

## 4.1 CAPACITOR

- As noted previously, very often, one prefer to disregard its sources and refers to the electrostatic field as a specific *physical object with no rest mass* in space. This field acts on charged particles ( $e^-$ ,  $p^+$  or ion) by an electric force. Then if a charged particle travels inside such a space region, the *electric field* provides work via the electric force it applies on the charged particle. This means that the **electric field contains energy**; so, one can model it even as a "**container of energy**" which can **provide** or can **receive energy**.

- In principle, one may "*store energy*" in a space region by placing any configurations of electric charges there. A battery **stores energy** by hosting electric charges on its electrodes for a "*long time*". The **capacitor** is a device that can store electric charges and energy "*for a while*". It is constituted by *a set of two conducting plates separated by an insulator*. The plates may be plane, cylindrical or spherical.

- One can model a **battery** (B in fig.1) as a "electric energy container" that **keeps constant** the difference of **potential  $V^1$**  between its terminals when a *device* or a *circuit is connected to them*. Once the plates of a capacitor C get connected (by metallic wires) to battery terminals, they get terminal potential and this builds the difference of potential **V** between the capacitor plates. The *electromotor force "emf"* of battery removes some free  $e^-$  from one plate of capacitor, this way charging it positively "+" and transfers the **same number** of  $e^-$  to the other plate of the capacitor (charging it negatively "-"). At the end of transfer, each plate gains the same amount of charge ( $+Q, -Q$ ). Each of these two charges is distributed uniformly on the inner surface of one **conducting plate**(fig.2). As the areas of the plates are equal, each of them get the same charge density  $\sigma$  (but opposite sign). It comes out that, **for a given potential difference V** between them, **the plates of a capacitor can host only a precise amount of charge "Q"**. Once this amount  $Q$  is stored on the plates, the battery does not remove any more charges and the process of the charge transfer stops. Assume that, after this moment, one disconnects the battery. The charges  $+Q, -Q$  will remain on conducting plates (Fig.2) and the capacitor will keep the same difference of potential  $V$  between its plates.

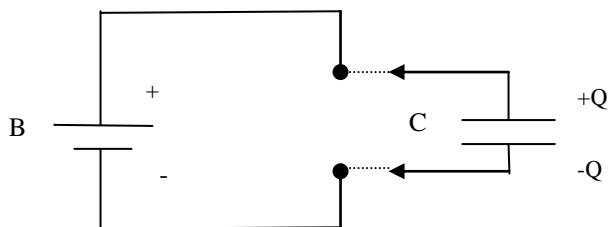


Fig.1

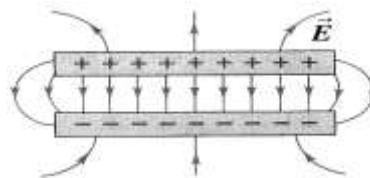


Fig.2

- Let's see the relationship between  $Q$  and  $V$  on capacitor plates. The **strength  $|E|$**  of electric field close to the surface of a flat conductor is proportional to the charge density ( $E \sim \sigma$ ) on its surface. Then, for uniform distribution, the charge density is  $\sigma = Q/A$ ; so, one gets  $|E| \sim Q$  where  $Q$  is the total charge on surface.

Also, the magnitude of potential difference  $\Delta V$  between two points inside field is proportional to the field strength ( $\Delta V = E \cdot d$ ). So, as distance between plates " $d$ " is constant, it comes out that potential difference  $\Delta V = V^+ - V^-$  between the two plates is proportional to the magnitude of charge  $Q$ . As one assigns  $V^{(-)} \equiv 0$  it comes out that  $\Delta V = V^+ \equiv V \sim Q$ . The **voltage "V"** is known as the **electric potential in capacitor**. Its magnitude is **proportional** to charge magnitude  $Q$  and **vice-versa**. Actually, one relates the amount of charge on each capacitor plate to the voltage " $V$ " by the expression  $Q = C \cdot V$  (1)

where  $C = \frac{Q}{V}$  (2) is the **capacitance** of the capacitor. Its unit in SI system is farad (1F);  $1F = 1Coulomb/Volt$

<sup>1</sup> One uses the symbol **V** for the difference of potential *between two points in a circuit* instead of  $\Delta V$  that one uses mainly in **electrostatic**.

