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5.1 ELECTRIC CURRENT

-In an electrostatics' experiment, the electric charges may move from one location to another (by friction, mechanical transport or by induction) but this motion lasts only for a *very short interval of time*. However, there is many electric experimental setups that concerns situations where the electric charges move for *a long time*. One says that there is *an* ***electric current*** *if a* ***non- zero net*** *electric* ***charge*** *pass through a flat surface.* **Attention**: When a neutral fluid (*say H_2O*) flows through a hose, there is a continuous movement of positive " p^+ " and negative charges " e^- " charges (*located inside its molecules*) through any hose section but the *net charge passing through a section is zero* and there is no current.

- One may distinguish three main situations where there is an **electric current**:

a) *Free electrons inside a conductor*. In absence of field, the electrons move inside the conductor irregularly and the net charge passing through any section of conductor is zero. But, in presence of an electric field, they get a common motion in opposite sense of $\vec{E}$ field and produce a current.

b) *The ions in the electrolyte of a battery*. The electrolytic solution in a battery contains *positive* **and** *negative ions* that have *irregular* motion (*current zero*) as long as the battery terminals are not connected to a closed circuit. When the battery is connected into a closed circuit, these ions get a directed motion and a build up an electric current inside the battery. A similar situation happens when ions inside a "*container of plasma*" get a directed motion.

c) *The free charges (e*-, p+, ions) in vacuum. This is the case of "e-" used in an oscilloscope screen or accelerated beams of charged particles *(e*-, p+, ions) used in particles' colliding experiments.

The first recorded experiments on electric current are performed by Alessandro Volta in 1799. Also, he built the first battery (*voltaic pile*) and used it in a closed circuit for heating thin wires till incandescence.

-Let's consider the flow of a **net positive charge** through a plane (Fig.1). Assume that the *net* electric *charge* **ΔQ** passes through a plane section during the time interval **Δt**. One measures the *flow of charge* by the *average current* (1) defined as the **net "positive"** charge passing through this section during **1 sec**, i.e.

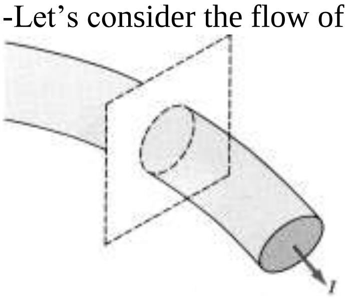

Fig.1

$$I = \frac{\Delta Q}{\Delta t} \qquad (1)$$

If the charge *flow is not steady*, one should refer to an infinitesimal interval of time " ***dt*** " and use the ***instantaneous current***

$$i = lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \qquad (2)$$

Expressions (1-2) define the ***electric current*** as a ***flow rate*** of ***net positive charge through a section***. The ***unit of current*** in SI system is 1Ampere (1A=1C/1s). Often, one uses current subunits; mA=10^{-3}A, 1μA=10^{-6}A, 1nA=10^{-9}A and 1pA=10^{-12}A . Note that the ***current is a scalar*** (*not a vector*). The current follows the wire direction when propagating through a wire. **By definition**, the *current starts at the positive terminal* of the source (battery) and flows ***through circuit*** versus the negative terminal (Fig.2).

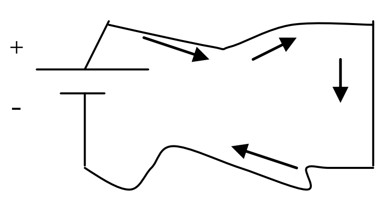

Fig.2

- The reference of current to the positive charge direction means that, in case of conductors in a circuit (*wires, resistors*,..), the direction of current is opposite to the electrons' motion. This definition, that simplifies the calculations, means that **current in a circuits is directed from higher potentials versus lower potentials.** So, whenever the moving charge is "-Q ", one refers to the motion of charge "+Q" in *opposite direction at same speed* (Fig.3). ***Inside*** a battery electrolyte, the opposite charge ions move in opposite direction and there are *two components* that add together to build up the ***net current***; one due to positive ions (moving towards ***"+" terminal***) and one due to negative ions (travelling in opposite direction i.e. versus terminal "-"). The direction of current inside a battery is ***from "-" to "+" terminal.***

