9. SOURCES OF MAGNETIC FIELDS

-Previously, we have noted that it does not exist *a single magnetic pole* and the quantitative definition of magnetic field vector $\vec{B}$ is referred to the magnetic action on "*an electric charge in motion* ". The detailed analysis of experimental records confirmed that the *origin of magnetic field is at the electric current*. Hans Christian Ørsted was the first to show experimentally that, just like a magnet, a *wire that carries a current exerts a magnetic force on another wire carrying current.*

9.1 MAGNETIC FIELD DUE TO A LONG, STRAIGHT WIRE CARING CURRENT

-One may verify easily, by use of iron particles, that a straight long wire carrying current creates around it a magnetic field with circular field lines. Also, by using a compass needle (fig.1), one may find out that the ***direction*** of magnetic ***field fits*** to the following ***rule of right hand*** :

If the thumb follows the current direction, the curled fingers of the right hand show the direction of magnetic field lines around it.

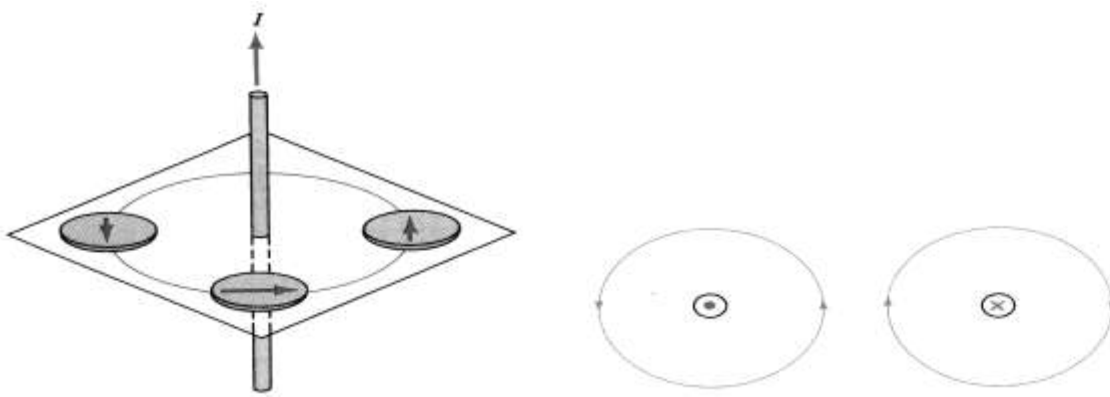


Fig.1

The ***magnitude*** of the ***field vector*** $\vec{B}$ is the same over a circle centered at wire and it depends only on the magnitude of current in wire "***I"*** and the circle radius "***R"***. The experimental measurements show that

$$B = \frac{\mu_0 I}{2\pi R} \tag{1}$$

where the constant $\boldsymbol{\mu_0 = 4\pi * 10^{-7}\ T*m/A}$ is known as the *magnetic permeability of vacuum.*

Note: As the *magnitude* of *magnetic* field *"B" decreases* with distance "R", it comes out that the *density of field lines decreases* with increase of distance from current, too.

9.2 MAGNETIC FORCE BETWEEN TWO PARALLEL WIRES CARRYING CURRENT

- Consider two long straight parallel wires at a distance "***d*** " from each other and carrying currents "I_1" and "I_2". The first wire creates a magnetic field with magnitude B_1 at distance "***d"***; $\vec{B}_1$ is *perpendicular* to the direction of current I_2 passing thought the second wire. By referring to a length " l_2 " on the second wire, one finds out that the *magnitude* of *magnetic force* applied on this length of wire is

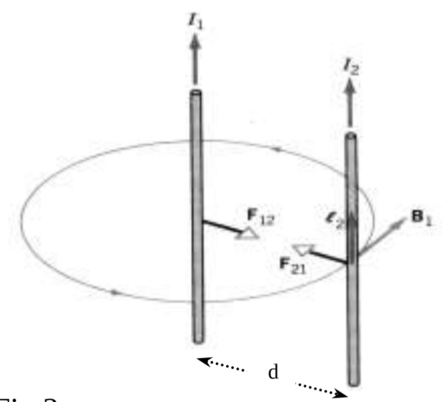


Fig.2

$$F_{21} = I_2 l_2 B_1 = I_2 l_2 * \frac{\mu_0 I_1}{2\pi d} = l_2 * \frac{\mu_0 I_1 I_2}{2\pi d} \qquad (2)$$

