MECHANICAL WAVES

3.1 GENERAL

-Any SHO is an oscillation in time at a constant period T (and frequency f_0 or ω_0) and a constant amplitude of a "*displacement*" parameter (position, angle, current, pressure, density..). The simple harmonic motion (SHM) is a SHO where the oscillating parameter is the displacement of a particle.
-A ***wave*** is the ***propagation process*** of a ***"disturbance" through a medium***. The disturbance is built up in a limited region of medium and then gets propagated through the "*medium points"* (ex: ripples propagating on still water of a lake, sound propagation in air, radio waves propagating in space..).
- During the wave propagation each point of "*propagating medium*" transports the "*disturbance*" to the next point and after a while returns to its equilibrium position. If one refers to motion of a single point of the medium, one may model its disturbance in time as an oscillation around its equilibrium.
- This chapter covers the harmonic mechanical waves. In this case, each "particle" of propagating medium performs a SHM(or DHM or FHM). Note that a wave can propagate through a matter medium (object of this chapter) or through a field medium(ex. Electromagnetic field in space).

3.2 MECHANICAL WAVE CHARACTERISTICS

-Let's consider a string kept straight with one end fixed. One up-down shift i.e. "*disturbance*" produced at the free end of string will propagate through string. During the propagation of this disturbance, each point of string repeats the displacement sequence of the free end point and then returns to its equilibrium position. One says that a **pulse wave** propagates through the string (fig.1).

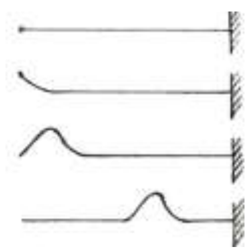


Fig.1

-From fig.1, one may note that, during the propagation of pulse, each **particle of** the **string moves perpendicularly** to the *direction of propagation* of the pulse shape. One says that a **transverse wave** (or **TW**) propagates through the string. If the particles of propagating medium move along the same direction as that of wave propagation, one says that a **longitudinal wave** (or **LW**) propagates through the medium. (*Example: When one presses shortly on the car brakes, the local pressure pulse moves the brake liquid particles along the same direction as that of propagation for the pressure pulse*).

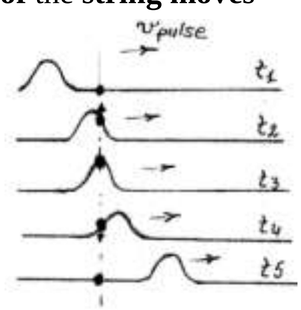


Fig.2

- The transmission of disturbance through adjacent points of propagating medium is due to **restoring forces** acting inside it. A solid medium, due to its three dimensional lattice structure, produces *restoring forces along three space directions* and can propagate TW and LW waves. A gas medium can propagate only LW waves. A liquid can transport LW waves *in volume* and TW waves *on* its *surface* (due to the surface tension action as a restoring force).

- During the propagation of a mechanical wave, the particles of medium oscillate at a small amplitude and after a while they return to their equilibrium position. So, there is *no transport of matter* during the propagation of a mechanical wave. What parameter is propagated ? One may figure out this by analyzing the motion of a leaf on the still water of a lake when a TW surface wave propagates on lake. Initially, the leaf is at rest and it has only a potential energy which may be assigned as equal to zero. When a ripple (pulse wave) passes through leaf position, the leaf will move i.e. it will gain a kinetic energy K. When the pulse moves away the leaf returns to its initial position (fig.2) and its energy is zero. *No energy remains to the leaf, but there is an* ***energy transport*** through its location. Note that the same analysis is valid for the water particle under the leaf. So it comes out that a transportation of kinetic energy accompanies the propagation of wave. As linear momentum is related to kinetic energy by the expression $K = \frac{mv^2}{2} = \frac{m^2v^2}{2m} = \frac{p^2}{2m} \to p = \sqrt{2mK}$ it comes out that there is linear momentum propagation, too. A wave transports ENERGY and MOMENTUM through propagating medium.

- In the next sections we will define the *wave function*. As a starting point, one may note that the *shape of initial disturbance* should be included in the wave function because this is what one can identify as the " *propagating object* " (see fig.2).