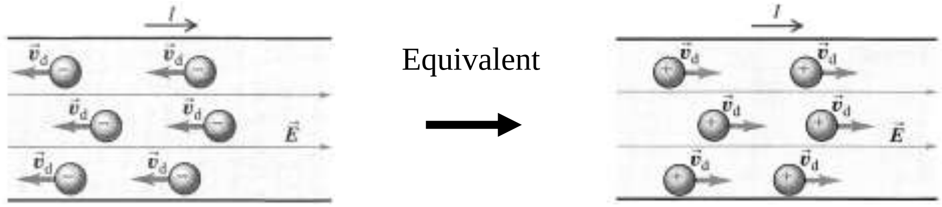

Fig.3

-A wire is constituted by a metallic conductor (Cu, Al,.). As long as it is not connected into a circuit, the free electrons move irregularly. If one connects a wire end to the terminal of a battery, "*some of charges pass fast to the wire*" and build the same potential (= $V_{terminal}$) at all wire point. Afterwards, the wire behaves as conductor in electrostatics; there is only a field perpendicular to wire surface and ***all charges*** are located "at rest" on its surface. If one follows on by connecting the other end of wire to the other battery terminal and *closes the circuit* (see fig.2), the free electrons in wire will get the action of an electric field due to difference of potential exerted between the two ends of wire. This action will make them move continuously in *opposite direction to the local electric field* . Their motion produces an **electric current (***opposite to e- motion sense***) along the local field direction**. Note: *While the current flows*, the <u>*net charge in wire remains zero*</u> *because, every second, the same amount of charge that get into wire through one terminal and goes out of it from the other terminal.*

5.2 CURRENT DENSITY

- One has defined the electric current as the net positive charge that passes "*through a section*". In this model one assumes an uniform flow distribution of charges through the conductor section. But, if one needs to know about the charge flow at different locations on a section of conductor, one has to refer to the ***current density***, a physical parameter that provides this type of information.

- Consider a beam of "+" charged particles all moving at same velocity $\vec{v}_d$ along current direction. Assume that each particle carries the same amount of charge (+q) and moves inside a cylinder with section area "A". If the ***volume density*** of these charges is "***n***", there is $N_{ch} = \boldsymbol{n * (v_d * \Delta t) * A}$ charges inside a cylinder with length $\boldsymbol{l = v_d * \Delta t}$. All those charges pass through a section "A" for Δt sec.

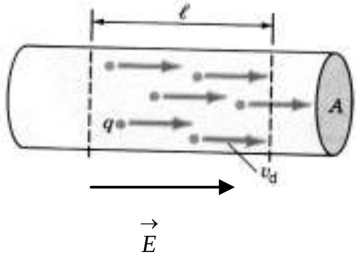

Fig.4

This means that the total charge $\Delta Q = q * N_{ch}$ or

$$\Delta Q = (n * v_d * A * q) * \boldsymbol{\Delta t} \qquad (3)$$

passes through the section ***A*** for time ***Δt.***

So, one gets the following expression for the current

$$I = \frac{\Delta Q}{\Delta t} = n * v_d * A * q \qquad (4)$$

Then, the ***current passing through*** *the* ***unit of area section*** **(** known as current density " J ") is

$$J = \frac{I}{A} = \frac{n * v_d * A * q}{A} = n * q * v_d \qquad (5)$$

The unit of current density in SI system is [***A/m²***]. Actually, the **current density** is defined as a **vector**

$$\vec{J} = nq\vec{\upsilon}_d \qquad (6)$$

- Note that $\vec{J}$ may have different direction at different points of the same section crossed by a current. For *negative* charges, $\vec{J}$ vector has the opposite direction to $\vec{\upsilon}_d$ [1] due to "-" sign of ***q***.

Remember*: The* ***current*** *is a* ***scalar*** *but the* ***current density*** *is a* ***vector****.*

5.3 RESISTANCE AND RESISTIVITY

- Experiments carried out with *conducting objects* showed that: *when one applies a difference of potential* ***V*** *at object boundaries, the current* ***I*** *passing through the object is* ***proportional to V****.* The proportionality constant depends on the *material*, the object *geometry* and *range of V*-values, but for any conductor, a <u>*relation of type*</u> $I = a * V$ holds on. One has defined the **resistance** of a conductor as the inverse of this constant " $R=1/a$ ". So, $R = \frac{V}{I}$ (unit 1ohm = 1V/1A) (7)

<u>*The resistance of a conductor defines how many volts one must apply at its boundaries so that a current with magnitude 1A passes through the conductor*</u>.

