LIGHT (Basic information)

- The ***geometrical optic*** models the light as "a set of *tiny particles that travel along* ***straight lines*** labelled as ***optical rays*** ". The ***optical ray*** is thought as an ***extremely thin beam of light***. The geometrical optics finds the ***image*** of an object by following the path of optical rays emerging from the points at object's borders.

- The experimental procedures showed that the ***diffraction*** of light becomes the major drawback when one tries to produce ***light rays*** in a lab by using very small opening diaphragms. The model of geometrical ***optics*** *cannot explain* ***diffraction***. The existence of diffraction shows that:
 a) The *optical ray* is not a physical reality (**but it remains a very useful physics modeling tool**).
 b) The geometrical optics model cannot explain all phenomena related to light.
One has explained diffraction by applying the wave model for light. The wave model of light attaches *specific* values of *wavelength* to different *colours* of light.

REFLECTION & REFRACTION OF LIGHT

If a *beam of light falls* on a ***rough*** interface, there is a ***diffused reflection*** (fig.1.a).

-If the interface (*between two **different** mediums)* is well ***polished***, there is a ***specular reflection***. In this case there is only one reflected ray (fig.1b) and the *specular* ***reflection law*** says that:

$$\theta_R = \theta_1 \qquad (1)$$

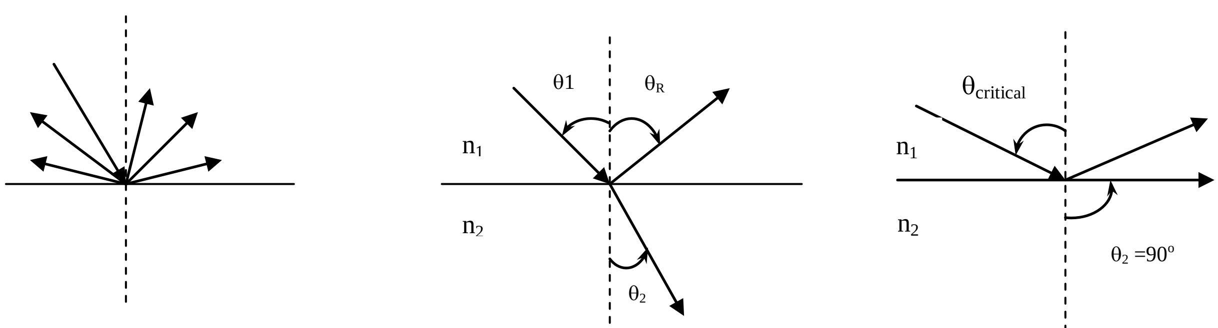


Fig.1a Diffused reflection   1b Reflected-refracted rays   1c Critical reflection

-If the medium beyond the interface is *transparent*, there is a ***refracted*** light ray, too. The ***refraction law*** says that: the direction of refracted ray (angle θ_2 *from vertical*) is related to incident angle θ_1 by Snell's law

$$n_1 sin\theta_1 = n_2 sin\theta_2 \qquad (2)$$

$n_1 = c/\upsilon_1$ and $n_2 = c/\upsilon_2$ stand for the refraction index of medium "0" and "1"
c, υ_1 and υ_2 stand for the speed of light in vacuum, medium "1" and medium " 2".

The incident, reflected and refracted rays are in the same plane.

TOTAL INTERNAL REFLECTION

- Suppose that a monochromatic (**one colour**) beam of light falls from a medium with larger refracting index ***n_1*** onto a medium with smaller refractive index ***n_2*** ($n_1 > n_2$). As $\frac{n_1}{n_2} > 1$, the expression (2) makes sense only for ***incident angles smaller*** than a given value $\theta_{1\text{-critical}}$ which can be found from the condition[1]

$$\frac{n_1}{n_2} sin\theta_{1-critical} = 1 \qquad (3)$$

For $\boldsymbol{\theta_1 = \theta_{1\text{-critical}}}$ one gets $\theta_2 = 90^0$ which means that there is a "***refracted ray directed along the interface***". For $\boldsymbol{\theta_1 \geq \theta_{1\text{-critical}}}$ there is ***no refracted beam*** inside the second medium. One says that, for $\boldsymbol{\theta_1 \geq \theta_{1\text{-critical}}}$ there is a " ***total internal reflection of light*** ".

Notes: a) ***Angles are measured relative to the normal at the impact point of incident ray***.
b) ***The intensity of refracted ray decreases while $\theta_{refracted}$ approaches 90^0***.

