOUS CHARGE DISTRIBUTION

Remember that Coulomb's law is valid for charged **point particles**. In order to find the electric field due to a continuous charge distribution on/in a macroscopic object, at a point of space "P" one must:

a) Divide the total charge into *infinitesimal small elements*, each of them with a small charge ***dq***.
b) Use the Coulomb's law to find the field due to each " ***dq*** " at point "P" as $d\vec{E} = k\frac{dq}{r^2}\hat{r}$ (8)
c) Apply the *principle of linear superposition* and get the sum of all these infinitesimal fields as

$$\vec{E} = \int_{charged\_object} d\vec{E} = k\int_{charged\_object} \frac{dq}{r^2}\hat{r} \qquad (9)$$

In these calculations, one *has* ***(always)*** *to express* **dq** charge as function of coordinates of its location, i.e. ***dq*(x, y, z)** so that one can calculate the value of integral (9).

2.6 ELECTRIC DIPOLE

-One calls **electric dipole** a set of two electric charges (-Q ;+Q) separated by a distance "***d*** ". The **electric dipole** model is widely used in physics (*radio / TV antennas, molecular interactions, crystal proprieties*..).
For example, in a ***polar molecule*** (like HCl, CO_2, H_2O..) the centers of positive and negative charges do not coincide and there is a ***permanent molecular electric dipole***. One uses this model even for ***non-polar molecules*** placed inside an external electric field because in this case they get an ***induced electric dipole***.

2.6a ELECTRIC FIELD PRODUCED BY A DIPOLE

- Let's consider two point charges **(+Q; -Q**) at **distance** $d \equiv 2a$ and let's find the electric field at a point on the perpendicular line that pass by the dipole center at **distance " *r* "** (see Fig.7). The net field at this point is the vector sum of electric fields built by each of two charges (+Q; -Q). At distance "***r***" from the dipole center, both fields have an equal **magnitude** calculated as (take q_t = +1C for calculation of E):

$$E_+ = E_- \equiv E = k\frac{Q}{r^2 + a^2} \qquad (10)$$

Their directions are shown. To find the net field we consider the components along each axis; Ox and Oy.

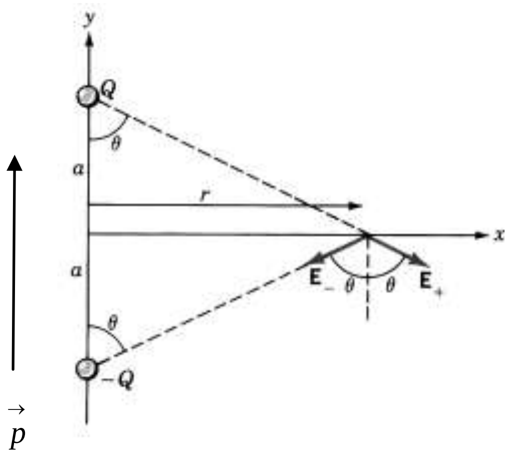


Fig.7

Along Ox; The two components have the same magnitude but are inversely directed. Their sum is zero.

Along Oy; The two components have equal components
$E_+^y = -E\cos\theta \quad E_-^y = -E\cos\theta$
As they have the same angle " θ ", along -Oy, their sum is

$$E^y = E_+^y + E_-^y = -2E\cos\theta \qquad (11)$$

From fig.7 one can see that $\cos\theta = \frac{a}{\left(r^2 + a^2\right)^{1/2}}$

So, the net electric field at "*r*" has only the y-component directed opposite to Oy; $\vec{E}_{net} = E_{net}\hat{j}$

where $$E_{net} = E^y = -2k\frac{Q}{r^2 + a^2} * \frac{a}{\left(r^2 + a^2\right)^{1/2}} = -k\frac{Q*2a}{(r^2 + a^2)^{3/2}} \qquad (12)$$

One has defined the **moment** of the **electric dipole** (*-Q; +Q at distance d*) as $\vec{\boldsymbol{p}} = (\boldsymbol{Q} * \boldsymbol{d})\hat{\boldsymbol{j}}$ (13)
The *electric dipole moment* $\vec{p}$ is **directed** from **negative to positive charge**(see fig .7).

