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7.1 THE MAGNETIC FIELD

- If one pours a powder of iron particles around a permanent magnet (natural "lodestones" or manmade) an *ordered pattern* appears (fig.1). One can easily discern the *presence of lines* that become very dense at two magnet extremes (labelled as *poles*). One says that the ***magnet builds up a magnetic field in the space around it*** and the alignment of iron particles "*just makes visible*" the ***magnetic field lines***.

- The *nomination of magnetic poles* comes from this arbitrary definition: *If one hangs the mid-point of a magnet by a string and leaves it horizontal, it will get always aligned somehow around to geographic direction north-south. The magnet pole that seeks the earths geographic North is called north magnetic pole "**N**"; the other one is called the south magnetic pole "**S**". As the experiments show that "N" pole of a magnet repeals "N" pole of another magnet and attracts its pole "S", it comes out that earth has its own* magnetic poles and the ***magnetic pole "S" of earth is located somewhere around its geographic north***.
(https://phet.colorado.edu/sims/cheerpj/faraday/latest/faraday.html?simulation=magnet-and-compass)

It's important to note that:
a) The location of poles "*to the extremes"* of a permanent magnet is somehow un-precise.
b) If one cuts a magnet in two pieces, each of the two pieces acts as a magnet with two poles "***N"*** and "***S"***.
No one has observed a monopole magnet, yet.

Those experimental facts pushed towards development of a slightly different (compared to that of electric fields) theoretical model for the study of magnetic action. Michael Faraday observed closely this *type of field* and had the idea to ***start*** the definition of its parameters from its ***field lines*** (instead of "*charges* " at *electricity*).

- So, the theoretical model for magnetism starts from "magnetic ***field lines***" and "***magnetic field vector*** ".

a) The ***magnetic field lines*** are ***closed loops***. In the space ***around*** the body of a magnet, they *get **out of pole N*** and go ***into the pole S***. ***Inside*** the magnet body, they are directed **from *pole S to pole N*** (fig.2).

b) The ***magnetic field vector*** $\vec{B}(x,y,z)$ is the measure of ***magnetic field*** action at location (x, y, z) of space. It is tangent to the field ***line*** passing by the point (*x, y, z*), it is directed the same way as this field line and its **magnitude** (***field strength***) is *proportional* to the local density of field lines (i.e. the ***number of field lines*** crossing ***1m²*** area ***perpendicular to field line direction*** at this point).

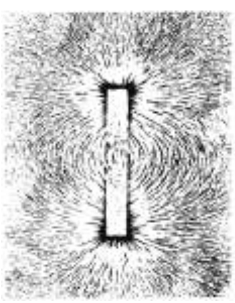

Fig 1

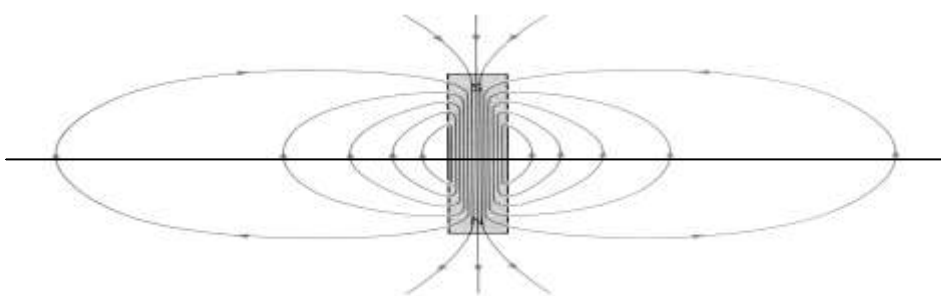

Fig 2

If one passes a plane P *perpendicular to* direction "***N–S***" and *midway* between poles (see fig 2), the lines of magnetic field will go into it (or out it) along the normal to this plane. If one observes the plane P from pole "N", one will see the field lines entering this plane. In the following we will *draw **a cross* " x "** *to show* ***a field line*** going ***into the plane*** and a ***dot* " • "** to show a field line going **out of a plane versus observer.**

- *Remember*: One defines *the electric field* vector $\vec{E}$ at a space location, by electric force exerted on an **isolated** charge ***+1C*** at rest at this point ($\vec{E} = \vec{F}_{El}/1C$) . One would prefer to apply a similar definition for vector $\vec{B}$ but ***there is no isolated magnetic charge*** ("*magnetic charges*" exist only in couple, as a dipole ***N – S*** **).**

Similarly one refers to a force ***:*** ***<u>The magnetic field exerts a force on an electric charge in motion.</u>***

