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6.1 ELECTROMOTIVE FORCE

-The terminal "+" of a charged battery contains unbalanced positive charges which build up an $\vec{E}$ field around it. If one connects one end of a conducting wire to terminal "+", this electric field will attract the free electrons from the wire into the battery. This charge motion (*that generates a current in the wire*) lasts for a very short time because all the wire volume gets fast the potential "V" of terminal "+" and the motion of electrons stops. But, if one connects the other wire end to terminal "-" of battery, there is a difference potential "V" applied on wire and a ***current*** "I=V/ R" will flow through it. Note that it will be a *steady current* in a *wire* as long as there is a *fixed potential difference between its two ends.*

- If the *current* **I** flows *only* in *one direction* through a *circuit* (Fig. 1) one deals with a **DC** *current* and **DC** *circuit.* Generally, in a DC circuit, the voltage and current do not change with time. At DC circuit shown in fig.1, the ***positive*** charges leave the higher potential (*terminal +)* and move through resistor R toward the lower potential(*terminal* -). Once **inside the battery**, the positive charges have to move from the lower (-) to the higher (+) potential so that the **current** in the circuit be steady. The electrostatic field inside the battery is opposite to the direction of motion for (+) charges; so, it tends to block the current. The fact that current goes on, means that the positive charges overcome this *electrostatic potential barrier* ("*hill"*) inside the battery. *How does this happen?* There is only one answer; the *action of pulling them up the barrier* is provided by external (***non-electrostatic***) sources of energy. For a common use battery, the *external source* contains " *chemical energy* ".

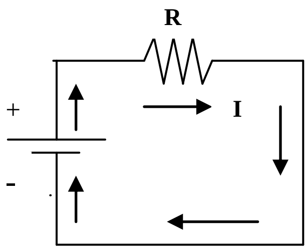


Fig. 1

- This *external (for system electric field - charge) action* is due to a set of *chemical reactions* that happen *in electrolyte*. In a lead–acid cell, one has immersed a **PbO_2 plate** and a **Pb plate** into an **aqueous solution of H_2SO_4**. The *strong* action of the electric field due to dipole moment of a H_2O molecule in solution dissociates some of H_2SO_4 surrounding molecules into *positive ions* (H^+) and *negative ions* (HSO_4^- or SO_4^{2-}). Next, due to the specific *chemical affinity* of those ions to **PbO_2** and **Pb plates,** the following reactions happen in solution *at the interfaces between the electrolyte and plates*:

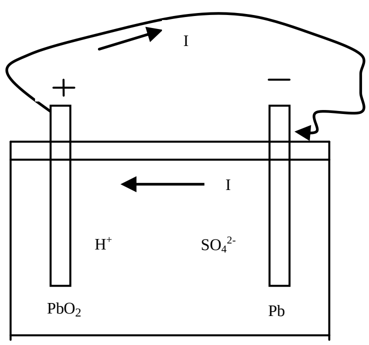


Fig.2

At **PbO_2 plate**: $PbO_2 + 3H^+ + HSO_4^- + 2e^- \rightarrow PbSO_4 + 2H_2O$ (1)

So, (2e-) are *removed from* **PbO_2 plate** and it becomes **Terminal "+"**

At **Pb plate** : $Pb + SO_4^{2-} \rightarrow PbSO_4 + 2e^-$ (2)

So, (2e-) are *stored at* **Pb plate** and it becomes **Terminal "-"**

So, the first reaction " *stores q = +2e* " on plate " + " while the second reaction " *removes q = +2e* " from plate " - ". The charged plates build up a local $\vec{E}$ field that pushes ions away from the same sign plates. So, the reactions (1, 2) tent to *decrease the concentrations of ions around the plates*. This effect is opposed by ***diffusion processes*** which tent to keep uniform density through electrolyte by bringing other ions close to plates " i.e. *by moving them against the electric potential barrier inside the battery*". In **an open circuit,** the chemical reactions at interfaces of plates *stop* when the stored charge on plates is such that the related field $\vec{E}$ can blocks further approach(due to diffusion) of the same sign ions . For a lead battery, this corresponds to the potential difference **2.05V** between the two plates **in open circuit** (*a 12V battery contains 6 pair of plates*).

