K.Angoni, Physics Dept., Vanier College

SOUND

GENERAL

- Sound is a ***mechanical longitudinal*** (***LW***) wave. During a LW wave, the particles of propagating medium oscillate around their equilibrium position along the direction of wave propagation. This type of waves:
    - can propagate in fluids (*liquids & gases*) and solids;
    - produce *oscillations of local density and local pressure* in the medium that propagates it.

In general, the *amplitude* of displacement due to a sound wave is *very small* and the same is true for the related oscillations of pressure and density in the propagating medium. For example, an ordinary sound wave in air produces pressure oscillations of order ~ ***1Pa*(= 1N/m²)** while normal air pressure is ***1Atm = 10⁵Pa*** .

- The *main characteristic* of *sound waves* is the ***frequency of oscillations***.
One labels an acoustic wave as:
A) ***Sound*** *wave* when the frequency of oscillations is between 20 Hz and 20000Hz.
B) ***Infrasonic*** *wave* when its frequency is less than 20 Hz.
C) ***Ultrasonic*** *wave* when its frequency is above 20000Hz.

-The *sound speed* is actually the *speed of a longitudinal* mechanical *wave(LW)* in the propagating medium.

a) In a fluid, it is calculated as $v_{LW} = \sqrt{\frac{B}{\rho}}$; B[N/m^2] is the " bulk modulus " in the role of *restoring force.*

b) In a solid, it is calculated as $v_{LW} = \sqrt{\frac{Y}{\rho}}$; Y[N/m^2] is the " Young " modulus in the role of *restoring force*.

ρ[kg/m^3] - the volume mass density (of fluid or solid) is in the role of *inertial factor*.

In the following, we will refer to the propagation of a sound wave in air, which is the main experience with sound waves. But, the *same model* is valid for any mechanical LW propagating in a fluid or a solid medium.

1. SOUND WAVE DISPLACEMENT

- A fluid in *equilibrium* means "macroscopic" uniform density and uniform pressure in its whole volume.
There is a *random* motion of molecules of fluid but the average of their displacement in time is zero.
For wave propagation studies, one ***models*** the ***air in equilibrium*** (i.e. still air) as a medium at constant density constituted by ***molecules at rest*** at their equilibrium locations. When a sound wave propagates through still air, each molecule ***oscillates*** around its own equilibrium position along the direction of wave propagation (fig.1).

- Assume that, due to a *harmonic* sound wave, the ***displacements*** y = ***y(x)*** of air molecules inside a pipe, *at a given moment*, are those shown by the graph in fig1.a. Those displacements shift the air molecules as shown by the dots in first raw down the graphs. Consequently, at this moment, the change of air pressure (*proportional to change of local air density*) at locations ***´x´*** along the pipe corresponds to graph shown in fig 1.b. By comparing graph "b" to "a", one see that, the ***maximal changes*** of ***pressure ΔP*** *(i.e. maxima of pressure "displacement")* happen half way ***between two consecutive maximal displacements*** with opposite sign and ***ΔP = 0*** corresponds to the locations with maximum shifts " + / - y_{max} " of air molecules from their equilibrium position.

General rule: In a harmonic sound wave, the **wave function that** describes the **pressure *"displacement "* (*i.e. its changes*) is shifted by λ/4** with respect to the **wave function of particles' displacements.** *So, if one knows the function* $y(x,t) = y_{max}\sin(kx-\omega t)$ *, than one can get* $\Delta P(x,t)=\Delta P_{max}\sin[(k(x-\lambda/4) -\omega t]$ *and vice versa.*

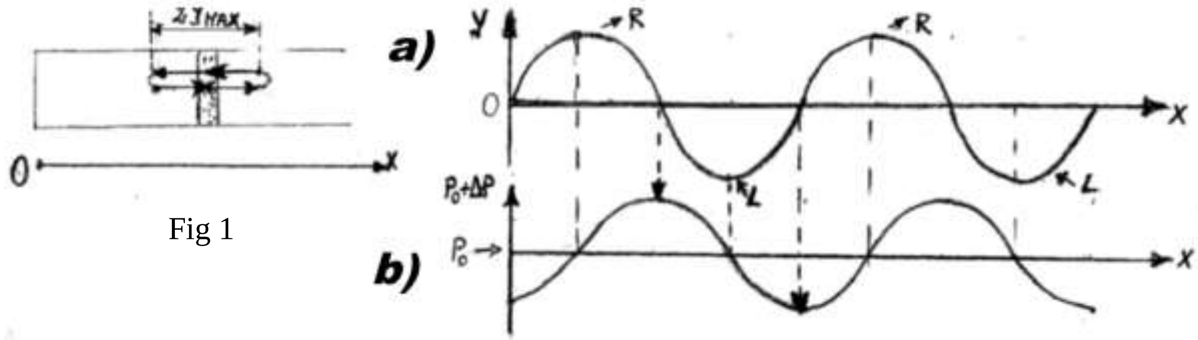


Fig 1

R - shift on the right
L - shift on the left

(*Air molecules locations, sound wave)*   *   * * ******* * *   *   * * ******** *   *   *
(*Air molecules locations*, equilibrium*)*   *

- The wave front is a very important concept for waves that travel in ***2D*** or ***3D*** space**.** One has defined the **WAVE FRONT** as the **locus of space points** at which the wave function has **the same phase at a given moment of time**. (*Ex. The same crest of ripples that travel on still water surface of a lake corresponds to one wave front).* If the wave front has a spherical shape, one call it a *spherical wave*. Spherical waves propagate only in uniform mediums. At a big distance from the source, a front wave *portion* of spherical wave transforms to a plane (see fig 2.a,b,c).

