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4.1 CAPACITOR

- As noted previously, very often, one prefer to disregard its sources and refers to the electrostatic field as a particular *physical object* in a region of space. This field acts on charged particles (e-, p+ or ion) by an electric force. Then if a charged particle travels inside this space region, the *electric field* provides work via the electric force it applies on charged particles. This means that the **electric field** <u>*contains energy*</u>; so, one can model it even as a "**container of energy**" which can **provide** or can **receive energy**.

- In principle, one may "*store energy*" in a space region by placing any configurations of electric charges there. The battery is a device that ***stores energy*** by keeping electric charges on its plates for a "*long time*". A common ***capacitor*** is a device that can store electric energy "*for a while*". It is constituted by *a set of <u>two charged conducting plates separated by an insulator</u>*. The plates may be plane, cylindrical or spherical.

- One can model a **battery** (B in fig.1) as a "electric energy container" that **keeps constant** the difference of **potential V**[1] between its terminals when a ***device*** or a ***circuit is connected to them***. Once the plates of a capacitor C get connected (by metallic wires) to battery terminals, they get terminal potential and this builds the difference of potential **V** between the capacitor plates. The *electromotor force* "***emf***" of battery removes some free ***e-*** from one plate of capacitor, this way charging it positively "+" and transfers the ***same number*** of ***e-*** to the other plate of the capacitor (charging it negatively "-"). At the end of transfer, each plate gains the same amount of charge (+Q,-Q). Each of these two charges is distributed uniformly on the inner surface of one **conducting plate**(fig.2). As the areas of the plates are equal, each of them get the same charge density $\boldsymbol{\sigma}$ (but opposite sign). It comes out that, ***<u>for a given potential difference V</u>*** between them, ***<u>the plates of a capacitor can host only a precise amount of charge "Q"</u>.*** *Once this amount Q is stored on the plates, the battery does not remove any more charges and the process of the charge transfer stops.* Assume that, after this moment, one disconnects the battery. The charges +Q, -Q will remain on conducting plates (Fig.2) and the capacitor will keep the same difference of potential V between its plates.

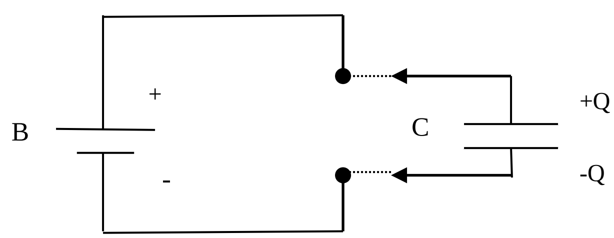


Fig.1

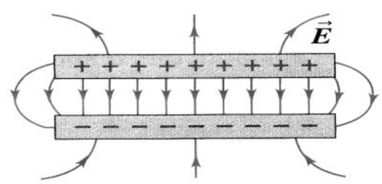


Fig.2

- Let's see the relationship between Q and V on capacitor plates. The **strength** $|E|$ of electric field close to the surface of a flat conductor is proportional to the charge density (***E~ σ***) on its surface. Then, for uniform distribution, the charge density is ***σ = Q/A***; so, one gets $|E|$ **~ *Q*** where **Q** is the total charge on surface. Also, the magnitude of potential difference ΔV between two points inside field is proportional to the field strength (***ΔV=E*Δ***). So, as distance between plates "Δ" is constant, it comes out that potential difference $\Delta V = V^{+} - V^{-}$ between the two capacitor plates is proportional to the magnitude of charge **Q**. Then, one assigns $V^{(-)} \equiv 0$ and get out that $\Delta V = V^{+} \equiv V$. The ***voltage*** "*V*" is known as the <u>*electric* ***potential in*** *capacitor*</u>. Its magnitude is ***proportional*** to charge magnitude ***Q*** and **vice-versa**. One operates mainly with the voltage "*V* " and the amount of charge on each capacitor plate is expressed as $Q = C * V$ (1)

Note that $C = \dfrac{Q}{V}$ (2) is the ***capacitance*** of the *capacitor*. Its *unity* in SI system is *farad* (1F); *1F = 1Coulomb/Volt*

[1] One uses the symbol **V** for the difference of potential ***between two points in a circuit*** instead of Δ**V** that is used mainly in **electrostatic**.