- For a given *difference of potential* " $V$ " between the plates, a *larger capacitance*  $C$  stores larger amount of charge " $Q$ " on plates and this *increases* the *strength of electric field* " $E$ " in the space between them. We will see that, this increases the amount of electric energy stored in a capacitor, too.

The **capacitance** is the main parameter of a capacitor. It **measures its efficiency in storing energy**. For general purpose capacitors, the **capacitance** values are in  $\mu\text{F}$ (microfarad) ,  $\text{nF}$ (nanofarad) and  $\text{pF}$ (picofarad).

- The Fig.2 presents the electric field in the empty space between two plates of a capacitor. One may neglect the small areas close to plate corners and consider only *the field in central region*. In this region one can use the model of field built by a charge  $Q$  *distributed uniformly on a large plane*. We have found that this field is **uniform**, directed perpendicular **to the plane** and its magnitude is  $|E| = \frac{\sigma}{2\epsilon_0}$ . Since

there is two plates with the *same charge density*  $\sigma$  producing fields with the same *magnitude and direction* (from "+" to "-" plate), it comes out that, for an area plate "A" the magnitude of net electric field in the space between them is

$$|E| = 2 \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \quad (3)$$

As the potential difference between the two plates<sup>2</sup> at distance "d" is  $V = E \cdot d$  (4)

it comes out that  $V = \frac{Q \cdot d}{\epsilon_0 A} = \frac{Q}{(\epsilon_0 A / d)}$  (5) and  $C = \frac{Q}{V} = \epsilon_0 \frac{A}{d}$  (6)

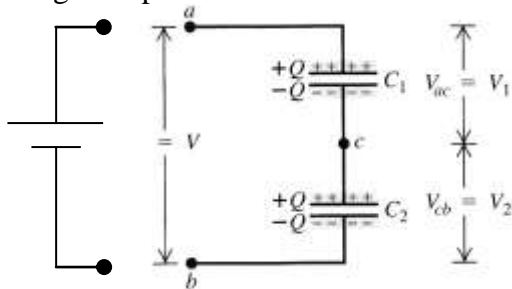
One may find out from (6) that  $[\epsilon_0] = \text{F/m}$ . So,  $\epsilon_0 = 8.85 \cdot 10^{-12} \text{C}^2 / \text{Nm}^2$  \_or\_  $8.85 \cdot 10^{-12} \text{F/m}$

- The expression (6) shows that, **in vacuum**, the capacitance of a given capacitor depends only on its geometry parameters. If the capacitor shape is not plate, the expression for its capacitance is more complex than (6) but in all cases capacitance depends only on geometrical parameters.

## 4.2 CAPACITORS IN SERIES AND CAPACITORS IN PARALLEL

- The two main parameters of a capacitor are: the **capacitance** and the **maximum potential it can support**. If the available capacitors do not fit with requirements of a given circuit, one may use a combination of several capacitors. There are two main ways to connect several capacitors; *in series* or *in parallel*.

- Figure 3 presents the scheme of two capacitors connected *in series*. If one connects the points a and b



to the battery terminals, the capacitors  $C_1$  and  $C_2$  acquire the same amount of charge  $Q$  on their plates because the "*exterior charges*  $\pm Q$ " transferred by the **emf** of battery **induce** the same magnitude of charge  $-/+ Q$  to the interconnected plates. As the difference of potential for each capacitor is:

$$V_1 = \frac{Q}{C_1} = V_{ac} \quad \text{and} \quad V_2 = \frac{Q}{C_2} = V_{cb} \quad (7)$$

Fig. 3

$$\text{it comes out that} \quad V = V_{ac} + V_{cb} = \frac{Q}{C_1} + \frac{Q}{C_2} = Q \frac{1}{C_{eq.}} \quad (8)$$

<sup>2</sup>  $E$  is the magnitude(*field strength*) of  $E$ -vector. The potential of (-) plate is zero. Do not forget that it is a "- charged plate.

This expression shows that the set of two capacitors connected *in series* behaves the same way as a single *equivalent capacitor* with capacitance  $C_{eq}$  such that

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \quad (9)$$

If  $N$  capacitors are connected *in series*, one gets the **equivalent capacitance**  $\frac{1}{C_{eq}} = \sum_{i=1}^N \frac{1}{C_i}$  (10)

Note that, for the **same amount of charge "Q"** stored on its plates, this "*equivalent capacitor*" can support **higher voltage** on its terminals than each of single capacitors in series " $V^{\max} = V^{\max}_1 + V^{\max}_2 + V^{\max}_3 + \dots$ ".