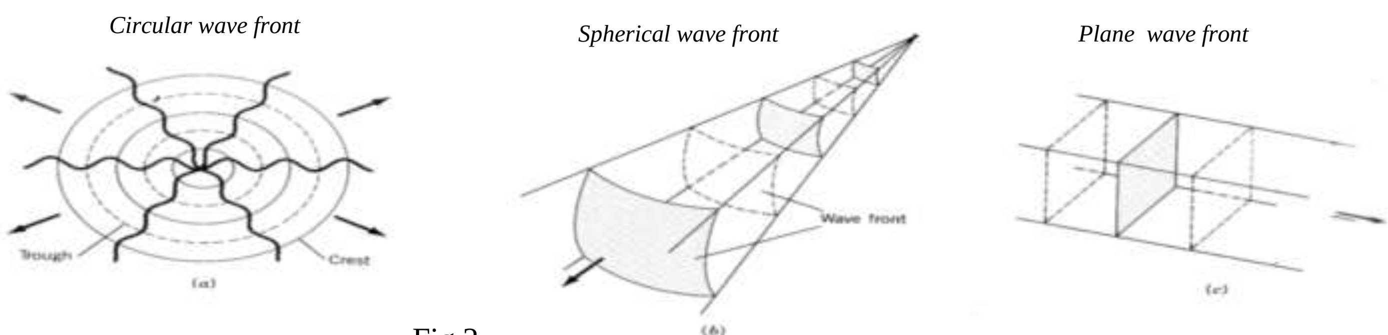


Fig 2

2. SOUND WAVE RESONANCE IN PIPES

Remember what happens with *displacement* at a *fixed end* and a *non-fixed end* when a TW wave travels along a string: If the end point of string is **fixed,** there is a **node(**zero value**) of displacement**; If it is **free** there is an **anti-node(***maximum value***) of displacement**. In both cases, there is a *reflected wave* at the *same frequency* and *wavelength* as the original one. In the case of a string with length " *L* " those two waves *travelling along the opposite* directions, *interfere* and build up a *resonant wave on string* if the ratio " $L / (\lambda/2) = n$ " .
This means that there is a resonance in a string if its length hosts a ***integer number of half-wavelengths***.

- Generally, one classifies a pipe as a **closed** pipe if ***one of its ends** is **closed*** and classifies it as an **open** pipe if ***both ends** are **opened***. To get a *resonance* ***in a pipe***, the length of the pipe should host a standing wave for the sound wave (***displacement***) in such a way that at the ***end point of pipe*** there is a:

a) ***Node (****zero displacement - it cannot oscillate) if this is a* **Closed end**(see fig.1a).
b) ***Anti-node*** (*maximum displacement* "+" or "-") if this is an **Open end**.

Based on criteria "a-b" for the displacements of air molecules, one may figure out that a *sound wave* will build a ***resonance in a pipe*** if the ratio of pipe *length* to sound *wavelength* fits to one of the following conditions;

a) for a **closed pipe,** the pipe length must host an **<u>odd</u> number of λ/4** (see fig 3.a):

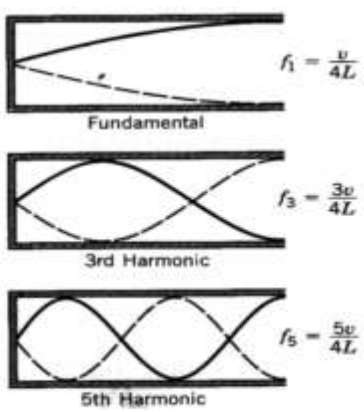
$$\lambda/4 \quad = L \rightarrow v * T = 4L \rightarrow\rightarrow \rightarrow 1/f = \frac{4L}{v} \rightarrow\rightarrow\rightarrow\rightarrow f_1 = (\frac{v}{4L})$$
$$3(\lambda/4) = L \rightarrow v * T = (4L)/3 \rightarrow 1/f = \frac{4L}{3v} \rightarrow\rightarrow\rightarrow\rightarrow f_3 = 3(\frac{v}{4L})$$
$$5(\lambda/4) = L \rightarrow v * T = (4L)/5 \rightarrow 1/f = \frac{4L}{5v} \rightarrow\rightarrow\rightarrow\rightarrow f_5 = 5(\frac{v}{4L})$$

b) for an **open pipe,** the pipe length must host an **<u>integer</u> number of λ/2** (see fig 3.b)

$$\lambda/2 \quad = L \rightarrow v * T = 2L \rightarrow \rightarrow\rightarrow 1/f = \frac{2L}{v} \rightarrow\rightarrow\rightarrow\rightarrow f_1 = (\frac{v}{2L})$$
$$\lambda \; = L \rightarrow v * T = L \rightarrow\rightarrow\rightarrow\rightarrow \frac{1}{f} = \left(\frac{L}{v}\right) \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow f_2 = \frac{v}{L} = 2(\frac{v}{2L})$$
$$3(\lambda/2) = L \rightarrow v * T = 2L/3 \rightarrow 1/f = \frac{2L}{3v} \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow f_3 = 3(\frac{v}{2L})$$