When considering the magnitude of magnetic force applied on the length "$\boldsymbol{l_1}$" on first wire by the magnetic field $\vec{B}_2$ due to the current in second wire, one gets the expression

$$F_{12} = I_1 l_1 B_2 = I_1 l_1 * \frac{\mu_0 I_2}{2\pi d} = l_1 * \frac{\mu_0 I_1 I_2}{2\pi d} \qquad (3)$$

Note that, if the *currents* have the ***same direction*** (fig. 2) the exerted *forces are* ***attractive*** and if the *currents* have ***opposite directions*** the exerted *forces are* ***repulsive***.

- By comparing the expressions (2) and (3) one finds out that a *force* with ***same magnitude*** is exerted on the ***unity of length*** on each wire

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi d} \qquad (4)$$

It is very interesting to mention that the standard of current unit (*Ampere*) is based precisely on this expression: *The* ***current passing*** *through any of two parallel wires at a* ***distance 1m*** *is* ***1A*** *when the* ***magnitude*** *of magnetic* ***force*** *exerted* ***on 1m length*** *of each of them* ***is*** *equal to* ***$2*10^{-7}$ N.***

9.3 BIOT-SAVART LAW: MAGNETIC FIELD DUE TO "*A CURRENT ELEMENT* "

- The expression (1) for magnetic field due to current through a long straight wire helps to get to the *basic expression* applied for calculation of magnetic field due to a current passing through *any shape of wire.* Essentially, this is an expression for magnetic *field due to a current through an infinitesimal short wire.* Next, by an integral calculation, one may find the field due to the whole wire shape. One may get to this ***basic expression***, by comparing the calculation steps of "***B*** *field* " due to an "*infinite long current*" to the calculation steps of "***E*** *field* " due to an electric charge distributed uniformly along an infinite long wire.

- Let's start by noting $\frac{\mu_0}{2\pi} \equiv 2k'$ at expression (1) and writing it in form $B = \frac{2k' I}{R}$ (5)
which is a similar expression to that of ***electric field magnitude*** built by an electric charge distributed uniformly on a long straight wire $E = \frac{2k\lambda}{R}$ (6) where $k = \frac{1}{4\pi\varepsilon_0}$ while $k' = \frac{\mu_0}{4\pi}$

Without going to detailed explanations, we note that the ***similarity between the magnetic and electric fields expressions*** *is general and much more profound than it appears at this mathematical comparison.*

- As a second step, one may note that the *elementary* ***field vector*** due to the *elementary charge* ***dq = λdl***

(see fig.3.a) is given by the expression

$$d\vec{E} = k\frac{dq}{r^2}\hat{r} = k\frac{\lambda dl}{r^2}\hat{r} \qquad (7)$$

$\hat{r}$ is the ***unit vector*** directed from the charge "***dq = λdl*** " to the point where the electric field is calculated.

Then, by using the integral calculation along the wire length (or Gauss theorem; see Lect.3b ex.#2) one gets the expression (6) for the magnitude of E - *field* at distance *"R"* from a uniformly charged wire.