Then one uses this ***force*** to define the ***magnetic field vector*** $\vec{B}$. The experiments records show that, if a particle with *electrical charge* " **q** " moves at *velocity* $\vec{\upsilon}$ inside an area covered by a ***magnetic field***, the magnitude of the ***magnetic force*** $\vec{F}_B$ depends on the charge of particle, its speed, its direction of motion and the local density of magnetic field lines. More precisely, the measurements confirm that :

$$\left|\vec{F}_B\right| \sim |q|\left|\vec{\upsilon}\right|\sin\vartheta \qquad (1)$$

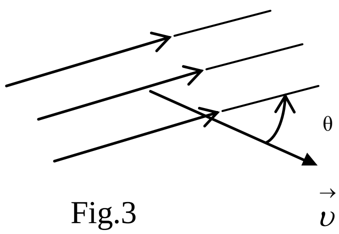

Fig.3

where ***θ*** is the angle ***measured from*** the *velocity* $\vec{\upsilon}$ direction onto the *direction of local magnetic field lines*. There is a *zero force* if the charged particle enters the magnetic field area *along the field lines* and a *maximum force* when it enters *perpendicularly* to them; this effect is included in expression (1) by sine function.

- Also, the experiments show that the ***magnitude*** of the ***magnetic force*** is proportional to the *density of magnetic field lines*. But, the ***<u>density</u> of*** field ***lines*** is proportional to local ***field <u>strength</u>,*** too**.**

So, one has
$$\left|\vec{F}_B\right| \sim \left|\vec{B}\right| \qquad (2)$$

and by selecting
$$\left|\vec{F}_B(x,y,z)\right| = |q|\left|\vec{\upsilon}\right|\left|\vec{B}\right|\sin\vartheta \qquad (3)$$

one can get the <u>*magnitude*</u> of ***field vector*** $\vec{B}$ from expression $B = F_B / q\upsilon\sin\vartheta$ (4)

- The experiments show that $\vec{F}_B$ *is directed* perpendicularly to the plane defined by vectors $\vec{\upsilon}$, $\vec{B}$ and its orientation is that of the *vector product* $\vec{\upsilon} \times \vec{B}$. So, simply put, it comes out that

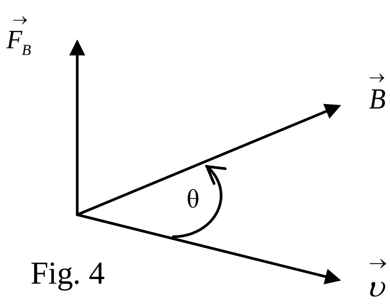

Fig. 4

$$\vec{F}_B = q\,\vec{\upsilon}\times\vec{B} \qquad (5)$$

- $\vec{B}$ direction is found by the direction "***S*** ►***N*** " of a compass.
- The **direction** of magnetic force $\vec{F}_B$ is defined by the rule of vector product (***right hand or bottle cap rule***) and the **sign** of the charge (***q*** *must have its* ***algebraic sign at formula 5***) .

<u>Note</u>: The *magnetic field does not achieve work on a charged particle moving inside it because all time,* $\vec{F}_B \perp \vec{\upsilon}$ *and the displacement* $\Delta\vec{s}$ *has same direction as* $\vec{\upsilon}$. *So* $\Delta A = \vec{F}_B * \Delta\vec{s} = 0$

The unit of magnetic field strength (B-vector) is derived from the expression (4); so, it is a derived unit. The ***SI unit*** of ***magnetic field strength*** is "***Tesla***". One deals with B ***= 1T*** if the magnetic field exerts a ***force with magnitude 1N*** on electric ***charge +1C*** moving at ***1m/s*** along a direction perpendicular to $\vec{B}$. So, ***1T*** =1N/(C*m/s) = 1N / (C/s)*m = ***1N/(A*m)***. But, the commonly used unit is "gauss" ***1G =10*** $^{-4}$ ***T.***

IMPORTANT:

a) One defines a magnetic field *without making reference to its sources*.
From the first step, the magnetic action is counted as a field action.

b) The *definition of magnetic field vector is related to the electric charge*. This is a first information about a ***profound relation between the electric and magnetic phenomena***.