So, for <u>*closed pipes*</u> only ***odd*** orders of harmonics are possible $\qquad f_n = n\left(\frac{v}{4L}\right);\ n = 1,3,5..$ (1)

and for <u>*open pipes*</u> ***all*** orders of harmonics are possible $\qquad f_n = n\left(\frac{v}{2L}\right);\ n = 1,2,3,4,5..$ (2)



THREE FIRST NORMAL MODES OF A PIPE

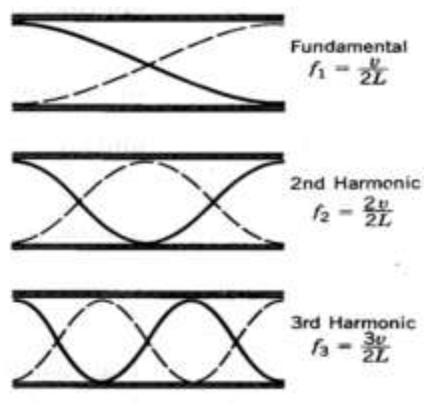


Fig 3.a fig 3.b

-Assume that a *set* of sound waves with *different frequencies* but equal amplitudes (i.e. equal intensities) gets into a pipe. If the ***frequency*** of one wave of the set matches a ***frequency*** " f_n " of a ***normal mode*** of the pipe, there is a ***resonance*** produced into the pipe. The function of this resonant wave is $y_n(x,t) = 2A\sin(k_n x+\varphi_n)\cos(\omega_n t)$ and the amplitude of displacement at pipe output (*open end*) is " 2A ". This means that the mechanical energy of air molecules located at the open end of pipe is ~ $4A^2$ and the intensity of sound wave emitted outside pipe gets amplified. The sound wave at this frequency " f_n " becomes the main component heard outside the pipe.

<u>3.DOPPLER EFFECT</u>

- Assume that the blowing horn of a train emits an harmonic sound wave at <u>*frequency*</u> f_0. This is the frequency of sound that would hear a person *at rest* in a station if the train is *at rest* in station, too.
If this train is ***approaching*** the station, this person would hear a ***higher frequency*** ($f' > f_0$).
If the train is ***moving away*** from the station, he would hear a ***lower frequency*** ($f' < f_0$).
The change of recorded frequency due to a ***relative motion*** between the ***source*** of a sound wave and the ***observer*** is called *Doppler Effect*. Note that a similar phenomenon happens to *E&M* waves, too.

K.Angoni, Physics Dept., Vanier College

Relative motion(*reminder*). Let´s consider a frame O "***at rest*** " and a frame O' moving at ***constant velocity*** $\vec{u}$ versus frame O. Assume that end point ***I*** of an object ***at rest*** versus O-frame, has the coordinate X_I in frame **O** and the coordinate X'_I in the frame **O´**. As one may figure out easily (see fig. 4) $X'_I = X_I - u*t$.
If ***F*** is the other end point of object, its coordinates in the two frames (X_F , X'_F) are related as $X'_F = X_F - u*t$.
So, the length of this object in O frame is $L_O = X_F - X_I$ and its length in O' is

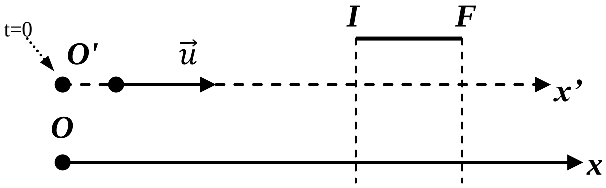


Fig.4

$$L'_O = X'_F - X'_I = [(X_F - u*t) - (X_I - u*t)] = X_F - X_I = L_O$$

The measured length of an object is the same in any two frames moving at constant velocity versus each other.
So, the length of an object at uniform motion versus a given frame is the same as if it was at rest versus this same frame.

In the following, the wavelength of a sound wave is in the role of length IF; so, $\lambda' = \lambda_0$.
- Let's consider the sound propagation in still (***equilibrium*** ***) air.*** Frame "O" is tied to ground and "***air*** *at rest versus the ground "* while O_s and ***O'*** are two frames *tied to* **source** and the ***observer***. Let ***V*** be the ***speed of sound*** in ***still air***, f_0 the *frequency* of sound wave *as measured at source*, ***V '*** and ***f ´*** the ***speed*** and ***frequency*** of ***sound wave*** *as measured by observer (in frame O'). Let's label by* υ_S the ***speed*** of sound ***source*** and υ_{Ob} the ***speed*** of the ***observer*** versus ***O*** frame (*ground & still air*). One may distinguish three different situations:

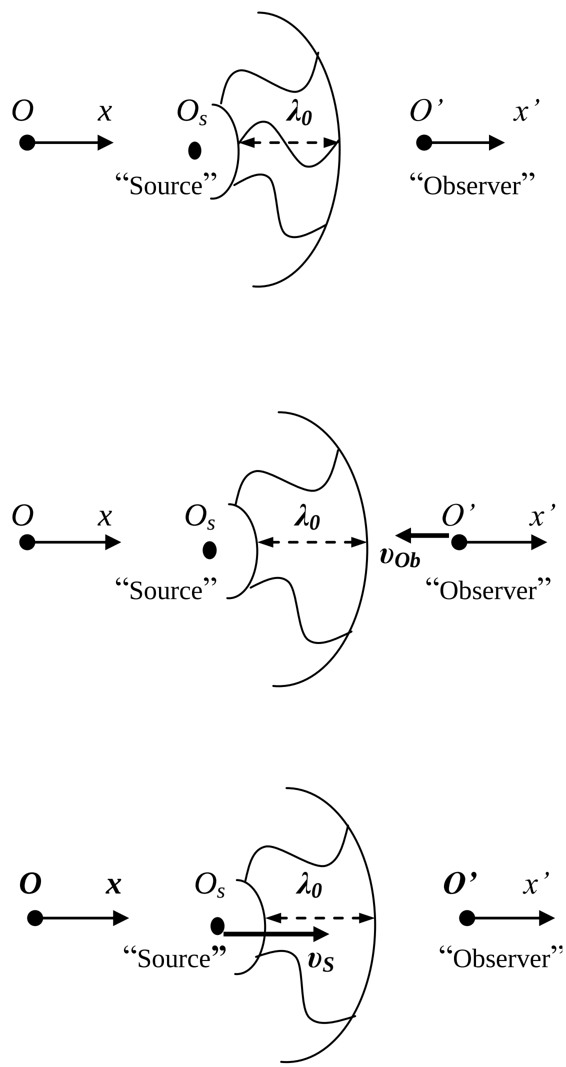


Figure 5

a) $\upsilon_S = 0; \upsilon_{Ob} = 0$ (*source and observer are at rest vs O-frame*)
The *distance between two consecutive wave fronts*, i.e. the **wavelength** of sound wave measured in frame O (and O_s)is

$$\lambda_0 = V / f_0 \qquad (3)$$

The distance between the same consecutive wave fronts, i.e. the **wavelength** of sound wave measured in observer's frame O' is $\lambda' = V' / f' = \lambda_0 = V / f_0$. As the speed of sound vs. both frames O and O' is the same, $V' = V$ ***it comes out that***

$$f' = f_0 \qquad (4)$$

b) $\upsilon_S = 0; \upsilon_{Ob} \# 0$ (*observer approaching the source at rest vs frame O*)
The **wavelength** of sound wave in frame O is $\lambda_0 = V / f_0$
The wavelength of sound wave in frame O' is $\lambda' = \lambda_0$ but in this case the ***speed of sound versus O' frame is*** $V' = V + \upsilon_{Ob}$
So, one get $\lambda' = V' / f' = (V + \upsilon_{Ob}) / f' = \lambda_0 = V / f_0$
and it comes out that $f' = [(V + \upsilon_{Ob}) / V] * f_0$
If the observer moves away from the source he would record the frequency $f' = [(V - \upsilon_{Ob}) / V] * f_0$
So, in general, depending on the ***motion of observer***

$$f' = [(V \pm \upsilon_{Ob}) / V] * f_0 \qquad (5)$$

c) $\upsilon_S \# 0; \upsilon_{Ob} = 0$ (*source approaching the observer at rest vs frame O*)
The **wavelength** λ_s in O_s frame is : $\lambda_s = (V - \upsilon_S) / f_0$
where $(V - \upsilon_S)$ is the speed of sound wave versus *frame* O_s *moving* at speed υ_S ; f_0 (= $1/T_o$) is the frequency of ***sound wave at source. The observer frame is at rest versus air; so, the speed of sound vs frame O' is V and*** $\lambda' = V / f'$.

As the **wavelength** is the same in two frames O_s and ***O'***, one writes

$\lambda_s = \lambda'$ or $(V - v_S)/f_0 = V/f'$ which brings to $f' = [V/(V - v_S)] * f_0$

If the source is moving away from the observer $f' = [V/(V + v_S)] * f_0$

So, depending on the ***motion of the source*** versus the observer he will record the frequency

$$f' = \frac{V}{V \pm v_s} * f_o \quad (6)$$

(-) approach; (+) moving away.

By combining (5-6) in a single expression, one gets the following general formula for Doppler effect:

$$f' = \frac{V \pm v_{ob}}{V \pm v_s} * f_o \quad (7)$$

a) Frequency increase
   1. Observer approach take $(+v_{ob})$
   2. Source approach take $(-v_S)$
   3. Source and observer approach take $(+v_{ob}$ and $-v_S)$

b) Frequency decrease
   1. Observer move away take $(-v_{ob})$
   2. Source move away take $(+v_S)$
   3. Source and observer move away take $(-v_{ob}$ and $+v_S)$

- Generally, v_s *and* v_{ob} are smaller than speed of sound V in still air and the frequency change is small.

