8.1 THE MOTION OF A CHARGED PARTICLE INSIDE A MAGNETIC FIELD

This situation happens frequently in nature(ex: *deflection of high-energy charged cosmic particles around earth*) and is widely used in technology (ex: *Confinement of particles in a plasma gas, deflection and focusing of charged particles in an accelerator or electrons in an old fashioned TV CTR tube*.)

UNIFORM MAGNETIC FIELD (mainly met in technological applications)

- One may distinguish between two situations:

The velocity $\vec{\upsilon}$ of the particle is perpendicular to $\vec{B}$ vector of field or $\vec{\upsilon}$ is not perpendicular to $\vec{B}$.

A] Particle velocity $\vec{\upsilon}$ is perpendicular to $\vec{B}$ (Fig.1). Let's take, for simplicity, the case when q > 0.

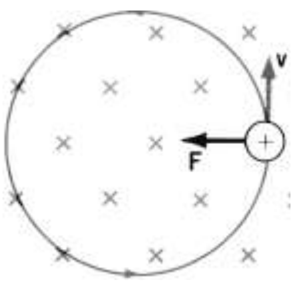


Fig.1

The magnetic force exerted on particle is perpendicular to $\vec{\upsilon}$. Let's assume that this is the only force exerted on the particle(no gravity, no electric force). As this force is all the time perpendicular to particle velocity, it will play the role of a ***centripetal force*** with magnitude

$$F_B = q\upsilon B \qquad (1)$$

Consequently, the particle will move around a circle.

By applying the second law of Newton for a net centripetal force,

one get the relation $$q\upsilon B = m\frac{\upsilon^2}{R} \qquad (2)$$

From this relation, one gets the radius of circular path as $$R = \frac{m}{q} * \frac{\upsilon}{B} \qquad (3)$$

The circle circumference is $C = 2\pi R$ and the particle makes a *full revolution* with a ***period***

$$T = \frac{C}{\upsilon} = \frac{2\pi R}{\upsilon} = \frac{2\pi}{\upsilon} * \frac{m\upsilon}{qB} = \frac{2\pi m}{Bq} \qquad (4)$$

Hence, a rotation frequency $$f_C = \frac{1}{T} = \frac{B}{2\pi} * \frac{q}{m} \qquad (5)$$

Note that if the magnitude of vector **B** remains **constant in time**:

a) The particle keeps rotating at **constant *speed*** (equal to that it ***enters into*** *magnetic field).*
b) The *period* and the *frequency* of rotations ***depend on q/m*** ratio but **not on** *particle* ***speed***.
c) The radius "*R*" of circle is proportional to *particle* ***speed (~ υ)*** and depends on ***q/m*** ratio.

- This kind of movement is characteristic for a type of charged *particle accelerators* called ***Cyclotrons.*** The frequency of revolutions "f_C" is known as *cyclotron frequency*. The magnetic field in a cyclotron *device does not provide work (* $\vec{F}_B \perp \vec{\upsilon}$ *) and the energy of the charged particle is not affected by the* magnetic field. Meanwhile, the main duty of an accelerator is the increase of particles' energy. For this purpose, i.e. the increase of particle energy, one uses an electric field inside a cyclotron.

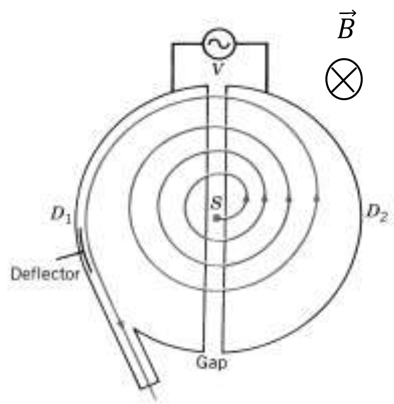


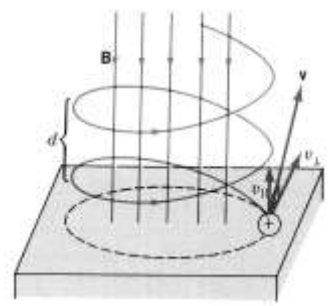
Fig.2

A cyclotron is constituted by two empty half cylindrical parts separated by a small gap. A uniform magnetic field is directed perpendicular to the base of cylinders and an alternating potential difference is applied between the two half cylinders of cyclotron. The charged particles are produced by an ion source "S" located at the center of this system. An electric field applied in the space between the two half cylinders accelerates the charged particles and gives them an initial velocity perpendicular to the magnetic field. Once the particle get inside a cyclotron half cylinder, there is no electric force on it. The action of magnetic field makes the particle rotate. Once the particle finishes the half circle path, one changes the polarity of electric potential and accelerates its motion through the "*gap*" in the opposite direction.

So, each time the particle passes through the gap, its kinetic **energy** (*i.e.speed*) increases by $\Delta K = qV$ (6) Therefore, the radius of the circle increases(see relation 3), too. When the particle reaches the desired energy it is moving around external circle and one uses a deflection mechanism to take it out of cyclotron.