-Once one switches a charged battery into a **closed circuit**, the " + *" charges* will go, *via the circuit,* into the terminal "-" (or electrons into terminal "+"). This **decreases** the net charge of each terminal and the related internal electric field at plate interface. As consequence, more **ions move** close to the plates and

the rate of chemical reactions increases instantaneously. These moving ions transport " + " charges toward " + " plate and " - " charges toward " - " plate. This means a current (***in direction of motion of "+"charges)*** from the lower potential to the higher potential inside the battery (Fig.2). Very fast, the rate of *chemical reactions* get set to a value that fits to the *principle of charge conservation* condition:

The magnitude of current must be the same outside in circuit and inside the battery.

- From electric modelling point of view, the **chemical source** (*external source to the system field-charge*) provides an **external work** that moves the positive charges uphill the electric potential inside battery. *The* ***work*** *provided by a* ***non-electrostatic source*** to shift the ***charge +1C*** from the ***lower potential*** *to the* ***higher potential*** *terminal is known as* ***emf***[1] *and is presented by the symbol* $\mathcal{E}$ *[J/C i.e. volts].* When moving a charge +***q*** *uphill the potential difference*, the *emf source* achieves the *external work*

$$W_{ext} = W_{emf} = q * \mathcal{E} \qquad (3)$$

- Both, the **electric potential** and the **emf** are related to the *work done during the displacement of +1C charge* but they have different physical origin and they shift +q charge in "opposite senses". The source of electric potential is the electrostatic field while the source *of* ***emf*** $\mathcal{E}$ is a ***non-electric phenomenon***. In an active electric circuit, there is one ***emf***, at least. *It provides the charge distribution at origin of the electrostatic field which drives the current through the circuit.* Note that the source of ***emf*** is not always chemical. It may be magnetic (electric generator), mechanical (Van de Graff generator), etc..

- When a charged battery is part of an ***open circuit,*** there is a potential difference[2] $V_{open} = (V^+ - V^-)$ between its terminals. In this case, if there is a shift of the charge +***q*** from "-" plate to "+" plate, the non-electric **emf** source would provide the *positive* amount of work $W_{emf} = W_{ext} = q * \mathcal{E}$ which goes to ***increase*** the electrostatic energy of battery by $\Delta U = q*(V^+ - V^-) = qV_{open}$.

So, as $W_{emf} = W_{ext} = \Delta U \dashrightarrow q * \mathcal{E} = q\,(V^+ - V^-) \dashrightarrow \mathcal{E} = V^+ - V^- \dashrightarrow \mathcal{E} = V_{open}$ (4)

- In the ***closed circuit*** shown in Fig.1, the *voltage V* applied between the ends of the resistor R is expressed as $V = I*R$ (*Ohms' law*) and it is equal to that on battery terminals. So, considering an ***ideal source*** (without internal resistance), one would find

$$\mathcal{E} = V = I*R \qquad (5)$$

Relation (5) tells that, in a closed circuit containing an *ideal source* **emf** and a resistor, the potential *rise* ($V=\mathcal{E}$) inside source *due to emf* is equal to potential *drop* ($V_R = I*R$) through the resistor.
A real battery in a closed circuit gets heated. This shows that it has a resistance-like behaviour, or in other words it presents an *internal resistance* to the current. So, it comes out that a real source in a circuit is equivalent to an ideal source plus a resistance (**r**) in series (Fig.3). In this case, one part of potential rise provided by emf $\mathcal{E}$ is spent to compensate for the ***drop*** of potential ΔV_r = **I*r** through the resistance and only the remaining difference (**$\mathcal{E}$ - I*r)** applies to the circuit outside the battery. So, the **potential difference between terminals** of a battery *in a closed circuit* (V_{closed}) is smaller than its emf $\mathcal{E}$.

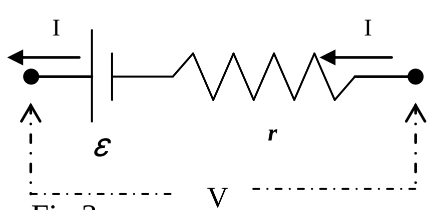


Fig.3

$$V_{closed} = \mathcal{E} - I*r \qquad (6)$$

Relation (4) shows that one can find $\mathcal{E}$ value by V_{open} measurements. For a real battery in a circuit with a resistance R (fig.1), the relation (5) transforms to $\mathcal{E} - I*r = V_{closed} \equiv V_R = I*R$ (7)

and $\mathcal{E} = I*r + I*R = I*(r+R) \longrightarrow I = \mathcal{E}/(r+R)$ (8)

[1] "Electromotive force". This wrong nomination (because $\mathcal{E}$ is *energy/charge* and not force) remained for historical purposes.