Exemple: If train approaches at v_s = 50Km/hrs = $50*10^3$ [m]/$3.6*10^3$[s]= 13.9 m/s and V = 344 m/s, an observer at rest, will hear the frequency $f' = \frac{344}{344 - 13.9} f_0 \approx 1.06 f_0$. So, only 6% increase of frequency.

4. Wavelength of waves emitted by a source in motion

-We introduced the model of harmonic travelling waves by referring to a source that performs SHM. Then, by referring to the oscillations *period* " *T* " *of the source* and the *propagation speed* " *V* " of the disturbance through the *medium* we introduced the wavelength by expression $\lambda = V * T$ (8)

We introduced the concept of the "*wave front*" as a *surface (or line) containing all points of space that have the same phase-value* by referring to the model of a *point source <u>at rest</u>* emitting spherical waves all around inside an uniform medium.
In this model, the **wavelength** is the **distance** between **two consecutive wave fronts**. In general, one shows it as the distance between the crests of two consecutive wave fronts (fig.6). In addition, we have noted that the *wavelength* is a *parameter* that *depends* both on the *source and* the *medium*.

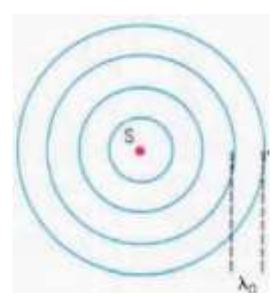


Fig.6 "λ" for a source at rest emitting spherical wave fronts.

- If one refers to the model of a source moving at constant speed, the distance between the consecutive wave fronts will depend on the source speed and the direction of source motion(fig.7). This distance is smaller in direction of source motion. So, in this case the wavelength "λ" of wave depends essentially on the space region where one measures it. An observer "*in front* " of an approaching source measures a *shorter* wavelength in comparison to the wavelength measured by an observer " behind " of this source even though the oscillations period (or frequency) at the source is the same.

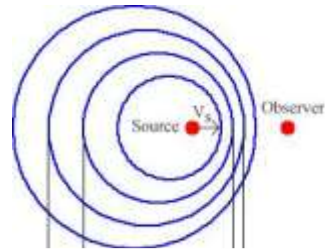


Fig.7 "λ" in case of a source moving at constant speed.

If the source speed " v_s " becomes equal to the speed of travelling waves " V ", the consecutive wave fronts in direction of motion superpose (fig.8). This means that λ = 0 and the expression $f' = f_0 * V/(V - v_s)$ for Doppler effect on sound waves predicts a frequency $f' = \infty$ for an observer versus which this wave approaches. The superposition of many crests means a constructive interference of their displacements. This produces a " big displacements " or a "shock of wave " *just in front* of moving source .

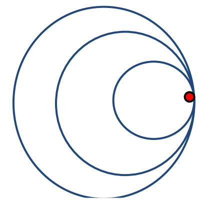


Fig.8 shock wave only at source location for $v_s = V$

-If the speed of the source is larger than the speed of wave propagation in the medium , i.e. if " $v_s > V$ " , the shock wave is produced at all space points which are located on a cone (known as "*Mach cone*") which has the summit at source . The shock wave propagates through the medium in a direction perpendicular to cone surface (fig.9). Figure 10 shows an example of a "*bow wake*" (i.e. a shock wave on the water surface) produced by a duck swimming at a speed larger than the speed of travelling TW waves on lake water.TW waves on water are generated by duck legs that hit on surface of water while the duck advances at a speed $v_D > V$. The V-shape of "shock wave" fronts shown in this picture corresponds to the cut of the Mach cone by a horizontal plane.

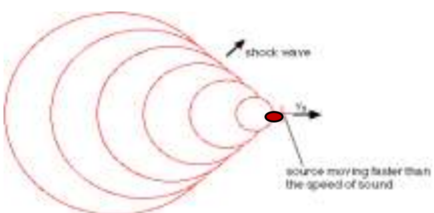


Fig.9 Mach cone of shock wave fronts for $v_s > V$

Fig.10 A "bow wake" produced when a duck swims at $v_D > V$

- One may find the angle "μ" of Mach cone (fig.11) by referring to the speed of source " v_S" versus the medium that propagates the waves and the speed " V " of propagation of waves in this medium from the

relation $sin\mu = \frac{V*t}{v_S*t} = \frac{V}{v_S}$ or $\mu = arcsin\frac{V}{v_S}$ (9)

One has introduced Mach number $M = \frac{v_S}{V}$ (10)

Often, one prefers to write expression (9) as $\mu = arcsin\frac{1}{M}$ (11)

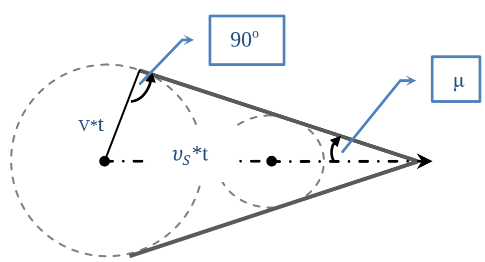


Fig.11 Mach cone of a shock wave. Only two wave fronts shown

In the practice of aviation, one uses the terms:

Subsonic for flights with M < 1;
Transonic for flights with M = 1;
Supersonic for flights with M > 1;
Hypersonic for flights with M > 5.