As d = 2a, one can use the magnitude of *electric moment* (***p = Q*d***) in expression (12) and find that

$$\vec{E}_{net} = -k\frac{Q*d}{(r^2+a^2)^{3/2}}\hat{j} = -k\frac{\vec{p}}{(r^2+a^2)^{3/2}} \qquad (14)$$

The vector of electric field at a point on the perpendicular line that pass by the dipole center has opposite direction to that of the *dipole moment* vector.

-For points on Ox *far from the dipole,* $r >> a$ (***far-field***) and the ***magnitude*** of electric field is

$$E_{net} \cong k\frac{p}{r^3} \tag{15}$$

Some more calculations show that the ***magnitude*** *of electric field due to an* ***electric dipole*** in the **far field** (r >> a) varies as $\sim\frac{1}{r^3}$ *for all locations* , i.e. *even for those that are not on line Ox.*

Note: When dealing with an electric dipole, one disregards charges that build it.
All calculations and conclusions are related to the *electric dipole moment* $\vec{p}$.

2.6b THE ACTION OF UNIFORM ELECTRIC FIELD ON AN ELECTRIC DIPOLE

- Let's consider an electric dipole inside an uniform electrostatic field along Ox (Fig.8). Assume that it has a *rigid structure* and the *dipole moment* $\vec{p}$ is directed at angle θ versus the direction of *uniform field* $\vec{E}$.

- The components of electric forces *along dipole axis* (8.b), exerted on each charge, have equal magnitude but opposite directions. As the dipole is rigid, the forces' components along dipole direction cannot extend it; they just sum up to produce a result zero. The components of electric forces *perpendicular to dipole axis* have the same magnitude but opposite directions. So, they produce two torque actions which tent to rotate the dipole clockwise around an axis Oz passing by dipole center and perpendicular to the plane Oxy (*this plane contains the vectors of electric field* $\vec{E}$ *and dipole moment* $\vec{p}$). One may figure out easily that both those torques are directed along "-Oz" (i.e. *into the plane*) and have equal magnitudes

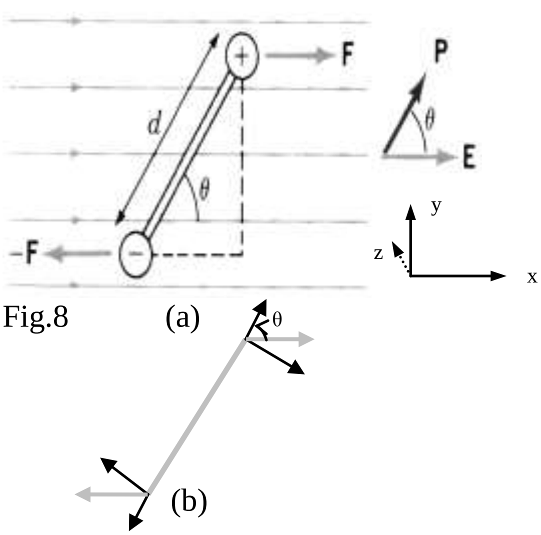


Fig.8 (a)

(b)

$$\tau_+ = \tau_- = \frac{d}{2}F\sin\theta \tag{16}$$

where $F= Q*E$

Consequently, the **magnitude** of net torque vector is

$$\tau = \tau_+ + \tau_- = 2*\frac{d}{2}F\sin\theta = dF\sin\theta = d(QE)\sin\theta$$

$$\tau = (dQ)E\sin\theta = pE\sin\theta \tag{17}$$

As this torque is directed opposite to Oz axe

$$\tau_z = -pE\sin\theta \tag{18}$$

Next, by referring to the cross product definition one finds out that

$$\vec{\tau} = \vec{p}\, x\, \vec{E} \tag{19}$$

This torque tents to ***align*** *the dipole moment* ***vector*** $\vec{P}$ *along the lines of field****, i.e. parallel to vector*** $\vec{E}$.