[2] In an electrical circuit, one uses the notation $V_{ab} = V_a - V_b$, where $V_a > V_b$

6.2 KIRCHHOFF RULES

-These are two basic rules that one applies to ***calculate the current and potential*** values in electric circuits. Actually, these rules are expressions of **conservation laws** for **charge** and **energy** in a circuit.

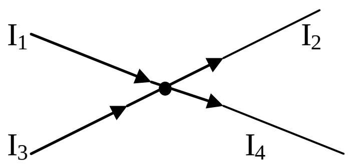


Fig.4

- A real circuit may contain several **junction** (Fig.4) **points**.
First Rule: *The algebraic sum of currents that enter and leave a junction must be zero. One considers* ***positive*** *a current that* ***enters*** *the junction and* ***negative*** *a current that* ***leaves*** *the junction.*
Essentially, this rule express the fact that the charge cannot get stocked or created into a circuit junction. ***The charge quantity that enters the junction is equal to that leaving it*** (*charge conservation law*).

- In a real circuit one may identify several loops (three loops and two junctions in Fig.5).

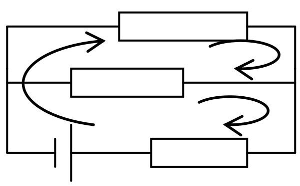


Fig. 5

Second Rule: *The algebraic sum of potential changes along a closed loop is zero.* ***Remember*** *that the* ***potential V*** *at a location corresponds to the potential energy of +1C charge at this location.* As the potential energy depends only on the location, it comes out that after the circulation through a loop, it gets the initial value; $V_{end} = V_{init}$ and $\mathbf{\Delta V = 0}$; Actually, this rule is the energy *conservation law* applied for a charge 1C.

-The Fig.6 shows the graph of potential evolution into a simple circuit. Note that one takes as zero[3] the potential of terminal "-" and it remains constant along an *ideal* wire ($R_{wire} \equiv 0$ *ohm*), i.e. $V_A = V_F = 0$.

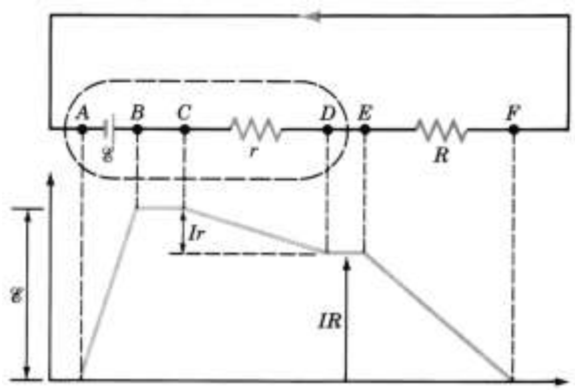


Fig. 6

One may refer to the energy of +1C charge while moving around the loop to prove the second rule of Kirchhof. When entering the terminal "-", this charge has *only drift kinetic energy*; its potential energy is zero ($\boldsymbol{V_A = V- = 0}$).
It gets the *potential energy* $\boldsymbol{\mathcal{E}}$ ($\boldsymbol{V_B (=V_C) = \mathcal{E}}$) while passing through the battery. It looses one portion ($\boldsymbol{V_r = I * r}$) of this energy inside the battery due to internal resistance ***r***. It keeps the same *potential energy* through the wire till the resistor ***R***. It looses the *potential energy* portion ($\boldsymbol{V_R = I * R}$) into resistor and gets to *potential energy* $\boldsymbol{V_F = 0}$.

- When applying the second rule, one may select the circulation sense in loop the way one likes.

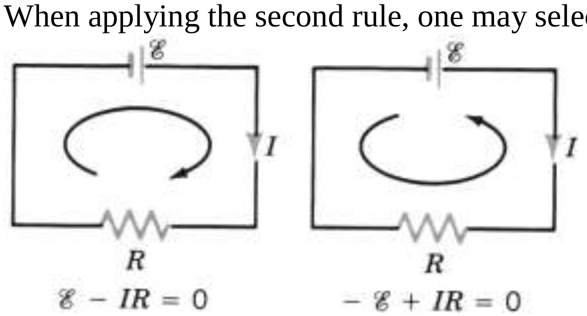


Fig.7

Meanwhile, one has to remember that the ***potential decreases following the direction of current*** in the circuit and increases along the opposite direction (see fig.7).