One uses the total internal reflection to make light travel inside the optical fibres (diameter 10 ~ 80μm).

CHROMATIC DISPERSION

-The **refractive index** "*n* $_{(>1)}$" of a transparent **medium** is the ratio between the ***light speed*** in *vacuum(c ~3*10^{8}m/s)* and its speed in the transparent *medium*. $\boldsymbol{n = c / v}$ (4)

Note that, in vacuum ***n =1*** no matter what is the colour (***wavelength***) of light but this is not true in other transparent mediums. In general, as shown in see fig.2, n-value is ***larger*** for blue colour **(short wavelengths).** The change of ***n**-value* with wavelength is at the origin of the ***chromatic dispersion:*** When a white (**constituted by many colours**) light ray falls from the air (or vacuum) on a transparent medium, there is a different angle *of refraction* in the medium for **each *color*.** So, ***different colour rays*** propagate along ***different directions*** (see fig.3a,b). One says that it happens " ***light dispersion***" inside the second medium. In fact, there is light dispersion any time a *parallel beam containing several colors* gets refracted on an interface.

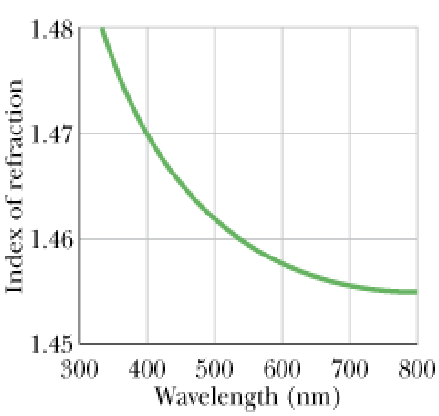


Fig.2 n-values in fused quartz

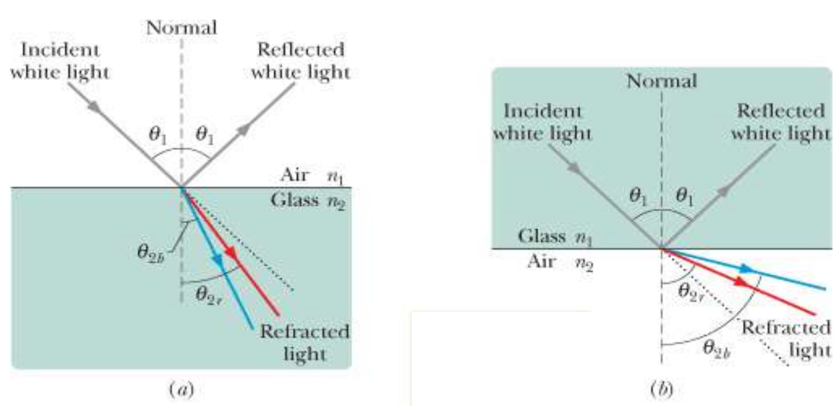
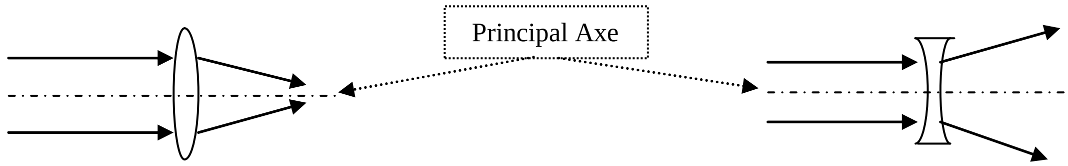


Fig.3 Chromatic dispersion of white light in glass

[1] Because the right side of eq. (2) i.e. $\sin\theta_1$ cannot exceed value 1.

LENSES

An optical lens is a transparent object that deviates the direction of optical rays.
A ***convergent*** lens is thicker at the center; ***a divergent*** lens is thicker at the *rim*.



Convergent lens   Divergent lens

The principal axe is perpendicular to lens shape and passes by its geometrical center.
The first step approximation of the lens modelling:
- refers to ***thin*** lenses (***thickness << diameter***) ;
- neglects spherical aberrations[2]; for this one consider only the paraxial[3] rays;
- neglects ***chromatic aberrations*** (i.e. **different focal point for different colours**).