- The fact that particles rotate all time at **constant rotation frequency** f_C is a big advantage for cyclotron function. For a *given type of particles* ***(same q/m value),*** one can easily calculate B magnitude so that particles rotate at a precise frequency f_C . *Then, by inverting the polarity of applied voltage between two half cylinders with a* ***frequency*** *equal* to f_C, one can increase the particle energy twice during each revolution. A modern cyclotron has a radius ~ 2m and is used *mainly to accelerate* ***protons*** or positive ions (instead of electrons). This is because the electron gets quickly to relativistic behaviour and this is associated with its mass increase. The straight consequence is a change in f_C values (eq. 5) and this would require a continuous modification of frequency for alternating potential. The ***synchrocyclotron*** is a more complicated device that can adjusts the frequency of alternating potential for ***relativistic particles***.

B] Particle velocity $\vec{\upsilon}$ is not perpendicular to $\vec{B}$ (Fig.3). One can decompose $\vec{\upsilon}$ into two components;



$\vec{\upsilon}_\perp$ (perpendicular to $\vec{B}$) and $\vec{\upsilon}_{||}$ (parallel to $\vec{B}$). The action of magnetic field ($F_b = q * \upsilon_\perp * B$) put the particle in a circular motion(as in fig.3 for q > 0) with period $T = \dfrac{2\pi m}{Bq}$. The component $\vec{\upsilon}_{||}$ is not affected by $\vec{B}$ field (as $\sin 180^0 = 0$) and it gives to particle a motion at speed $\upsilon_{\|}$ parallel to the field lines. So, it comes out that for an interval of time equal to one period of rotation " T "

Fig.3

the particle is shifted parallel to $\vec{B}$ direction by $d = \upsilon_{||} * T = \upsilon_{||} * \dfrac{2\pi}{B}\dfrac{m}{q}$ (7)

The net movement of the particle fits to a helicoidally path.

<u>Non-UNIFORM MAGNETIC FIELD</u> (charged particles in a plasma gas)

- A plasma gas is a medium with such a high temperature that any material used to confine it would be melted. So, one uses a specific configuration of magnetic fields to keep particles inside a restricted space region. These fields are known as "*magnetic bottles"*. As the radius of circular motion is $R \approx \dfrac{1}{B}$, it comes out that, R decreases at space region where magnitude of "B" increases(*close to two coils at fig.4*). Also, due to $\vec{B}$ orientation (which is tangent to field lines), the magnetic force is directed toward the region of weaker field ("seeking the **bottle center**" at fig. 4) for any direction of particle velocity. In general, this force has two components; one perpendicular to the axe magnetic field pattern which works as centripetal force and one component directed versus the "bottle center" that pushes the charged particles versus the "center of magnetic bottle". So, once a charged particle is inserted in a magnetic bottle it remains trapped there.

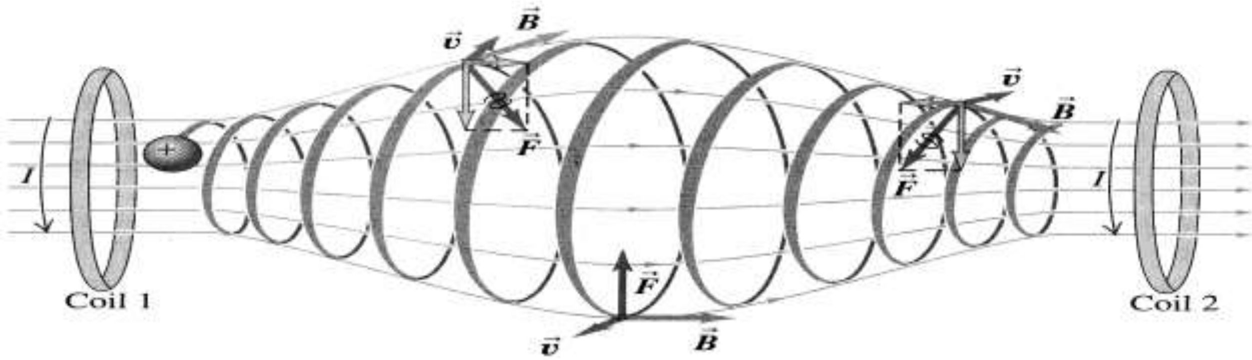


Fig.4

8.2 CHARGED PARTICLE INSIDE A SET OF FIELDS " MAGNETIC AND ELECTRIC"

-When a charged particle enters a space region where an electric field $\vec{E}$ and a magnetic field $\vec{B}$ superpose, there are two forces exerted on the particle; electric $\vec{F}_{el} = q\vec{E}$ and magnetic $\vec{F}_B = q\vec{\upsilon} x \vec{B}$. So, the **net force** (known as ***Lorentz force***) applied on the particle is $\vec{F}_{Lor} = q(\vec{E} + \vec{v}x\vec{B})$ (8)

- In general, it is difficult to predict the path followed by a charged particle under the effect of *Lorentz force*. The simplest cases concern two fields either parallel or perpendicular to each other. We will consider the case of two fields perpendicular to each other. Let's select a reference system where the **electric field** is directed **along Oy** axis, the **magnetic field** is directed **along Oz** axis and the charged particle "$+q$ " enters this space region at origin with **velocity** $\vec{\upsilon}$ directed **along Ox** axis (see Fig. 5).