[3] - In a circuit that contains more than one source, one assigns V = 0 to the (-) plate of one of them.
- If a point of circuit is grounded, one assigns V = 0 at this point.

6.3 COMBINATIONS OF RESISTORS

- Often, it is useful to know " *the equivalent resistor to a set of resistors into a circuit* ". To find the equivalent resistor one starts by grouping them into sets of *resistors in series* and *resistors in parallel*.

a) Resistors in series in a circuit.

If two resistors in series are connected to the battery, the difference of potential between terminals (V) applies to the end points (1,4) of set and the ***same current*** passes through each resistor. Ohm's law tells that, following the current sense, the potential drops by ***I*R₁*** between points (1-2), remains constant in wire section (2-3) and drops by ***I*R₂*** in section (3-4).

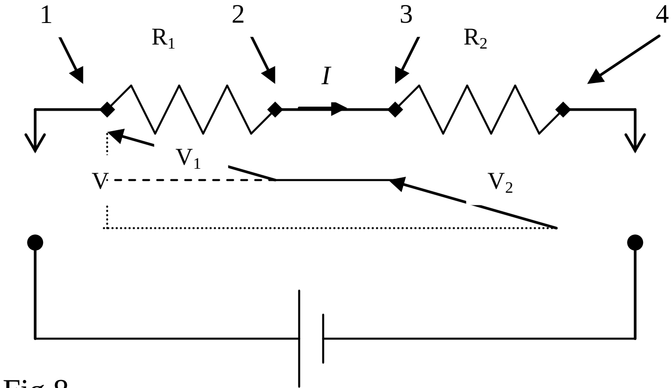


Fig 8

As the total potential drop (***V***) between points (1- 4) is the sum of two consecutive potential drops;

$$V = V_1 + V_2 = I * R_1 + I * R_2 = I * (R_1 + R_2) = I * R_{eq} \quad (9)$$

So, $R_{eq} = R_1 + R_2$. For N resistors in series one gets $R_{eq} = \sum_{i=1}^{N} R_i$ (10)

Remember that if $\boldsymbol{R_{large}}$ is the largest resistance of the set, then $\boldsymbol{R_{eq} > R_{large}}$.

b) Resistors in parallel in a circuit.

In this case, the first Kirchhoff's rule gives:

$$\boldsymbol{I = I_1 + I_2} \quad (11)$$

Next, one expresses the current at each resistor through the potential drop at its ends (the same V)

$$I_1 = \frac{V}{R_1}\_and\_I_2 = \frac{V}{R_2} \quad (12)$$

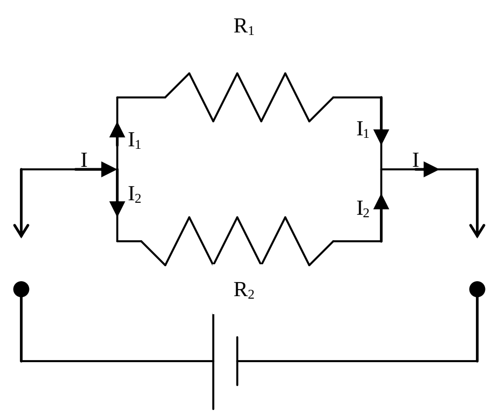


Fig.9

By substituting at (11) one gets $I = \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} = V * (\frac{1}{R_1} + \frac{1}{R_2}) \Rightarrow\Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$ (13)

For a set of N resistors in parallel one gets the equivalent resistor value as $\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}$ (14)

Remember that if $\boldsymbol{R_{small}}$ is the smallest resistance of the set, then $\boldsymbol{R_{eq} < R_{small}}$.

6.4 DIRECT CURRENT INSTRUMENTS

- ***The Ammeter*** is an instrument that measures the current passing by a ***given point of circuit***. This device has a very ***low resistance***. One measures the current by introducing the ammeter ***in series*** with the circuit (Fig.10) at the point of interest. As its resistance is very low, its introduction in circuit does not alter significantly the current in circuit; the value given by the instrument is practically the same as the current in the closed circuit without ammeter.