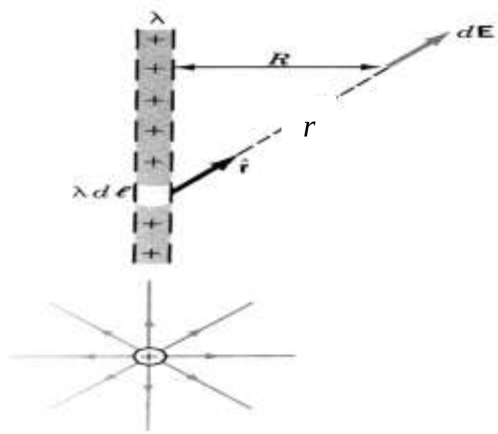


Fig.3 (a) Electric field due to an elementary charge ***dq = λdl***

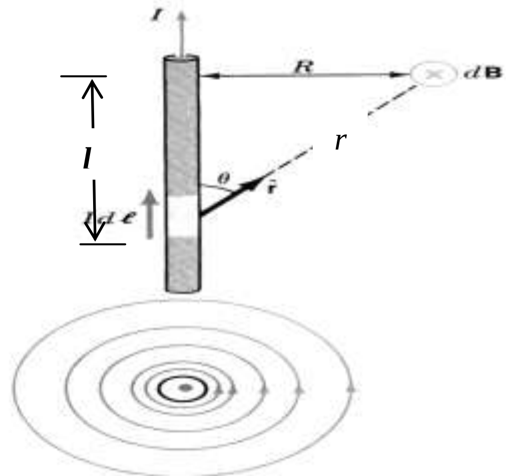


Fig.3 (b) Magnetic field due to an elementary current ***dI = Idl***

In a similar way, one can get to the expression (5) for **magnitude** of magnetic field B via the integral of the *elementary magnetic* ***fields*** **vectors** $d\vec{B}$ due to ***elementary currents dI = Idl*** in wire (fig.3.b)**.** By comparing the expression (6) to (7), one notes that the "*distance factor* " appears as ***1/R*** at expression (6) and ***1/r²*** at expression (7). So, due to symmetries between (5) and (6), one should expect a factor " ***1/r²*** " at the expression for ***elementary magnetic field*** $d\vec{B}$, too. Also, similarly to " ***dq = λdl*** " in (7) one should expect proportionality to " ***dI = Idl*** ". Meanwhile, one should remember that the ***directions*** of $\vec{E}$ and $\vec{B}$ fields depend on different rules; $\overrightarrow{dE} \uparrow\uparrow \hat{r}$ (see fig.3.a) but $\overrightarrow{dB} \perp \hat{r}$ due to right hand rule (see fig.3.b).

This way one gets to the expression

$$d\vec{B} = k'\frac{I\,\vec{dl}}{r^2}\,x\,\hat{r} = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}}{r^2}\,x\,\hat{r} \qquad (8)$$

In this expression, $\vec{dl}$ is the *infinitesimal length* vector with same *direction as this of local current* in wire and $\hat{r}$ is the *unit vector directed from the elementary current to the point where the magnetic field is calculated*. The expression (8) is known as ***Bio-Savart law*** for the magnetic field due to elementary current "$\boldsymbol{I\overrightarrow{dl}}$ **".** From the expression (8), one may see that the *magnitude of elementary magnetic field* is

$$dB = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}\sin\theta \qquad (9)$$

θ - the angle between $\vec{dl}$ direction and $\vec{r}$ direction at location of the infinitesimal length vector.

Important note: The elementary current " *dI* " is only a *mathematical model* that helps for calculations. There is no way to isolate or measure an elementary current in laboratory.

Three Important Exercises: Use the Bio-Savart law to find the magnetic field at **distance *R*** from:

a) a ***finite length wire;*** **b)** an ***infinite long wire*** (expr.1); **c)** an ***arch shape wire*** carrying a current ***I.*** In a, b, c cases, all infinitesimal contributions $\overrightarrow{dB}$ are directed *perpendicularly to the page(fig.4,5)*. So, the net magnetic field will be directed the same way and its magnitude is the sum of all elementary vectors.

a) The net magnetic field by a wire to a point P (see fig.4) at distance R is due to two contributions; one due to the part of wire lower P- point " *level* " and one due to the part over this "level". We will calculate each of these contributions and sum them up. Let's start by the contribution due to the lower half. The magnitude of dB (*due to a current element " Idl" located at distance " l "*, down *P-point* level (see fig.3b)

is $dB = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}\sin\theta$ From the figure 3.b one may see that $\sin\theta = R / r$ and

$r = [l^2 + R^2]^{1/2}$ So, one gets
$$dB = \frac{\mu_0}{4\pi}\frac{Idl}{r^2} * \frac{R}{r} = \frac{\mu_0}{4\pi}\frac{IRdl}{r^3} = \frac{\mu_0}{4\pi}\frac{IRdl}{[l^2 + R^2]^{3/2}} \qquad (10)$$