- For a given *difference of potential* "***V***" between the plates, a *larger capacitance* ***C*** stores larger amount of charge "***Q*** " on plates and this *increases* the *strength of electric field* "***E*** " in the space between them. We will see that, this increases the amount of electric energy stored in a capacitor, too.
*The **capacitance** is the main parameter of a capacitor*. It ***measures its efficiency in storing energy***. For general purpose capacitors, the ***capacitance*** values are in μF(microfarad) , nF(nanofarad) and pF(picofarad).

- The Fig.2 presents the electric field in the empty space between two plates of a capacitor. One may neglect the small areas close to plate corners and consider only *the field in central region*. In this region one can use the model of field built by a charge **Q** *distributed uniformly* on a **large plane**. We have found that this field is **uniform**, directed perpendicular **to the plane** and its magnitude is $|E| = \frac{\sigma}{2\varepsilon_0}$. Since there is two plates with the *same charge density* σ producing fields with the same *magnitude and direction (from "+" to "-" plate)*, it comes out that, for an area plate "A" the magnitude of net electric field in the space between them is

$$|E| = 2\frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \qquad (3)$$

As the potential difference between the two plates[2] at distance "d" is V = E*d (4)

it comes out that $$V = \frac{Q*d}{\varepsilon_0 A} = \frac{Q}{(\varepsilon_0 A/d)} \qquad (5)$$ and $$C = \frac{Q}{V} = \varepsilon_0 \frac{A}{d} \qquad (6)$$

One may find out from (6) that [ε_o] = F/m . So, $\varepsilon_0 = 8.85*10^{-12} C^2/Nm^2$ _or_ $8.85*10^{-12} F/m$

- The expression (6) shows that, **in vacuum**, the *capacitance of a given capacitor depends only on its geometry parameters*. If the capacitor shape is not plate, the expression for its capacitance is more complex than (6) but in *all cases capacitance depends only on geometrical parameters*.

4.2 CAPACITORS IN SERIES AND CAPACITORS IN PARALLEL

- The two main parameters of a capacitor are: the ***capacitance*** and the ***maximum potential*** *it can support.* If the available capacitors do not fit with requirements of a given circuit, one may use a combination of several capacitors. There are two main ways to connect several capacitors; *in series* or *in parallel*.

- Figure 3 presents the scheme of two capacitors connected *in series*. If connects the points a, b to the battery terminals, the capacitors C_1 and C_2 acquire the same amount of charge Q on their plates because the exterior charges +/- Q transferred by battery emf **induce** the same magnitude of charge -/+Q to the interconnected plates.
As the difference of potential for each capacitor is:

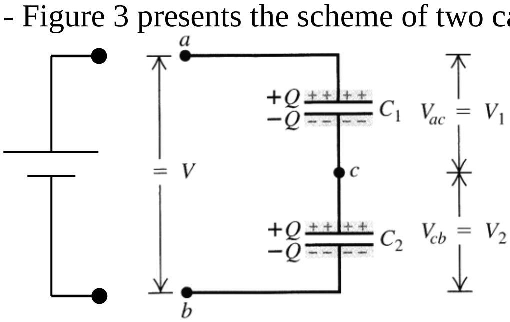


Fig. 3

$$V_1 = \frac{Q}{C_1} = V_{ac} \quad \text{and} \quad V_2 = \frac{Q}{C_2} = V_{cb} \qquad (7)$$

it comes out that $$V = V_{ac} + V_{cb} = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\frac{1}{C_{eq.}} \qquad (8)$$

[2] E is the magnitude(*field strength*) of E-vector. The potential of (-) plate is zero. Do not forget that it is a "-" charged plate.

This expression shows that <u>*the set*</u> of two capacitors connected *in series* behaves the same way as a single *equivalent capacitor* with capacitance **C_{eq}** such that $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad (9)$$

If N capacitors are connected ***in series***, one gets the ***equivalent capacitance*** $$\frac{1}{C_{eq}} = \sum_{i=1}^{N} \frac{1}{C_i} \qquad (10)$$

Note that, for the **same amount of charge "**Q**"** stored on its plates, this "*equivalent capacitor*" can support **higher voltage** on its terminals than each of single capacitors in series " $V^{max} = V^{max}_1 + V^{max}_2 + V^{max}_3 + \ldots$".