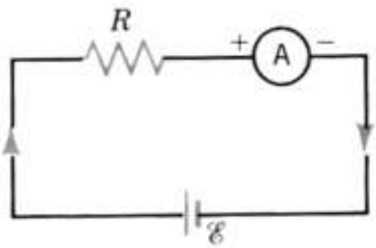


Fig.10

Example. Consider that a closed circuit contains an *ideal* source with $\mathcal{E} = 12V$ and a resistor with $R = 20\Omega$. The current in circuit without ammeter is $I = 12/20 = 0.6A$. If we use an ammeter with $R_{am} = 0.1\Omega$ the current becomes $I_1 = 12/20.1 = 0.597A$, i.e. only 0.003A smaller. This means an *inaccuracy* (0.003/0.6)*100%= 0.5% which is inside range of precision ***ε = (0.5÷1)% used in general for electric measurements.*** Note that to change the scale range of an ammeter, one changes the value of R_{am} so that it *keeps the* <u>similar accuracy</u> *of measurements*.

-One uses a ***voltmeter*** to measure the ***potential difference*** between ***two points*** in a ***circuit***. This type of instrument has a ***very large resistance***. To measure the difference of potential between two points one has to insert the voltmeter *in parallel* to the part of circuit between these two points (Fig.11).

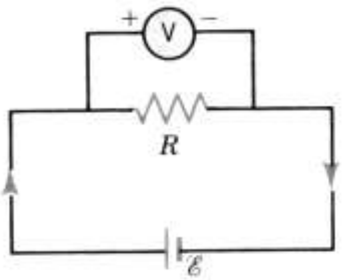


Fig.11

As the resistance of voltmeter is large, its introduction does not affect significantly the current in circuit. Thus, the potential drop to the resistor R (V=I*R) is essentially the same as if the voltmeter were not in circuit. <u>*Example*</u>: For a **12V** source *and* $R = 20\Omega$ in circuit, $I = 12/20 = 0.6A$. When using a voltmeter with $R = 10K\Omega$, $1/R_{eq} = (1/20) + (1/10^4) = 0.0501$ $R_{eq} = 1/0.0501 = 19.96\Omega$ and $I_1 = 12/19.96 = 0.601A$. As the current through voltmeter is 12/10000 = 0.012A, it comes out that the current through the resistance R is *practically unchanged* (0.601-0.012~0.6A).

- One uses an ***ohmmeter*** to measure the resistance. This instrument contains a *battery* $\mathcal{E}$, *a variable resistance* R_s and one ammeter ***A*** (Fig.12) with resistance R_A. In fact, this instrument measures the current but its ***scale*** is ***calibrated to corresponding values of resistance***. When one selects a given range, R_s is set to a fixed value. If one would connect the terminals and produce " *a short or court-circuit* ", one would measure the current that corresponds to the resistance $R_A + Rs$ in circuit

$$I_0 = \mathcal{E} / (R_A + Rs) \qquad R_A \text{ - ammeter small resistance} \qquad (15)$$

If one court-circuit the two terminals of ohmmeter, its ***reading*** *(or the needle****) will go to the maximum (****end of scale****)***.

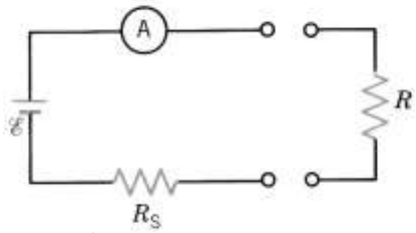


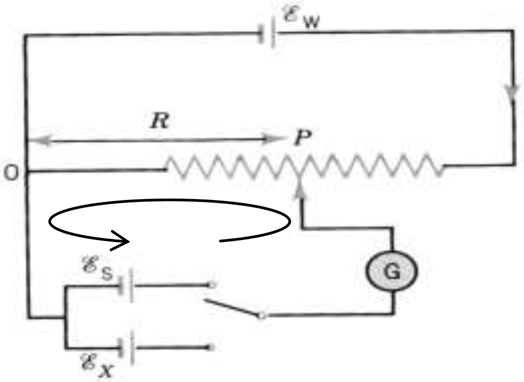
Fig.12

Next, with ***R*** in closed circuit $\qquad I_R = \mathcal{E} / [(R_A + Rs) + R] \qquad (16)$

With R is in circuit, $I_R < I_0$ and the reading falls inside the scale. The measurement is based on the "difference" ($I_0 - I_R$) and appears as resistance ***R*** in (Ω) on the pre-calibrated scale of instrument.