3.3 WAVE SUPERPOSITION

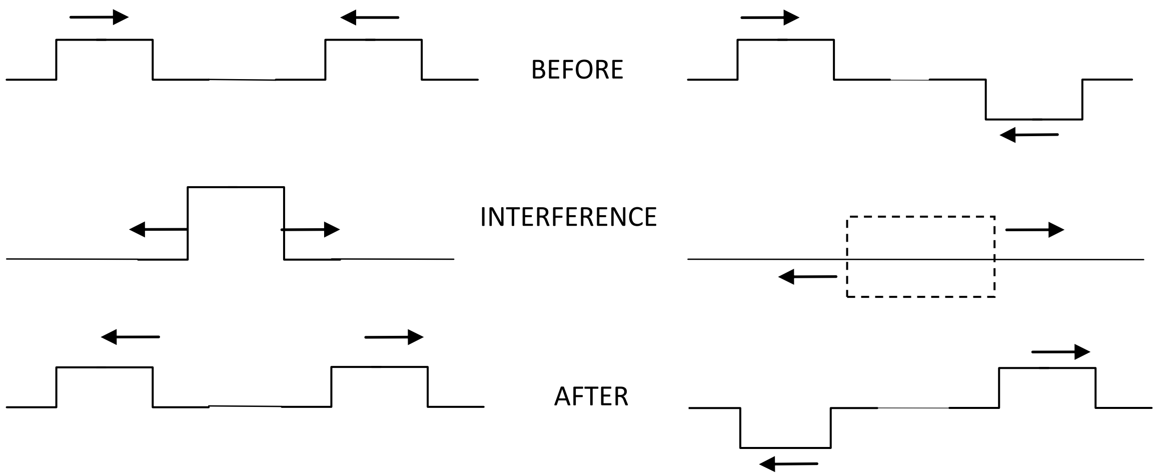
-Let's consider a string pulled on both ends; there is a tension along the string. Assume that one builds up simultaneously two pulse disturbances on its both ends. These two pulses will propagate along the string and at a given moment they will overlap; it happens a ***SUPERPOSITION OF PULSE WAVES***. What happens to the medium particles located at a region where two waves superpose? Here it works the ***PRINCIPLE OF LINEAR SUPERPOSITION***: If the first wave alone would produce the displacement $\mathbf{y_1}$ and the second wave alone would produce the displacement $\mathbf{y_2}$, then, the net displacement of the particle will be $\mathbf{y = y_1+y_2}$. In general, if a set of waves with displacements $\mathbf{y_1, y_2, y_3, y_4 \ldots\ldots y_n}$ superpose at a given point of propagating medium, then the displacement of the particle at this point is

$$y = \mathbf{y_1 + y_2 + y_3 + y_4 + \ldots\ldots + y_n} \qquad (1)$$

Note: The linear superposition principle is valid in the limits of Hook's law (*small displacements*)

-The principle of linear superposition is valid no matter what is the physical nature of parameter (*mechanical waves propagating in space, electric fields propagating through the same location, pressure waves propagating through a liquid...)* that propagates through a medium and no matter what is its mathematical form (scalar or vector) of parameter. In the case of "scalar waves" the expression (1) is the algebraic sum of scalars; in case of "vector waves" the expression (1) is a vector sum.

- If two or more **scalar waves** of same type (ex. pressure oscillations) superpose (see fig. 3) an **INTERFERENCE** phenomena happens. If the "*displacements of superposing waves*" add up there is a "CONSTRUCTIVE INTERFERENCE" and if they subtract there is a "DESTRUCTIVE INTERFERENCE" which may even produce a zero value for the "*parameter displacement*".



CONSTRUCTIVE INTERFERENCE Fig.3 DESTRUCTIVE INTERFERENCE

-When dealing with superposition of vector waves (*like mechanic or electromagnetic waves*), the interference is possible only if the displacement vectors(T_1,T_2) are aligned along the same direction in space or , at least, have components along the each other space direction(in fig.4a, both waves propagate along same direction Oz). If the displacements of two vector waves are aligned along perpendicular directions, even though there is a *superposition* of waves, no *interference* is produced (fig. 4b). The second wave does not have any effect on the displacement produced by the first wave and vice versa. In this case, the particles of propagating medium move on elliptical or on circular paths (for equal magnitudes of displacements of superposing waves), around their equilibrium position.