-Figure 4 presents the scheme of two capacitors connected *in parallel*. In this case, the same difference of potential applies on the two capacitors and the battery ***emf*** has shifted the total charge amount " $Q_1 + Q_2$ " from/to both of them. The magnitudes of *charges* Q_1 and Q_2 *depend on the capacitance of capacitors as follows* $C_1 = \frac{Q_1}{V}$ and $C_2 = \frac{Q_2}{V}$

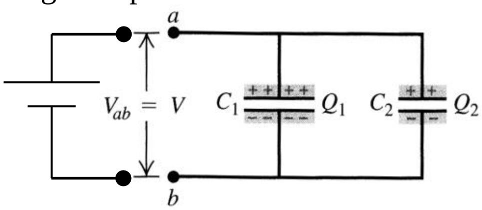


Fig.4

In this case, the *equivalent capacitor* would get the charge $$Q = Q_1 + Q_2 \qquad (12)$$

and its capacitance would be $$Ceq = \frac{Q}{V} = \frac{Q_1 + Q_2}{V} = \frac{Q_1}{V} + \frac{Q_2}{V} = C_1 + C_2 \qquad (13)$$

By connecting ***in parallel*** N capacitors, one gets the ***equivalent capacitance*** $$C_{eq} = \sum_{i=1}^{N} C_1 \qquad (14)$$

This equivalent capacitor " offers **higher storage capacity of charge** " Q_1+Q_2+.. for the **same voltage.**

4.3 ENERGY STORED IN A CAPACITOR

- The ***stored energy*** in a capacitor is ***concentrated in the electric field*** created by plates' charges in the space between them. This *stored energy* is due to the *work spent by emf of* battery when shifting some "free electrons" *from* one plate (that becomes charged "+") and *onto* the other plate (that becomes charged "-"). Note that the electron path passes through the wires and battery (not through space between the plates). This transfer process lasts for a short interval of time until the charge in capacitor get to value ***Q=C*V***. Next, there is no more charge motion in circuit. *Remember that W_{ext} (=- W_{int}) needed to shift a charge inside an electrostatic field "at constant K "depends only on the initial and final locations and not from the path shape.* In the following, to simplify the calculations, one uses a model in which, instead of "-Q" charge, the action of *battery emf* transfers the same amount of "+Q " charge (*from "-" plate to..battery.. to "+" plate*).

- Let's consider the *circuit battery – capacitor* at an intermediary moment when the plates' charge is "***q***" and the ***difference of potential*** between them is "υ". The work ***dW*** done by ***efm*** of battery, for the transport of the next small charge amount "+***dq*** " from "-" plate (with $V_- \equiv 0$) to "+" plate (with $V_+ = \upsilon$) is an *external work* for the ***system electric field-charge***. As $dW_{ext} = dU = dq*(\upsilon - 0)$ one get $$dW_{ext} = \upsilon * dq \qquad (15)$$

As $\upsilon = \frac{q}{C}$, from (15) one gets $$dW_{ext} = \frac{q}{C} * dq \qquad (16)$$

The *total work* done by ***battery emf*** to shift the total charge Q is $$W_{ext} = \int_0^W dW_{ext} = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C} \qquad (17)$$

The principle of energy conservation for ***system electric field-charge*** tells that $W_{ext} = \Delta U_E = U_E - 0 = U_E$ *where* U_E is ***energy electric stored*** inside the capacitor. Next, by using relations (1), (2), one finds out that

$$U_E = \frac{Q^2}{2C} = \frac{Q}{2} * \frac{Q}{C} = \frac{Q}{2} V = \frac{1}{2} C V^2 \qquad (18)$$

- This energy is distributed uniformly inside the volume " ***d*A*** " of space between two capacitor plates.