- In normal conditions, the *free electrons* inside a neutral conductor *move irregularly* due to their **thermal energy** at a **speed** *up to* ***10^5-10^6m/s***. When a potential difference is applied on conductor, the electric field gives them an ***ordered movement*** against the field direction (i.e. $\vec{v}_d \uparrow\downarrow \vec{E}$). The magnitude of " $\vec{v}_d$ *is kept under control* " and fixed to a certain value by the number of their "*collisions*" to positive ions that form the crystal lattice. It it is *proportional to magnitude of the electric field in the wire (* $\boldsymbol{v_d \sim E}$) but it depends on the *ion density* of crystal lattice, too. Shortly, the ***effect of the applied voltage on the free e- in a conductor, consists to an additional very small***[2] (~ 10^{-4}m/s) but directed <u>***drift speed***</u> **.** By referring to the direction of motion for "+" charges and by using the relation (6) one find out that $\vec{J} \sim \vec{\upsilon}_d \sim \vec{E}$. *The relation between* ***local current density*** *and the* ***local electric field*** *is written as*

$$\vec{J} = \frac{1}{\rho}\vec{E} = \sigma\vec{E} \qquad (8)$$

where ρ [Ω*meter] is the **resistivity** and σ[1/Ω*m] is the **conductivity of conductor.** ***Resistivity or conductivity*** is a property of the <u>***material***</u> that constitutes the conductor.

Let's consider again fig.4 and assume that *electric field is uniform* in the piece of wire with length ***l*** . Then, the magnitude of potential difference across its borders is $V = E * l$ (9)

[1] This would be a **drift velocity for e⁻ on a wire**.
[2] compare to the magnitude of velocity due to thermal motion

Form (9) one get $E = V/l$ and after substituting this at (8) $J = \frac{E}{\rho} = \frac{V}{l} * \frac{1}{\rho}$ one find out that $I = J * A = \frac{V}{l * \rho} A$ (10). From expression (7) one gets $I = \frac{V}{R}$ (11)

Next, by comparing (10) to (11) one finds out that $R = \rho \frac{l}{A}$ (12)

ρ - resistivity of wire material; ***l***- wire length ***A***-wire section

- The experimental measurements show that ***<u>above 20^{o}C</u>***, the **resistivity** of **<u>metals</u>** *increases linearly* with temperature as

$$\rho = \rho_0 \left[1 + \alpha(T - T_0))\right] \quad (13)$$

The *temperature coefficient* **α** [1/C^0] *of resistivity* and the reference values of *resistivity* ρ_0 at 20^{o}C (273^{o}K) for **metals** are given in tabulated way. The experiments show that the expression (13) is not valid for low temperatures (T < 20^{o}C) and ρ ≠ 0 even for T = 0^{o}K (see Fig 5.a).

- The **<u>superconductors</u>** behave *somewhat similarly* at high temperature and completely differently at <u>*low*</u> <u>*temperatures*</u>; their resistivity ***drops to zero*** *under a certain temperature* (Fig. 5.b).
Note that the **resistivity** of **<u>semiconductors</u>** *decreases* while temperature increases (Fig.5.c)

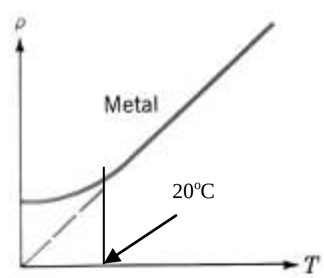

Fig.5a

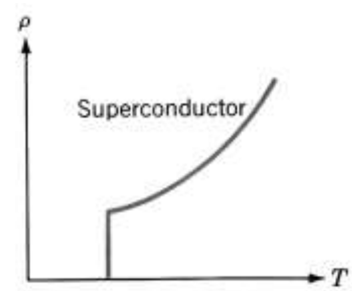

Fig.5b

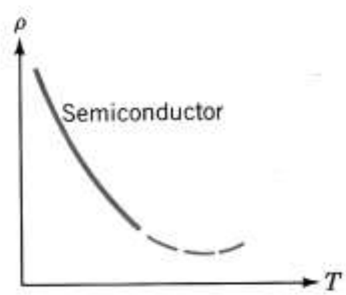

Fig.5c

5.4 OHM'S LAW

- **In general**, the numerical value of **resistance** R of a conductor (*defined as* $R = \frac{V}{I}$) depends on the range of "V" values. But, there is a group of conductors for which it is a *constant that does not depend on V value*. **The Ohm's law** applies for all devices where **R is a constant and it says:** *The difference of potential V across the device is direct proportional to current through it*

$$V = R * I \quad (14)$$

Remember: This relation presents Ohm's law <u>*only if **R is a constant** independent on V and I*</u>.