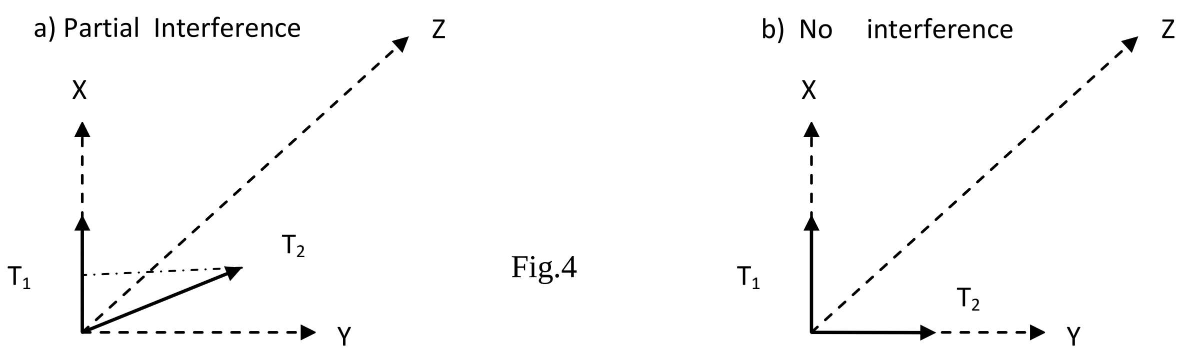


Fig.4

3.4 PULSE WAVE ON A STRING; CALCULATION OF PROPAGATION SPEED.

- We will refer to the propagation of a small pulse through a string under tension (fig.2) to get the expression for the speed of a mechanical wave. Let's assume that the *magnitude* of restoring force(*tension in string*) remains the same all time and at any point of string. In the lab frame, the pulse wave is travelling to the right *at constant speed* "***V*** " **while** any of string particles oscillates ***up*** and ***down*** its equilibrium position(this is a TW wave).

- One may simplify calculations by attaching *a coordinative frame Oxy to the pulse shape* (fig.5). In this frame, the pulse shape remains fixed all time while the **particles of string move to the left**

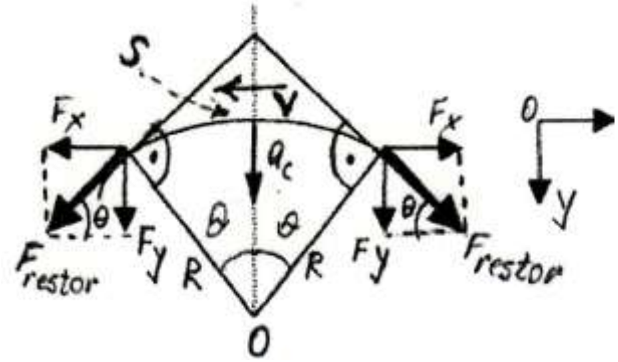


Fig.5 Small string piece at top of pulse

(like water particles inside a curved piece of hose) at constant velocity "-***V*** ". Note that as the problem is considered in an inertial frame Oxy (*moving at constant velocity versus lab frame)* one does not have to make any correction when applying the second law of Newton.

- In this frame, as seen from the figure, "*the particles of string are travelling to the left along a curved path*". At any point on pulse profile, the *particle velocity* fits to the tangent and it´s magnitude "**V**" is constant.

Mechanics tells that, in this circumstances, a **string particle**(*small piece of string at pulse top)* moving along an arch (*with extension 2θ*) undergoes a centripetal acceleration with magnitude

$$a_c = \frac{V^2}{R} \qquad (2)$$

where R is the radius of curvature of pulse profile at top. The **net force** $\vec{F}_{net} = m\vec{a}_c$ acting on this particle "*piece of string*" is the vector sum of the two string tensions (*restoring forces*) applied on both sides of this piece of pulse and it is directed along Oy axe (*x components cancel each other*).One can neglect the effect of gravity force on "*piece of string*" because its magnitude is much smaller than the tension in string. From the figure 5, one may find out that

$$F_y = F_{restor}\sin\theta \qquad (3)$$

Then, the magnitude of $\vec{F}_{net}$ is

$$F_{net} = 2F_y = 2F_{restor}\sin\theta \qquad (4)$$

For small angles $\sin\theta \approx \theta$ and one gets

$$F_{net} = 2F_{restor} * \theta \qquad (5)$$

Next, by applying the second law of Newton

$$F_{net} = 2F_{restor} * \theta = ma = ma_c = m\frac{V^2}{R} \qquad (6)$$

From this expression one finds out that

$$V = \sqrt{\frac{2\theta F_{restor} R}{m}} \qquad (7)$$

If the mass density of string is ***μ***[kg/m], one may express the mass of " *this particle - string piece* " with length $s = R*2\theta$ as

$$m = \mu * s = \mu * 2R\theta \qquad (8)$$

and by substituting (8) to expression (7) one gets

$$V = \sqrt{\frac{2R\theta F_{restoring}}{\mu * 2R\theta}} = \sqrt{\frac{F_{restoring}}{\mu}} \qquad (9)$$

The result at expression (9) is valid for any mechanical wave: ***The propagation speed of mechanical waves in a medium is proportional to square root of a restoring force*** *(T at a string)* ***and inverse proportional to the mass density of the medium*** (often labeled as *inertia parameter of medium*).