So, the energy per unit of volume, i.e. the **energy density u,** is
$$u = \frac{\frac{1}{2} C V^2}{d * A} \qquad (19)$$

By substituting $C = \varepsilon_o * \frac{A}{d}$ in (19) one gets
$$u = \frac{\frac{1}{2}(\varepsilon_o * A/d)V^2}{d * A} = \frac{\varepsilon_0}{2} * \frac{V^2}{d^2} = \frac{\varepsilon_0}{2} E^2$$

So, the electric field in space between plates has the ***energy density***
$$u[J/m^3] = \frac{\varepsilon_0}{2} E^2 \qquad (20)$$

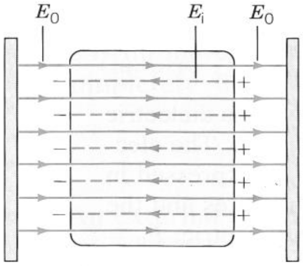
Notes: - Although the expression (20) is derived for the case of a plane plate capacitor in vacuum, it is valid for the ***energy density of any configuration of electric field in vacuum***.
- Even the ***"empty space" may contain energy, for example electric energy.***

Note that the "older interpretation" considered that the electric energy belongs only to a set of electric charges.

4.4 DIELECTRICS

- The actual capacitors contain a dielectric (insulator) material between the conducting plates. One uses the dielectric to: a) *avoid* the *contact* and keep fixed the distance between the plates.
b) *increase* the *maximum difference of potential* V^{max} supported by the capacitor.
c) *increase* the *capacitance* of the capacitor compared to empty configuration.

- The ***dielectric constant*** (**κ**-kappa) of a substance is a measure of "***how much it get polarized*** " under the effect of an external electric field. A polar molecule (H_2O, NH_3..) has a *permanent electric dipole moment*. In presence of an external electric field, there is a torque effect which tents to align the dipole moments $\vec{P}$ along the direction of exterior field ($\vec{E}_0$). At non-polar molecules (H_2,CH_4..), the *exterior field induces*, at first, an electric moment and then the torque tents to align it along $\vec{E}_0$. In both cases, the dielectric surface in front of capacitor plates get opposite sign charge to that of the plate in front(fig.5).



(+) (-)

Fig.5

This is the origin of an additional internal field $\vec{E}_i$ which has opposite direction to $\vec{E}_0$. So, the magnitude of net field inside the dielectric is decreased to
$$E_D = E_o - E_i \qquad (21)$$

Note*: For the same charge amount (Q) on the plates, the magnitude of net electric field inside the dielectric is smaller than the electric field in vacuum.*

- Consider a dielectric substance between the plates of a capacitor with Q charge. If the ***field magnitude*** between plates in vacuum was E_0, in presence of the dielectric, it will become

$$E_D = E_0 - E_i \equiv \frac{E_0}{\kappa} \qquad \kappa \geq 1\_has\_no\_units \tag{22}$$

The magnitude of field inside the capacitor *in vacuum* is $E_0 = \frac{\sigma}{\varepsilon_0}$. In presence of the dielectric it

becomes $$E_D = \frac{E_0}{\kappa} = \frac{\sigma}{\kappa\varepsilon_0} = \frac{\sigma}{\varepsilon} \tag{23}$$

ε_0 is the ***permittivity of vacuum***; **$\varepsilon = \kappa\varepsilon_0$** is known as ***permittivity of dielectric material***

By similar steps, as in case of an electric field inside vacuum capacitor, one may find that the density of electric energy inside a dielectric in capacitor is $$u\left(\frac{J}{m^3}\right) = \frac{\varepsilon}{2}E_D^2 \tag{24}$$

-As the field inside capacitor is uniform, from the relation $V_D = E_D * d = (E_O/\kappa) * d = (E_O * d)/\kappa$

for the *same charge* Q on plates, the potential difference between them is $V_D = \frac{V_0}{\kappa}$ (V_0 in vacuum).

So, if the **capacitance of a capacitor** in vacuum is $C_0 = \frac{Q}{V_0}$, in presence of a dielectric material

between the plates it becomes $$C_D = \frac{Q}{V_D} = \frac{Q}{V_0/\kappa} = \kappa * C_0 \tag{25}$$

This way, one gets another definition of dielectric constant as $$\kappa = \frac{C_D}{C_0} \tag{26}$$

Note: C_D is always larger than C_0 and $\varepsilon > \varepsilon_o$.