- **Galvanometer.** Both old versions of voltmeters and ammeters have a needle and their function was based on a sensitive current measuring device called a **galvanometer**. A galvanometer is characterized by two main parameters; its ***resistance*** (very *low*) and the ***current value*** *that* produces a *maximum scale reading* (or maximum needle *deflection*).
To convert a galvanometer to a voltmeter, one connects a *large resistance in series* and gets a device with large resistance.
To convert a galvanometer to an ammeter, one connects a ***low resistance*** (*called the "****shunt****" resistance) in parallel* in order to get a device with a very low resistance.

- The ***potentiometer is*** a device that provides different V- values by " *dividing* " a given potential difference V_{max} . It is often used as part of scheme given in fig.13 to measure the ***emf*** of a battery.



$\mathcal{E}_W$ supplies a current through a *variable resistance* R (≤ 1m long slide wire) and one can get out *variable values of potential difference* through points OP. The galvanometer G (*sensible ammeter for small current*) is at first connected to standard battery $\mathcal{E}_S$ (***Cd cell 1.018V***). One fits sliding for no current through G . The 2nd rule of Kirchhoff applied for the small loop that contains $\mathcal{E}_S$ and the galvanometer G gives

Fig.13

$$\mathcal{E}_S - I^*R_S = 0 \qquad (17)$$

Next, one switches in the circuit the unknown battery with **emf** $\mathcal{E}_X$ and fits anew sliding for no current through galvanometer G. In this case the 2nd rule of Kirchhoff will give

$$\mathcal{E}_X - I^*R_X = 0 \qquad (18)$$

Note that the current ***I*** through the section OP is ***the same in the two cases*** because there is no current coming from $\mathcal{E}_S$, $\mathcal{E}_X$ sources in the "small circuit" that contains G. So, by isolating ***I*** at (17-18) one gets

$$I = \mathcal{E}_S / R_S = \mathcal{E}_X / R_X \qquad \text{and} \qquad \mathcal{E}_X = \mathcal{E}_S * R_X / R_S \qquad (19)$$

Then, as $R_s = \rho * L_s / A_s$ and $R_x = \rho * L_x / A_x$, it comes out that $\mathcal{E}_X = \mathcal{E}_S * L_x / L_s$ (20)

6.5 RC CIRCUIT

- When a capacitor is connected to a battery in a closed circuit, at a given time " *t* ", there is a charge ***q(t)*** and a potential difference $\upsilon(t) = q(t) / C$ between its plates; the electric charges continue to store onto its plates until its charge gets to a certain maximum value $\mathbf{Q_{max}}$. Once the capacitor gets the charge $\mathbf{Q_{max}}$ the current through the circuit stops; the potential difference between plates becomes equal to $\boldsymbol{\varepsilon}$ (*battery emf*) and the charge of capacitor $Q_{max} = C * \varepsilon$. Let's analyze the charging process for a circuit with an ideal *emf* $\boldsymbol{\varepsilon}$ and a capacitor ***C in series*** with a resistor R (see fig. 14). In the following, *the quantities that do change with time are noted by lowercase letters and those that do not change by uppercase letters*.

a) CHARGING THE CAPACITOR (in a RC in series circuit)

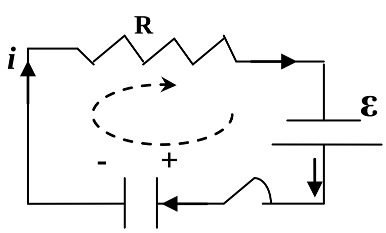


Fig.14 C

By using the 2nd Kirchof's rule we get $\varepsilon - \upsilon_C - \upsilon_R = 0$ (21)

$\upsilon_C = q / C$ is the potential drop through the capacitor (*q* is the charge in capacitor) and $\upsilon_R = R * i$ is the potential drop in resistor at time " t ".

One can rewrite relation (21) in the form $\varepsilon - \frac{q}{C} - R * i = 0$ or $C * \varepsilon - q = (RC)i$ (22)
As $i = dq/dt$, eq.22 takes the form $C * \varepsilon - q = (RC)dq/dt$ and as $\boldsymbol{C*\varepsilon = Q_{max}}$, one can rewrite it in the form

$$\frac{dq}{Q_{max}-q} = dt/(RC) \qquad \text{or} \qquad \frac{-d(Q_{max}-q)}{Q_{max}-q} = dt/(RC)$$

After noting $Z = Q_{max} - q$ and taking the integral of both sides as $\int dZ/Z = -(\frac{1}{RC}) \int dt$
one gets $lnZ = -\frac{t}{RC} + k$ i.e. $ln(Q_{max} - q\ ) = -\frac{t}{RC} + k$ where "k" is a constant.