-Figure 4 presents the scheme of two capacitors connected *in parallel*. In this case, the same difference of

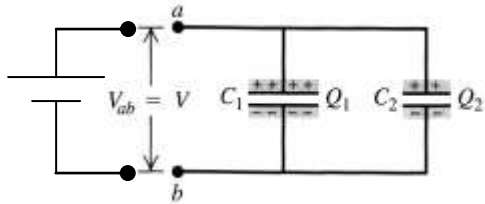


Fig.4

potential applies on the two capacitors and the battery **emf** has shifted the total charge amount " $Q_1 + Q_2$ " from/to both of them. The magnitudes of charges  $Q_1$  and  $Q_2$  depend on the capacitance of capacitors as follows  $C_1 = \frac{Q_1}{V}$  and  $C_2 = \frac{Q_2}{V}$

In this case, the *equivalent capacitor* would get the charge  $Q = Q_1 + Q_2$  (12)

and its capacitance would be

$$C_{eq} = \frac{Q}{V} = \frac{Q_1 + Q_2}{V} = \frac{Q_1}{V} + \frac{Q_2}{V} = C_1 + C_2 \quad (13)$$

By connecting *in parallel*  $N$  capacitors, one gets the **equivalent capacitance**  $C_{eq} = \sum_{i=1}^N C_i$  (14)

This equivalent capacitor " offers **higher storage capacity of charge** " $Q_1 + Q_2 + \dots$ " for the **same voltage**.

### 4.3 ENERGY STORED IN A CAPACITOR

- The **stored energy** in a capacitor is **concentrated in the electric field** created by plates' charges in the space between them. This **stored energy** is due to the *work spent by emf of battery* when shifting some "free electrons" *from* one plate (that becomes charged "+") and *onto* the other plate (that becomes charged "-"). Note that the electron path passes through the wires and battery (not through space between the plates). This transfer process lasts for a short interval of time until the charge in capacitor get to value  $Q = C \cdot V$ . Next, there is no more charge motion in circuit. *Remember that  $W_{ext} (= -W_{int})$  needed to shift a charge inside an electrostatic field "at constant K" depends only on the initial- final locations and not from the path shape.* In the following, to simplify the calculations, one uses a model in which, instead of "-Q" charge, the action of *battery emf* transfers the same amount of "+Q" charge (*from* "-" plate to..battery.. to "+" plate).

- Let's consider the *circuit battery – capacitor* at an intermediary moment when the plates' charge is " $q$ " and the **difference of potential** between them is " $v$ ". The work  $dW$  done by *efm* of battery, for the transport of the next small charge amount " $+dq$ " from "-" plate (with  $V_- \equiv 0$ ) to "+" plate (with  $V_+ = v$ ) is an *external work* for the **system electric field-charge**. As  $dW_{ext} = dU = dq \cdot (v - 0)$  one get

$$dW_{ext} = v \cdot dq \quad (15)$$

At this moment  $v = \frac{q}{C}$ . So, the relation (15) transforms to  $dW_{ext} = \frac{q}{C} \cdot dq$  (16)

and the **total work** done by **battery emf** to shift the total charge  $Q$  is

$$W_{ext} = \int_0^Q dW_{ext} = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{C} \quad (17)$$

The principle of energy conservation for **system electric field-charge** tells that  $W_{ext} = \Delta U_E = U_E - 0 = U_E$  where  $U_E$  is **energy electric stored** inside the capacitor. Next, by using relations (1), (2), one finds out that

$$U_E = \frac{Q^2}{2C} = \frac{Q}{2} * \frac{Q}{C} = \frac{Q}{2} V = \frac{1}{2} CV^2 \quad (18)$$

- This energy is distributed uniformly inside the volume " $d * A$ " of space between two capacitor plates.