7.2 ACTION OF MAGNETIC FIELD ON A CURRENT HOSTING CONDUCTOR

- Let's calculate the *magnetic force exerted on a straight wire* with *section "**A**"* and *length "**l**"* carrying the *current "**I**"* when it is placed perpendicularly to an ***uniform magnetic field*** $\vec{B}$ (direction along "x" in fig. 5). If there is no current in wire, the free electrons move irregularly and the net magnetic force exerted on the wire is zero. But, if there is a current in wire, all electrons have the same *ordered drift velocity* $\vec{v}_D$ and on each of them is applied the same force (magnitude and direction). The sum of magnetic forces exerted on each of them is transmitted to the whole wire. The magnetic force exerted on each electron is

$$\vec{F}_{el-B} = q_{el}\vec{v}_D \times \vec{B} = -e\vec{v}_D \times \vec{B} = -e(-v_D\hat{\imath}) \times \vec{B} = ev_D\hat{\imath} \times \vec{B} \qquad (6)$$

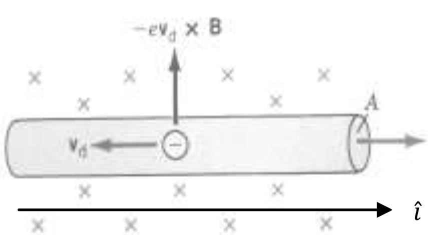

Fig.5

If the *volume density* of " *free electrons* " in wire is ***"n"***, there are "***n *A*l*** " such electrons in a length "***l"*** of wire. The force (6) applies on each of them. So, the net magnetic force on length "l" of wire is

$$\vec{F}_{w-B} = (nAl)\vec{F}_{e-B} = nAl * ev_D\hat{\imath} \times \vec{B} = e(nAv_D)(l\hat{\imath}) \times \vec{B}$$

and finally $$\vec{F}_{w-B} = e(nAv_D)\vec{l} \times \vec{B} = I * \vec{l} \times \vec{B} \qquad (7)$$

where $I = e(nAv_D)$ is the ***current magnitude*** and $\vec{l}$ *is a vector with magnitude "l = wire length" and same direction as that of current (i.e. of positive charges or "$-\vec{v}_D$ ").*
Remember that the ***magnetic force*** $\vec{F}_B$ on a wire that hosts current is always ***perpendicular to wire*** and the ***magnetic field*** $\vec{B}$. The *magnitude* of this force ***<u>does depend</u>*** *on the angle* between them, though.

7.3 MAGNETIC FIELD ACTION ON A LOOP HOSTING CURRENT

- What happens when a wire with current, shaped as a loop, is placed inside an uniform magnetic field ? *The direction of exerted magnetic force will change from one piece of wire to another due to change of direction of "* $\vec{l}$ *" vector*. Let's consider closely the case of a wire with *current "* ***I*** *"* shaped as a *plane rectangular loop* with *sides* ***a, c*** and placed inside an *uniform magnetic field* $\vec{B}$. When the <u>plane of the loop is perpendicular</u> to $\vec{B}$ (directed versus the observer in Fig.6), the directions of magnetic forces on each of wire sides are such that would tent to stretch the loop. If one rotates CCW the loop plane by angle "***α***" around two central points (S, S' ,see Fig.7), the exerted forces on the upper and lower sections will follow stretching the loop. But, the forces exerted on c-sides (*besides stretching*) have a component that builds up a *torque tenting to rotate the loop versus the initial orientation* where its ***plane*** *is* <u>perpendicular</u> *to* $\vec{B}$.

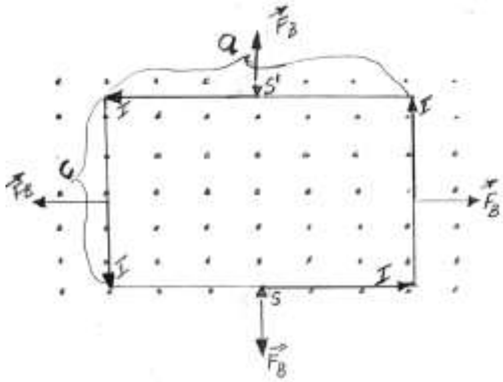

Figure 6 (Loop plane perpendicular to field)

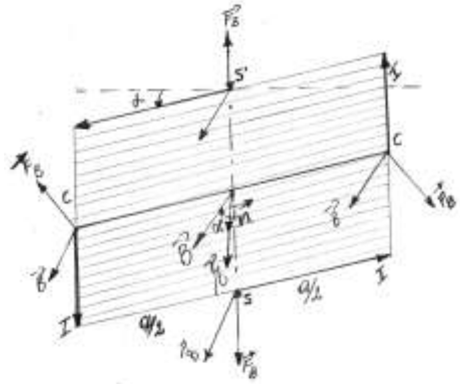

Figure 7 (Loop plane not perpendicular to field)