2.6c THE POTENTIAL ENERGY OF AN ELECTRIC DIPOLE

- An object is at **equilibrium** when there is a **minimum of potential energy** of system (remember mechanics). An electric dipole inside electric field is at equilibrium when it is aligned along field lines; this means that it has a minimum of electric potential energy if its moment $\vec{P}$ is aligned at same direction as $\vec{E}$ field lines. So, if $\vec{p}\uparrow\uparrow\vec{E}$, one must spend some "*positive*" external work for rotating it to an angle " **θ** " versus direction of $\vec{E}$ as this work must increase the potential energy of *system electric field - dipole (Wext = ΔU > 0)*.

-Also, *only the differences of potential energy have a physical meaning and one ties the* ***zero of potential energy*** *to a* ***configuration that simplifies*** *the mathematical* ***calculations***. For an electric dipole inside an electric field, the calculations are easier if one chooses **U(*θ = 90⁰*) = 0** *(not minimum)***,** that is for $\vec{p} \perp \vec{E}$ **.**

-Next, to find the potential energy **U(θ)**, for any angle θ, one has to calculate the ***work done by*** the ***electric field*** $\vec{E}$ while the ***dipole moment*** $\vec{p}$ get **rotated** from the angle $\boldsymbol{\theta_o = 90^0}$ to that angle **θ** versus $\vec{E}$ direction**.** By referring to the *work by an* ***internal*** *force (Wint = - Wext = -* ΔU *, see mechanics)* in case of the ***system electric field - dipole,*** it comes out that

$$W_{in} = -\Delta U = -[\,U(\theta) - U(90^0)] = -U(\theta) + U(90^0) = -U(\theta) + 0 = -U(\theta) \quad \text{or} \quad \boldsymbol{U(\theta) = -W_{in}} \qquad (20)$$

The **infinitesimal** ***work*** $\boldsymbol{dW_{in}}$ *done by the torque* (<u>due to $\vec{E}$ field action</u>) on the dipole momentum during a rotation by the angle $d\theta$ is (*see mechanics*)

$$dW_{in} = \tau * d\theta = \tau_z * d\theta \qquad (21)$$

The total work *made by electric field* when rotating $\vec{p}$ *from* the angle $\boldsymbol{\theta_o = 90^0}$ to angle $\boldsymbol{\theta}$ (see 18) is

$$W_{in} = \int_{90}^{\theta} \tau_z d\theta = \int_{90}^{\theta} (-pE\sin\theta)d\theta = pE\cos\theta \Big|_{90}^{\theta} = pE\cos\theta$$

Then, from (20) one gets

$$U(\theta) = -W_{in} = -pE\cos\theta \qquad (22)$$

and in vector form

$$U(\theta) = -\vec{p} * \vec{E} \qquad (23)$$

-The graph in Fig.9 presents the evolution of the *potential energy* of electric dipole depending on *angle θ versus the electric field direction.* It is ***minimum for θ = 0***, it is ***zero for θ = π/2*** and ***maximum for θ = ± π.***

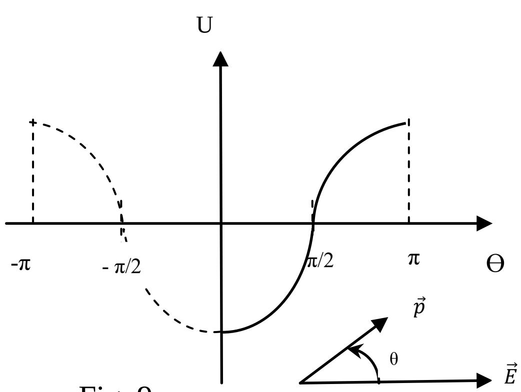


Fig. 9

-The water molecules possess a *large* dipole electric moment (p=**6.2*10⁻¹⁰C*m**) and they interact strongly with electric fields. In microwave cooking, an external *alternating field* makes those dipole moments (*and molecules*) oscillate at a high frequency (~10^9Hz). The water molecules dissipate this kinetic energy through all other molecules in the meal and this increases the temperature of surrounding volume (*meal constituents*). By contrast, the glass interacts very weekly with the electric field because it has not a permanent dipolar moment. So, it does not receive any meaningful energy for E-field and its temperature does not increase.