So, the energy per unit of volume, i.e. the **energy density  $u$** , is

$$u = \frac{\frac{1}{2} CV^2}{d * A} \quad (19)$$

By substituting  $C = \epsilon_0 * \frac{A}{d}$  in (19) one gets

$$u = \frac{\frac{1}{2} (\epsilon_0 * A / d) V^2}{d * A} = \frac{\epsilon_0}{2} * \frac{V^2}{d^2} = \frac{\epsilon_0}{2} E^2$$

So, the electric field in space between plates has the **energy density**  $u [J / m^3] = \frac{\epsilon_0}{2} E^2 \quad (20)$

**Notes:** - Although the expression (20) is derived for the case of a plane plate capacitor in vacuum, it is valid for the **energy density of any configuration of electric field in vacuum**.

- Even the "**empty space**" may contain energy, for example electric energy.

*Note that the "older interpretation" considered that the electric energy belongs only to a set of electric charges.*

#### 4.4 DIELECTRICS

- The actual capacitors contain a dielectric (insulator) material between the conducting plates. One uses the dielectric to:

- avoid the contact and keep fixed the distance between the plates.
- increase the maximum difference of potential  $V^{max}$  supported by the capacitor.
- increase the capacitance of the capacitor compared to empty configuration.

- The **dielectric constant** ( $\kappa$ -kappa) of a substance is a measure of "**how much it get polarized**" under the effect of an external electric field. A polar molecule ( $H_2O$ ,  $NH_3$ ..) has a **permanent electric dipole moment**. In presence of an external electric field, there is a torque effect which tends to align the dipole moments  $\vec{p}$  along the direction of exterior field ( $\vec{E}_0$ ). At non-polar molecules ( $H_2, CH_4$ ..), the **exterior field induces**, at first, an electric moment and then the torque tends to align it along  $\vec{E}_0$ . In both cases, the dielectric surface in front of capacitor plates get opposite sign charge to that of the plate in front (fig.5).

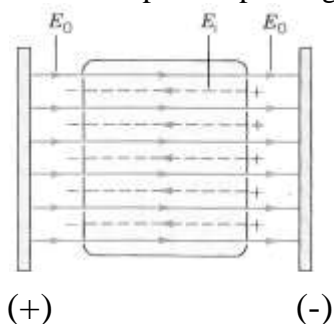


Fig.5

This is the origin of an additional internal field  $\vec{E}_i$  which has opposite direction to  $\vec{E}_0$ . So, the magnitude of net field inside the dielectric is decreased to

$$E_D = E_0 - E_i \quad (21)$$

**Note:** For the same charge amount ( $Q$ ) on the plates, the magnitude of net electric field inside the dielectric is smaller than the electric field in vacuum.

- Consider a dielectric substance between the plates of a capacitor with Q charge. If the **field magnitude** between plates in vacuum was  $E_0$ , in presence of the dielectric, it will become

$$E_D = E_0 - E_i \equiv \frac{E_0}{\kappa} \quad \kappa \geq 1 \text{ has no units} \quad (22)$$

The magnitude of field inside the capacitor *in vacuum* is  $E_0 = \frac{\sigma}{\epsilon_0}$ . In presence of the dielectric it

becomes

$$E_D = \frac{E_0}{\kappa} = \frac{\sigma}{\kappa \epsilon_0} = \frac{\sigma}{\epsilon} \quad (23)$$

$\epsilon_0$  is the **permittivity of vacuum**;  $\epsilon = \kappa \epsilon_0$  is known as **permittivity of dielectric material**

By similar steps, as in case of an electric field inside vacuum capacitor, one may find that the density of electric energy inside a dielectric in capacitor is

$$u \left( \frac{J}{m^3} \right) = \frac{\epsilon}{2} E_D^2 \quad (24)$$

-As the field inside capacitor is uniform, from the relation  $V_D = E_D * d = (E_0 / \kappa) * d = (E_0 * d) / \kappa$

for the *same charge* Q on plates, the potential difference between them is  $V_D = \frac{V_0}{\kappa}$  ( $V_0$  in vacuum).

So, if the **capacitance of a capacitor** in vacuum is  $C_0 = \frac{Q}{V_0}$ , in presence of a dielectric material

between the plates it becomes

$$C_D = \frac{Q}{V_D} = \frac{Q}{V_0 / \kappa} = \kappa * C_0 \quad (25)$$

This way, one gets another definition of dielectric constant as

$$\kappa = \frac{C_D}{C_0} \quad (26)$$

**Note:**  $C_D$  is always larger than  $C_0$  and  $\epsilon > \epsilon_0$ .