-A material that obeys to Ohm's law is called **ohmic material**. Provided that the **temperature** is kept **constant** during the measurements, the **metals** are ohmic materials. **Carbon and some alloys behave as ohmic** materials *even when the temperature varies over a wide range*. This is the main reason why one uses **carbon to build high quality resistors.** Note that, *inside a certain range of temperatures, a resistor built by metals or ceramic does provide a constant resistance , too.*

5.5 POWER IN A DC CIRCUIT

REMINDER: When the weight pulls an object down an inclined plane, the *force* $\vec{F}_G$ achieves a **positive work** . *This* ***positive work*** *done* ***by " internal " force*** $\vec{F}_G$ ***diminishes the potential*** *energy of object (more precisely of* ***the system*** *gravitation field-object).* If U_{up} and U_{down} are the potential energies for object " up " and "down", then the **gravitational field** of earth achieves *a positive work*

$$W_{int} = -\Delta U = -(U_{fin} - U_{in}) = -(U_{down} - U_{up}) = U_{up} - U_{down} = U_{ini} - U_{fin} \qquad (15)$$

- The ***electric field*** in a circuit pushes the charge ***"+q"*** along **local** $\vec{E}$ direction and this way ***builds the current*** inside the circuit. As $\vec{F}_{el}$ and displacement of ***"+q"*** charge are directed in same way, the **electric field** delivers a **positive work "W_{int}"** while shifting this charge *from a higher to a lower* potential along the circuit. This work can be expressed through the initial and final potential energies of the charge " **q** " in the electric field (*or that of the system electric field-charge*) as follows

$$W_{int} = -\Delta U = -(U_{fin} - U_{in}) = -(U_{Low} - U_{High}) = U_{High} - U_{Low} = (V_{High} - V_{Low}) * q$$

which generally is written as $$W_{int} = V * q \qquad (16)$$

Note: *In electric circuits, one uses the notation V for the difference of potential* $V_{High} - V_{Low} \equiv V$.

During an infinitesimal interval of time "***dt",*** the ***electric field moves the amount of charge "+dq"*** through potential difference "*V* " in circuit *and* ***delivers*** *the infinitesimal internal work " dW(> 0)"*

$$dW = V * dq \qquad (17)$$

So, it comes out that the **electric field** ***provides a power (*** *i.e.* delivers energy *at a rate****)*** *in circuit*

$$P = \frac{dW}{dt} = V * \frac{dq}{dt} = V * I \qquad (18)$$

"V " is the potential difference <u>between two points</u> in circuit; "I" is the current passing through both of them (it must be the same current); "P" is the power delivered by electric field in the portion of circuit between the two points.

-As explained, the electric field delivers work to put charges in an *ordered motion* and build a current **I in circuit**. Assume now that an *electric device* is connected into the circuit (see fig. 6).

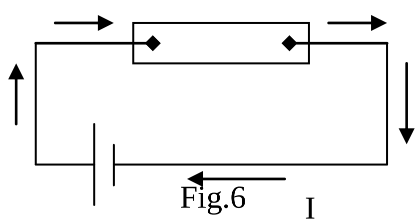

Fig.6

a) If the device is a **resistor**, all the delivered **energy by field in it is transferred into thermal energy**. As ***e-*** has a ***<u>constant drift speed v_d</u>***, its average kinetic energy remains constant. This means that, once it gets the *drift speed* v_d, it transfers all the amount of energy received from the field to ions of crystal lattice of resistor by "*collisions"*. This energy increases the temperature of resistor.

b) If the device is an electric motor, the product ***V*I*** (*both measured at motor terminals*) gives the power delivered by electric field into the motor. It is ***mainly*** converted into *mechanic work* provided by the electric motor. But, always, a portion (*small in a high quality device*) is converted into heat.

c) If the device is a recharging battery, the energy delivered by electric field is transferred ***mainly*** into *chemical energy* of electrolyte solution. But a small portion is converted into heat, too.
Note: In any situation a portion of "electric energy" is transferred into thermal energy.

- The general law of energy conservation tells that: *the power delivered by the electric field is transferred with the same rate into other forms of energy." No part of energy can disappear".*

Remember: *Once getting to speed* υ_d *, the electrons do not store energy any more*. So, once the parameters "***V** and **I*** " in circuit get fixed values , the *electrons behave as "a switch that converts the electric energy provided by field into other forms of energy"*.

If the circuit consists of *only ohmic conductors* , the electrons will convert all energy provided by electric field into thermal energy. If an ohmic conductor has the resistance R, it comes out that the amount of *thermal power* delivered into the conductor is

$$P_R = V_R * I_R = (I * R) * I = I^2 * R = \frac{V^2}{R} \qquad (19)$$

Note: *The expression (18) is valid for any kind of electric device connected between the two points of circuit ; the expression (19) is a particular application of expression (18) that concerns the conversion of electric energy into thermal energy inside a resistor.*

As the SI unit of power is watt [W] one can deduce that, in electricity, 1W = 1V*1A.

Note that SI energy unit 1[J] = [W*s] = [V*A*s] =[V*(C/s)*s]=1V*1C