- A main application of the shock wave model deals with the sound. In particular, when an object moves through the air it produces pressure changes in air. Those pressure changes propagate through air at speed of sound wave (LW wave). If the object moves at a speed larger than the speed of sound in air, the displacement of shock wave builds up extreme changes of pressure over the surface of the Mach cone. The result appears as a " *sonic boom* " for an observer located on Mach cone. This sonic boom effect is heard by people when a supersonic jet flies near ground(Fig.12). Also, when there is humidity in air, the increase of local pressure on the surface of Mach's cone condensates the water vapors and builds up a cloud with the same shape (fig.13).

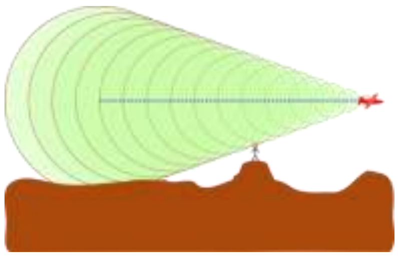


Fig.12 Sonic boom is heard when the Mach cone swipes the location of person

Fig.13 Sonic boom pressure condensates the water vapor in shape of Mach cone.

5. BEATS

The principle of linear superposition is valid for sound waves, too.

$$y_{Tot} = \sum_{i=1}^{n} yi\_for\_i = 1,2,....n \quad (12)$$

We used this principle to explain how two or more waves can produce standing waves and resonance inside a "region of space" when they interfere. The standing waves and resonance are results of *waves interference* in "***space domain***" (displacement "bubbles" in graph y = y(x) correspond to oscillation amplitudes at *different x-values*).

- Now, let's see how the same principle can explain the interference in "***time domain***" (*same space location but different t-values*). This type of interference is known as " ***beats*** ". Let´s consider two sound waves:
  - that propagate along the same direction (*say "- Ox"*);
  - having the same amplitude "A" and *zero phase constant*;
  - with ***slightly different frequencies*** (say $f_1 \gtrsim f_2$); $y_1 = A\sin(k_1 x + \omega_1 t)\_and\_y_2 = A\sin(k_2 x + \omega_2 t)$
  - that interfere at a given space location. *(to simplify math calculations, assign x = 0 at this place).*

So, at x = 0, the wave functions give $y_1 = A\sin(\omega_1 t)\_\omega_1 = 2\pi f_1$ and $y_2 = A\sin(\omega_2 t)\_\omega_2 = 2\pi f_2$

- By use of superposition principle at this point (x = 0) one can get its ***resultant displacement*** as follows

$$y_{Tot}(x=0) = y_1 + y_2 = A\sin(\omega_1 t) + A\sin(\omega_2 t) = 2A\sin\left(\frac{\omega_1 + \omega_2}{2}\right)t * \cos\left(\frac{\omega_1 - \omega_2}{2}\right)t \text{ and}$$

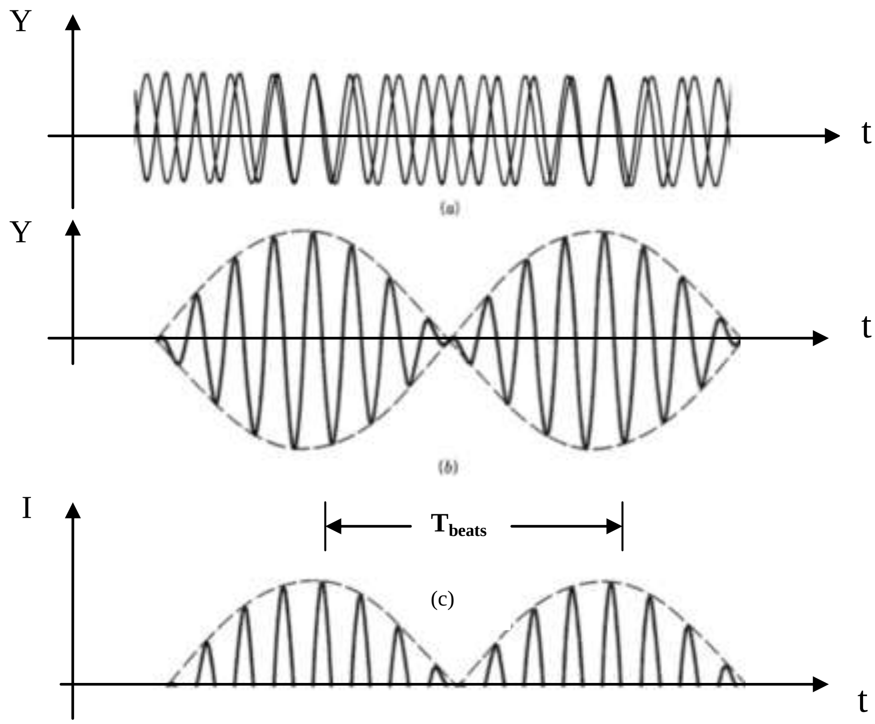
$$y_{Tot}(x=0) = 2A\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t * \sin 2\pi\left(\frac{f_1 + f_2}{2}\right)t \quad (13)$$

Note that the expression (13) describes the ***time evolution of displacement*** at a ***given point*** in space.