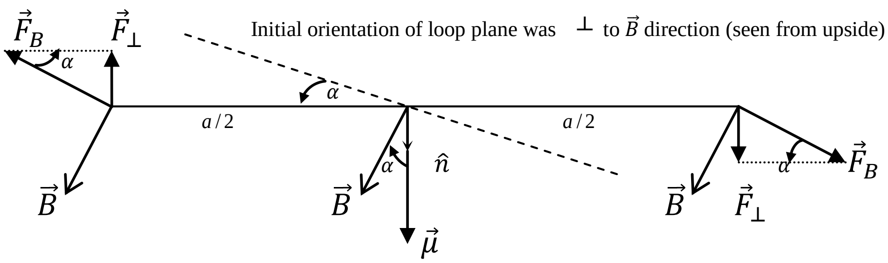

Fig. 8 Observation of rotated loop from upside (S´S direction)

The magnitude of each lateral force is $|F_B| = IcB$. Its rotating component is $F_\perp = |F_B|\sin\alpha$ (see fig. 8).As the lever arm of force $F_\perp$ is $\left(\frac{a}{2}\right)$, it comes out that the related torque action of the magnetic force on one side of loop is

$$\tau_1 = F_\perp * {}^{a}\!/_{2} = IcB * sin\alpha * \frac{a}{2} = 0.5*(ac) * IBsin\alpha \qquad (8)$$

Then, the rotational action of the two lateral forces produces the net torque about axis SS' with magnitude

$$\tau = 2 * 0.5(ac)IBsin\alpha = (ac)IBsin\alpha = AIBsin\alpha \qquad (9)$$

"A=ac" is the area of loop

-We derived the expression (9) for the case of a *single rectangular plane loop*. Similar calculations show that the *same expression is valid for any shape of a plane loop*. On may figure out easily that if a ***set with N loops*** is placed inside an *uniform magnetic field*, the net torque exerted on the set has the magnitude

$$\tau = N * AIB\sin\alpha \qquad (10)$$

- Actually, one uses a more compact way to express the action of magnetic field <u>*on current hosting loops*</u>. One starts by introducing a *loop unit vector* $\hat{n}$, perpendicular to loop plane, placed at its center and directed as the thumb in right hand rule (*other fingers curled in current sense*, see fig. 9). Next, one defines the

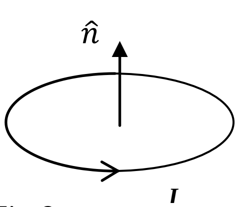

Fig.9

the ***<u>magnetic dipole moment</u>*** of loop as $\vec{\mu} = IA\hat{n}$ (11)

For a coil with N similar loops this expression get the form $\vec{\mu} = NIA\hat{n}$ (12)

Then, one expresses the net torque on the *set of loops hosting current* by a cross product as

$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad (13)$$

- The expression (13) is similar to that of the torque ($\vec{\tau} = \vec{p} \, x \, \vec{E}$) exerted on an electric dipole $\vec{p}$ inside an uniform electric field $\vec{E}$. We found the expression $U_p = -\vec{p} * \vec{E}$ for the potential energy of electric dipole inside an electric field by using the work done by the torque $\vec{\tau}$ (due to electric field) for the rotation of the dipole $\vec{p}$. The reference U = 0 corresponds to the angle 90^0 between the vectors $\vec{p}$ and $\vec{E}$.

By applying exactly the same mathematical procedure one gets the expression $U_\mu = -\vec{\mu} * \vec{B}$ (14)

for the potential energy of magnetic dipole moment $\vec{\mu}$ inside a magnetic field $\vec{B}$. Note that this energy is :

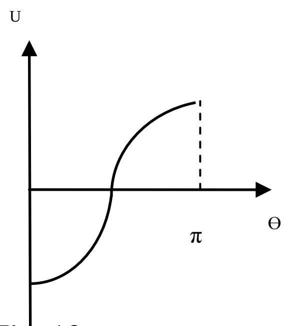

Fig. 10

- **zero** for $\vec{\mu} \perp \vec{B}$ (angle 90^0) and this is an " *unstable orientation* ";
- **negative** and **minimum** for $\vec{\mu} \uparrow\uparrow \vec{B}$ (angle 0^0) which is "*the stable orientation* ";
- **positive** and **maximum** for $\vec{\mu} \uparrow\downarrow \vec{B}$ (angle 180^0) which is "*unstable orientation*".