REFLECTION AND TRANSMISSION OF A PULSE AT BOUNDARIES

-Now, let's consider what happens with a pulse wave when it get reflected at the end of the string. The end point of string may be "free to move" or fixed and two different situations may be produced:

a) If the ***end point*** of string is ***fixed***, the reflected pulse will be ***inverted***. One may figure out that the *displacement phasor* of the *end point gets* inverted. So, there is an instantaneously **phase change by π** produced during reflection at a fixed end. One may explain this effect by the third Newton's law: the last particle of string pushes up the particle of wall in contact and this one reacts with same magnitude but in opposite direction on the string particle. Then, the reflected pulse travels to the left.

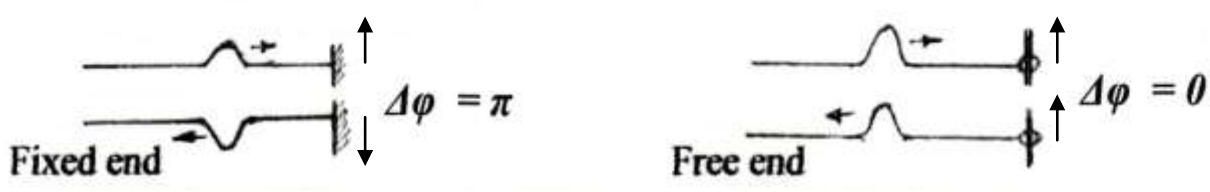


Fig.6 Phase of reflected wave

b) If the end point of string is **free**, the reflected pulse is not inverted because the last point of string gets to its max shift up before pushing wall particle; so, its motion follows pulse profile and the reflected wave is not inverted. So, the **phase change** (*of phasor*) during the reflection is **zero**.

-In the case of a single string there is only one ***secondary wave***; the reflected wave which is inverted or not depending on the type of contact at boundary. If another different ($\mu_2 \# \mu_1$) string is tied to the end point, two secondary waves will appear after the pulse hits on the end of first string. It will be a reflected wave which travels back along first string at same speed $\mathbf{V_1}$ as primary wave and a transmitted wave that will propagate along the second string at speed $\mathbf{V_2}$. If the linear density $\mu_2 > \mu_1$ then the ***reflected wave*** is **inverted** ($\Delta\varphi = \pi$) and if $\mu_2 < \mu_1$ it is *not inverted* ($\Delta\varphi = 0$).

In all cases, the transmitted wave propagates through the second string without a phase change at boundary. When two strings are connected to each other the *same tension applies at any point of each of them*. Based on this fact and taking into account that the linear densities are different,

one may find out the ratio of wave speeds through them as:

$$\frac{V_1}{V_2} = \sqrt{\frac{\mu_1}{\mu_2}} \qquad (10)$$

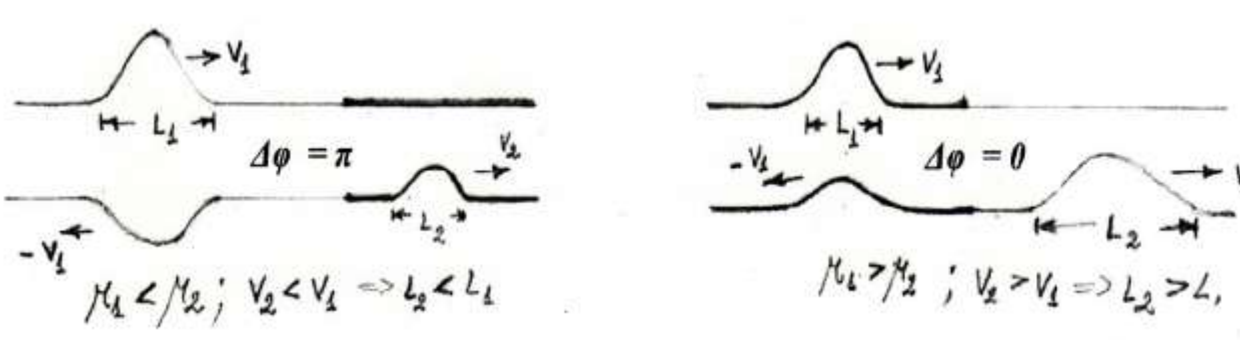


Fig.7 Pulse reflection / transmission through the boundary of two strings with different mass densities