- As we assumed that $f_1 \gtrsim f_2$, it comes out that $\frac{f_1-f_2}{2} \equiv f_{Amp}$ is a small frequency and the factor $2A\cos 2\pi\left(\frac{f_1-f_2}{2}\right)t$ is *an **amplitude*** that *varies slowly;* i.e. with a large period $T_{Amp} = \frac{1}{f_{Amp}} = \frac{2}{f_1-f_2}$.

The factor $\sin 2\pi\left(\frac{f_1+f_2}{2}\right)t$ presents an oscillation with high frequency; i.e. comparable to f_1 and f_2.

Fig.14



-The graph in figure 14.b corresponds to function (13) and represents " *a standing wave in time domain* ". This type of ***variation*** for " *displacement-Y* " ***<u>in time domain</u>*** is characteristic for a phenomenon called ***wave " BEATS "***. Note that the *intensity* (or **loudness**) of sound waves is *proportional* to the energy of oscillations which means that it is proportional to " Y^2 ". This explains the shape of sound wave ***intensity*** (shown in 14c). So, one perceives the " beats " of sound waves as a periodic sound signal with a period (*compare 14.c to 14.b*) $T_{beats} = T_{Amp}/2$ and this means a beat frequency

$$f_{beats} = \frac{1}{T_{beats}} = \frac{1}{T_{Amp}/2} = \frac{2}{T_{Amp}} = \frac{2}{2/(f_1-f_2)} = f_1 - f_2 \qquad (14)$$

To avoid the negative sign of f_{beats} (when $f_1 < f_2$) one writes this expression as $f_{beats} = |f_1 - f_2|$ (15)
You may simulate " beats" for different frequencies by changing f_1, f_2 values at https://ophysics.com/waves10.html

6. SOUND INTESITY

Let´s start by the calculation of *power transported* by a TW harmonic mechanical *wave* along a string. Next, we will apply the same model to calculate the power transported by a sound wave through a pipe.

*A] THE **POWER** TRANSPORTED BY A **TW TRAVELLING ON A STRING***

Assume that one moves a point of string (***wave source***) *up* and *down* with amplitude Y_{max} and *frequency* f. We fix the *origin* of Ox axis *at source* and align its direction to that of wave propagation (fig.15). Due to the wave, any *string element with length "**dx**"* will oscillate at amplitude Y_{max} (***assuming no energy loss***) and the same frequency " f ". If the *linear density* of the string is μ, the mass of "*string element* " with *length dx* is

$$dm = \mu dx \qquad (16)$$

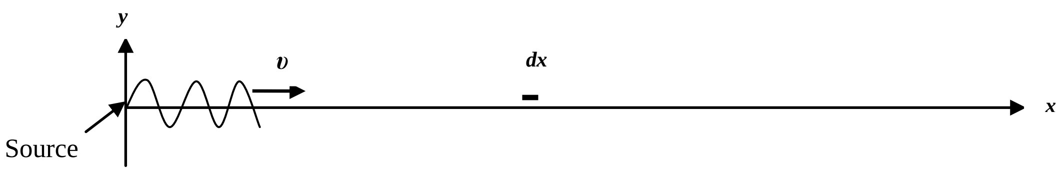


Fig. 15

The travelling wave brings to this element a SHM oscillation with energy (*remember SHM energy*) ;

$$dE = \frac{1}{2}kY_{max}^2 = \frac{1}{2}(\omega^2 dm)Y_{max}^2 = \frac{1}{2}\omega^2 Y_{max}^2 dm = \frac{1}{2}\omega^2 Y_{max}^2 \mu dx \qquad (17)$$

This element transports the same amount of energy to the next element of string. If this transport happens for a time interval **dt**, the amount of energy passing through *string element* **dx** during *a second* is ***dE/dt*** . This ratio represents the *power transported* by wave through the string. One can get expression of power (18) after the substitution ***dx/dt = V*** where ***V*** is the ***wave speed***

$$P = \frac{dE}{dt} = (\frac{1}{2}\omega^2 Y_{max}^2 \mu)\frac{dx}{dt} = \frac{1}{2}\omega^2 Y_{max}^2 \mu V = \frac{1}{2}(2\pi f)^2 Y_{max}^2 \mu V \qquad (18)$$

Example: The ***power*** transported by a TW wave with amplitude ***2mm*** and frequency ***50Hz*** that propagates at V=***10m/s*** on a string with linear density ***10g/m*** is $P = 0.5*(2*3.14*50)^2*(0.002)^2*(10*10^{-3})*10 = 1.97*10^{-2}W$

Note: This calculation model is valid for the power transported by a ***LW***, too.