At t = 0 the charge of capacitor is zero "i.e. *q(0) = 0* " and this brings to $lnQ_{max} = k$. Hence, one can rewrite the last expression as

$$ln(Q_{max} - q\ ) - lnQ_{max} = -\frac{t}{RC} \quad or \quad ln[(Q_{max} - q\ )/Q_{max}] = -\frac{t}{RC} \quad or \quad \ln\left(1 - \frac{q}{Q_{max}}\right) = -\frac{t}{RC}$$

From this last expression, one get $1 - \frac{q}{Q_{max}} = \exp\left(-\frac{t}{RC}\right)$ *or* $\frac{q}{Q_{max}} = 1 - \exp\left(-\frac{t}{RC}\right)$

which can be written as

$$\boldsymbol{q = Q_{max}[1 - exp(-t/\tau)]} \qquad (23)$$

by introducing the " ***time constant*** " $\boldsymbol{\tau = RC}$ of the circuit that contains ***RC in series***.

Graph of charge in the capacitor during the charge of capacitor (expression 23).

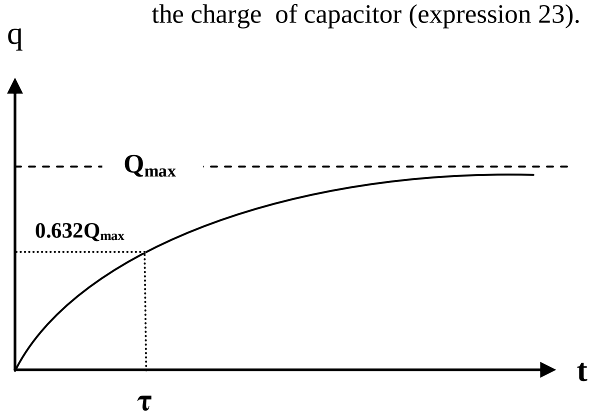


Fig.15.a

Graph of current in the circuit during the charge of capacitor (expression 24).

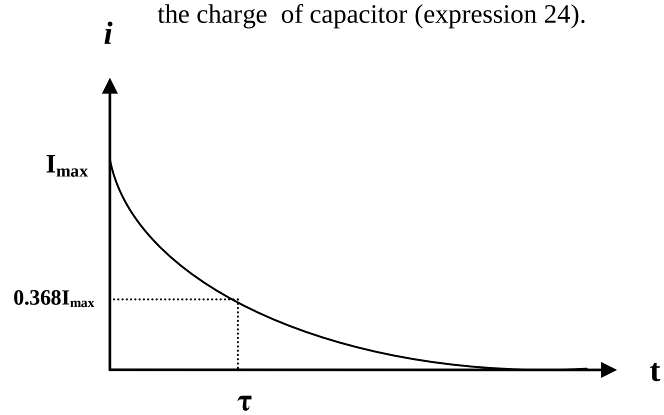


Fig.15.b

At $\boldsymbol{t = \tau}$ the charge in capacitor plates gets to $q(\tau) = Q_{max}\left(1 - \frac{1}{e}\right) = Q_{max}\left(1 - \frac{1}{2.73}\right) = 0.632Q_{max}$

From expression (23) one may derive the ***evolution of current*** in the circuit. As $\boldsymbol{i = dq/dt}$ it comes out

that $i = \frac{dq}{dt} = (-)\left(-\frac{Q_{max}}{\tau}\right)\exp\left(-\frac{t}{\tau}\right) = \left[\frac{C*\varepsilon}{R*C}\right]\exp\left(-\frac{t}{\tau}\right) = \left(\frac{\varepsilon}{R}\right)\exp\left(-\frac{t}{\tau}\right)$

Then, by noting that $\boldsymbol{I_{max} = \varepsilon / R}$ one gets the expression $\boldsymbol{i = I_{max}\exp\left(-\frac{t}{\tau}\right)}$ (24)

This expression tells that at *t=0s* the capacitor acts as a " *zero resistance* " and the current value (*i(0)* = I_{max}) is dependent only on resistance R. With time, it starts to " *act as an increasing resistance* " which decreases exponentially the current in circuit.