Then, the contribution by whole ***lower part*** of wire with ***length L_1*** (fig.4) is

$$B_{L_1} = \frac{\mu_0 IR}{4\pi}\int_{-L_1}^{0}\frac{dl}{[l^2 + R^2]^{3/2}} = \frac{\mu_0 IR}{4\pi} * \frac{1}{R^2}\left[\frac{l}{[l^2 + R^2]^{1/2}}\right]_{-L_1}^{0} = \frac{\mu_0 I}{4\pi R}\frac{L_1}{\left[L_1^2 + R^2\right]^{1/2}} \qquad (11)$$

Noting that $\frac{L_1}{[L_1^2 + R^2]^{1/2}} = \frac{L_1}{r_1} = \sin\alpha_1$ (see fig.4) one gets
$$B_{L_1} = \frac{\mu_0 I}{4\pi R}\sin\alpha_1 \qquad (12)$$

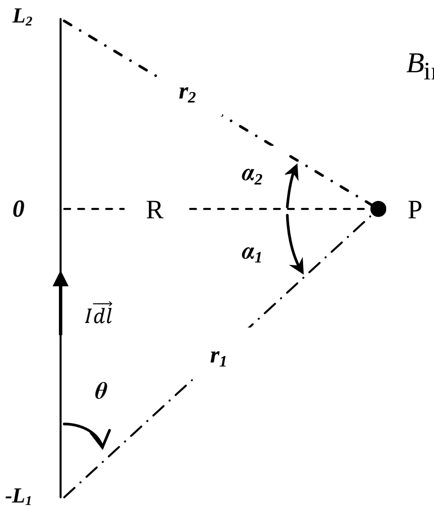
Similarly, the contribution to field magnitude from all elements in the upper part of wire is

$$B_{L_2} = \frac{\mu_0 IR}{4\pi}\int_{0}^{L_2}\frac{dl}{[l^2 + R^2]^{3/2}} = \frac{\mu_0 I}{4\pi R}\frac{L_2}{\left[L_2^2 + R^2\right]^{1/2}} = \frac{\mu_0 I}{4\pi R}\frac{L_2}{r_2} = \frac{\mu_0 I}{4\pi R}\sin\alpha_2 \qquad (13)$$

Finally, the ***net magnitude*** of magnetic field due to a ***finite length wire*** ($L = L_1 + L_2$) is given by the

expression
$$B = B_{L_1} + B_{L_2} = \frac{\mu_0 I}{4\pi R}(\sin\alpha_1 + \sin\alpha_2) \qquad (14)$$

b) Note that, for an ***infinite long wire***, $\sin\theta_1 = \sin\theta_2 = \sin 90^0 = 1$ and the expression (15) gives



$$B_{\inf\_long} = \frac{\mu_0 I}{4\pi R}(1+1) = \frac{\mu_0 I}{2\pi R}$$ i.e. the expression (1) given at first section.

c) The magnetic field due to a current "***I***" passing through a piece of arch "***θ***" on a circle with radius "***R***" *at its center P* is perpendicular to the plane of arch (*sense* *in* *or* *out* *the plane depending on current direction*). The contribution of each arch elementary current is $dB = \frac{\mu_0}{4\pi}\frac{Idl}{R^2}$ and their sum

is $B = \frac{\mu_0}{4\pi}\frac{I * l}{R^2} = \frac{\mu_0 I}{4\pi R} * \frac{l}{R} = \frac{\mu_0 I}{4\pi R} * \theta$

because arch length $l = R*\theta$; *θ is the angle in radians.*

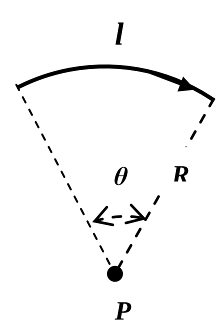


Fig. 5

So, $$B_{arch} = \frac{\mu_0 I}{4\pi R} * \theta \qquad (15)$$

Fig. 4