*B] THE **POWER** TRANSPORTED BY A **SOUND WAVE (LW) THROUGH A PIPE***

Let's apply the same model for calculation of power transported by an harmonic sound wave (***LW***) that propagates inside a ***pipe***. Assume that a sound wave at constant ***displacement amplitude*** Y_{max} and ***frequency*** f propagates inside a ***pipe*** with ***cross area*** *"**A**"*. One will need to update only the expression for "*oscillating mass*". In this case, it represents the mass inside the cylinder with elementary length ***dx***.
If the *volume density of mass* is ρ[kg/m^3], then $dm = \rho dV = \rho A dx$ (19)

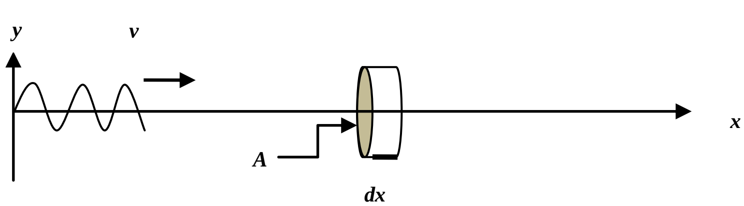


Fig. 16

Then, the two expressions (17-18) transform to

$$dE = \frac{1}{2}\omega^2 Y_{\max}^2 dm = \frac{1}{2}\omega^2 Y_{\max}^2 \rho A dx \qquad (20)$$

$$P = \frac{dE}{dt} = \frac{1}{2}(2\pi f)^2 Y_{max}^2 \rho A V \qquad (21)$$

Example: The ***power*** transported by an harmonic sound wave at frequency ***50Hz*** and ***0.1mm amplitude of displacement*** *inside* ***a pipe*** *with* ***radius 2cm*** *filled with* ***air (V = 340m/s, ρ =1.21 kg/m³)*** is
P *= 0.5*(2*3.14*50)²*(0.0001)²*(1.21)* [3.14*(0.02)²]* 340 =* ***0.000255W = 0.255*10⁻³W***

C] *<u>DEFINITION OF INTENSITY FOR A SOUND WAVE</u>*

A sound wave transports energy along its direction of propagation. The *intensity* ***I*** *[W/m²]* of this wave is " the amount of ***<u>power</u>*** (***P****[W=J/s]*) that falls on ***unit area*** *(****A****[m²])* ***<u>perpendicular</u>*** to its travelling ***direction "***.

$$\boldsymbol{I}\ [\text{W/m}^2] = \boldsymbol{P/A} \qquad (22)$$

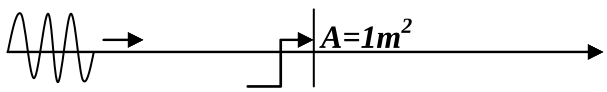


Fig. 17

In case of example given upside $I = 0.255*10^{-3}W / 3.14*(0.02)^2 = 0.203 W/m^2$

<u>Note : This definition for intensity is valid for any kind of wave no matter its type or physical nature.</u>

- The human ear can hear sound waves with intensity inside an extremely wide range [**10^{-12}** to **1 W/m²**] but it does not have a linear response to the sound intensity; for example *it does not perceive 2 times louder a sound with intensity of 0.6 W/m² when compared to a sound of intensity 0.3 W/m²*.
Experiments have shown that the <u>*human hearing " follows better " the logarithm of sound intensity*</u>. So, one has related the *sound loudness to " the* ***<u>level</u> of sound*** *intensity β " defined by expression:*

$$\beta = 10\log_{10}\frac{I}{I_o} \qquad (23)$$

where ***"I"*** is the intensity of sound in SI units [W/m²] and $I_0 = 10^{-12}\ W/m^2$ **.** The coefficient ***β has no real units*** but, in practice, one calls it the sound ***level*** in "***decibel*** (dB) units".

- With this definition, one finds out that the sound ***level of hearing threshold*** is

$$\beta = 10log_{10}\frac{I_0}{I_0} = 10log_{10}1 = 0dB \qquad (24)$$

and the ***pain threshold*** is

$$\beta = 10log_{10}\left(\frac{1\ W/m^2}{10^{-12}W/m^2}\right) = 10log_{10}10^{12} = 12*10log_{10}10 = 120dB \qquad (25)$$

Note that the sound level (0 ÷ 120 dB) avoids the extremely wide range of sound intensity (10^{-12} ÷ 1 W/m²) .

Common Sound Levels

30 db - Whisper ; 50÷60 db - Normal conversation ; 80 db - Ringing telephone; 98 db - Hand drill ;
105 db - Bulldozer; 110 db - Chain saw ; 120 db - Ambulance siren ; 140 db - Jet engine take-off

For the example given upside, one get a sound ***intensity level*** $\beta = 10log(0.203/10^{-12}) = 10log(2.03*10^{11})$
$= 10log(2.03)+10log10^{11} = 3.075+110\ log10 = 3.075+110 = 113.07dB$

- The power emitted by a sound source may be ***concentrated*** along ***one direction*** or ***distributed uniformly*** in all space directions. The first case concerns the majority of technical applications; one has to use some advanced models for the study of these applications.
In this course, we will consider only the cases where the energy propagates by ***uniform plane waves*** or spreads ***uniformly*** *over* ***spherical waves***.
In this ***last case***, if the ***source*** produces a sound wave with ***power P****[W]* and this wave propagates without energy loss as a spherical wave, at a ***distance R****[m] from the source*, the sound wave will produce the intensity

$$I[W/m^2] = \frac{P}{4\pi R^2} \qquad (26)$$