b) DISCHARGING THE CAPACITOR (in a RC circuit without emf)

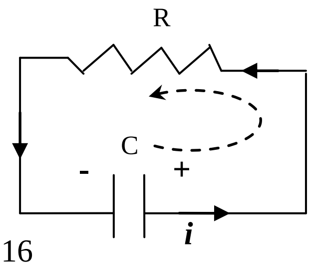


Fig 16

By using the 2nd Kirchof's rule one gets $\boldsymbol{v_C - v_R = 0}$ (25)

$\boldsymbol{v_C = q / C}$ is the potential jump through the capacitor terminals and $\boldsymbol{v_R = R * i}$ is the potential drop through the resistor at time " t ".

One can rewrite relation (25) in the form $\frac{q}{C} - R * i = 0$ or $\frac{q}{RC} = i$ (26)

By substituting $i = -dq/dt$ *to eq.26*, one get $\frac{q}{RC} = -dq/dt$ which can be transformed to $\frac{dq}{q} = -dt/RC$ and by taking the integral on both sides $\int dq/q = -(\frac{1}{RC}) \int dt$ one get

$lnq = -\frac{t}{RC} + k$ where k is a constant. Knowing that at t = 0 the charge of capacitor is $q = Q_{max}$ one gets $lnQ_{max} = k$ and this expression can be written as $lnq - lnQ_{max} = -\frac{t}{RC}$ *or* $\ln(\frac{q}{Q_{max}}) = -\frac{t}{RC}$ and $q = Q_{max}\exp(-\frac{t}{RC})$ which is written as $\boldsymbol{q = Q_{max}\exp(-\frac{t}{\tau})}$ (27)

For $\boldsymbol{t = \tau}$ one gets $\boldsymbol{q = Q_{max} / e = 0.37Q_{max}}$. The " ***half-time***" noted as " $\boldsymbol{T_{1/2}}$ " is the time it takes for the charge of capacitor to get to half of its initial value ($\boldsymbol{q = 0.5Q_{max}}$); so, $\boldsymbol{Q_{max} / 2 = Q_{max}exp(-T_{1/2} / \tau)}$ and $\exp(\boldsymbol{T_{1/2}/\tau}) = 2$ which brings to $\boldsymbol{T_{1/2}/\tau = ln2 = 0.693}$. So, one gets $\boldsymbol{T_{1/2} = 0.693 * \tau}$ (28)

Bearing in mind that ***during the discharge*** of capacitor the current in circuit is $\boldsymbol{i = - dq/dt}$, from the expression (27) one gets $i = -\frac{dq}{dt} = \left(\frac{Q_{max}}{\tau}\right)\exp(-\frac{t}{\tau})$ and as $Q_{max} = \varepsilon * R$ and $\tau = R * C$ it comes out that $i = \left(\frac{\varepsilon}{R}\right)\exp(-\frac{t}{\tau})$. As $\boldsymbol{\varepsilon / R = I_{max}}$. one get $\boldsymbol{i = I_{max}\exp(-\frac{t}{\tau})}$ (29)

Note that for $\boldsymbol{t = \tau}$ one gets $\boldsymbol{i(\tau) = I_{max} / e = 0.37I_{max}}$ and for $\boldsymbol{t = T_{1/2}}$ $\boldsymbol{i(T_{1/2}) = I_{max}\, e^{-0.693} = 0.5I_{max}}$

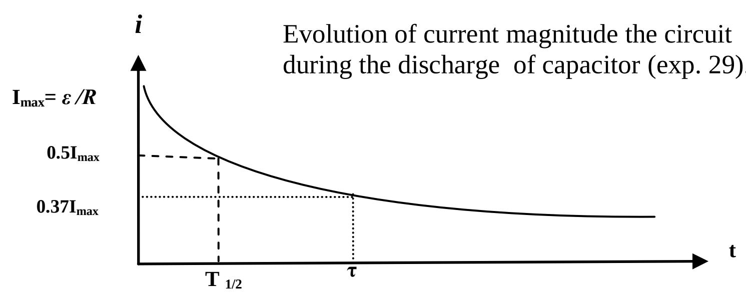


Fig.17

Note: Assume that one records a positive current in ammeter during the capacitor charge. Then, during the capacitor discharge, the current in circuit has opposite sense and the ammeter will record negative values of current. In this case, the <u>graph of current would appear as the reflected</u> of this in fig.17 versus the axe of time.