So; $\vec{E} = E\hat{\jmath}$; $\vec{B} = B\hat{k}$; $\vec{v} = v\hat{\imath}$

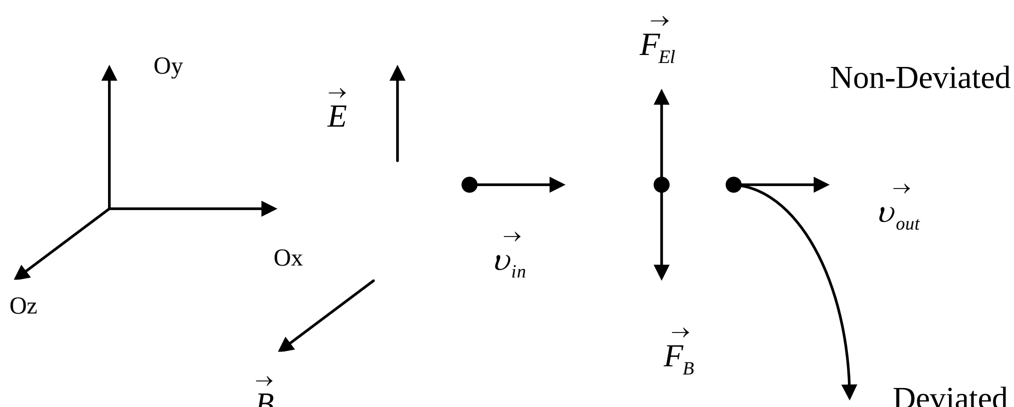


Fig.5

One can easily find that the *electric* and *magnetic* ***forces*** lie on Oy axis but in opposite directions.

So, the magnitude of the net force acting on the particle is $$F_{net} = qE - q\upsilon B \quad (9)$$

For a *given speed (velocity magnitude)*, it happens that $$F_{net} = qE - q\upsilon B = 0$$

Therefore, this speed is $$\upsilon = \frac{E}{B} \quad (10)$$

So, if a parallel beam of charged particles travelling at such speed enters this space region, all particles follow their way without deviation from the initial direction (Fig. 5). If the beam contains particles at different velocities, only those that fulfill the condition (10) will get out of region in the same direction. The other particles will deviate their path out of initial direction. A ***velocity selector*** device uses this effect to ***measure particle velocity*** or to **select particles** travelling at a ***given velocity***.

- The mass spectrometer (fig.6) is a device that make use of a *velocity selector* to *analyse* the abundance of **different isotopes** of the **same element** in a sample. At first, the atoms get equally ionized (mostly +1e or +2e) and are given a velocity by an electric field. Then, the device aligns these charged particles (*ions*) through a set of slits S_1, S_2 to build a collimated beam. Next, it directs the collimated beam through a velocity selector ($\vec{E} \perp \vec{B}_1$) section that leaves to "*get out* " only ions moving at speed $\upsilon = \frac{E}{B_1}$.

After that, the collimated beam containing ions at same speed enters a second section where a magnetic field $\vec{B}_2$ directed perpendicular to their velocity exerts a centripetal force with magnitude

$$F_{B2} = q\upsilon B_2 = m * a_C = m\frac{\upsilon^2}{R} \quad (11)$$

These ions follow circular paths with radius $$R = \frac{m}{q} * \frac{\upsilon}{B_2} = \frac{m}{q}\frac{E}{B_1 B_2} \quad (12)$$

As the ***different isotopes*** of same element have ***different masses*** (but ***q is the same*** *for all ions*) the radius is different for each isotope and they hit the detector (photographic plate or electronic counter) at different positions. So, based on density of traces or the records' intensity one calculates the relative abundance of different isotopes into the sample under study. The figure 6 presents the scheme of a mass spectrometer.

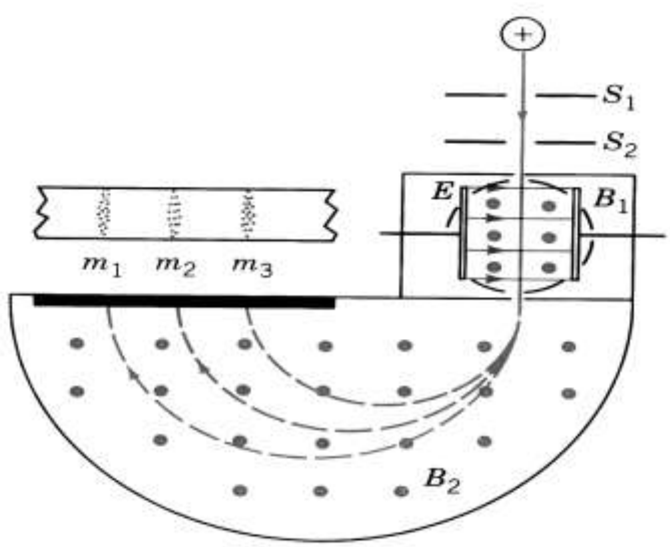


Fig.6