- To find an image, one traces ***three main rays*** that leave a point on object border:

a) The ***central ray*** passing through the **lens center**. On the other side of lens, the central ray follows a ***non-deviated path.***

b) One **ray** ***parallel to the principal axis of lens.*** *On the other side of lens;*
* *For a* **convergent** *lens, it passes by* ***the*** other ***side* focal** (3.a) point.
* F*or a* **divergent** lens, its *extension passes by* **focal** point ***closest to object*** (3.b).

c) * *For a* ***convergent lens;*** One ray ***directed*** toward the ***closest focal point (3.a).***
On the other side of lens it follows ***parallel to lens principal axis.***
* *For a* ***divergent lens***; One ray ***directed*** toward the **other side** ***focal point (3.b).***
On the other side of lens it follows ***parallel to lens principal axis.***

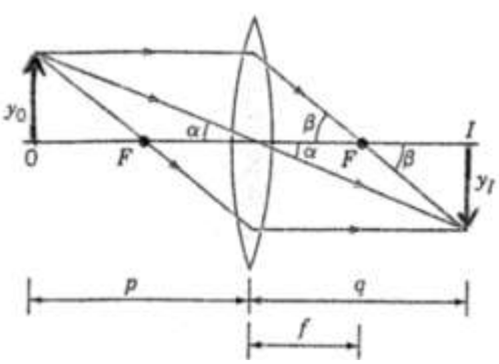


Fig.3a Convergent Lens

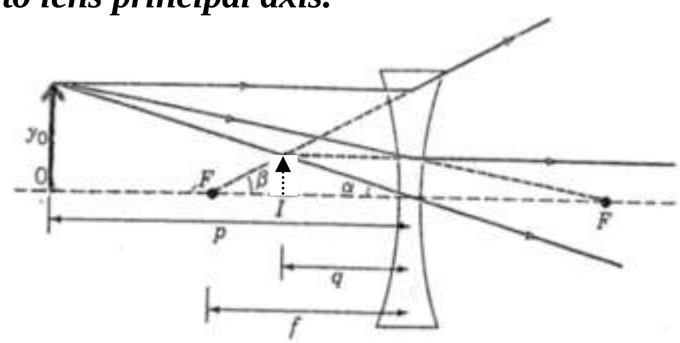


Fig. 3b Divergent Lens

[2] Different focal point location for rays passing close to lens rim
[3] A **paraxial** ray is a ray which makes a small angle (θ) to the main axis of the optical system, and lies close to the main axis throughout the system.

-How to find the image of an object "*at the output of a lens*" (or lens system)? In general, one should combine the ***graphical method*** and the ***analytical calculation***. Starting by a clear drawing allows to visualize the approximate location of images and avoid sign related errors in calculations; also, it informs about image orientation.
For a system of lenses, one applies this method several times. After the drawing/calculation for first lens, one follows by drawing the scheme of second lens *by paying special attention to the location of the image from first lens versus the second lens position*. If the image from first lens falls beyond second lens or inside its " left " focal distance, one may avoid mixing up of rays related to different lenses by shifting down the drawing of second lens.

One suggests to apply the following rules during the drawing of optical rays:

1. Locate the **object** always to **the left** side of the drawing.
2. Use the symbol $\updownarrow$ for ***convergent*** lenses and the symbol for the ***divergent*** lenses.
3. Use main rays (at least two) to locate the image; use **arrows** to show ray direction.
4. Draw in ***fullness real objects, real images and real rays***.
5. Draw in ***dashed line virtual objects, virtual images and virtual rays***.
6. For a system of lenses "the first lens image is the object for second lens and so on".

-Based on the geometrical rules and the paraxial rays approximation ($\tan\theta \approx \theta$) one gets (see textbook) the relations (5) for object and image positions and (6) *for linear (or lateral) magnification of image*

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \qquad (5) \qquad\qquad m=\frac{y_1}{y_0}=-\frac{q}{p} \qquad (6)$$

p, q are the distances of object and image from the lens; ***f*** is the focal distance of lens, y_1, y_0 are the image height and the object height.
One uses the ***same formulas*** for ***convergent*** and ***divergent*** lenses but has to respect rigorously the following ***rules for the sign*** of parameters p , q and f :

1- Light travels always from left to right (***arrows always directed right side***)

2- $p > 0$ for a ***real object*** (left of lens) and $p < 0$ for a ***virtual object*** (right of lens).

3- $q > 0$ for a ***real image*** (right of lens) and $q < 0$ for a ***virtual image*** (left of lens).

4- $f > 0$ for a ***converging*** lens and $f < 0$ for a ***diverging*** lens.

The "-"sign at (6) is included to inform on the ***image sense*** *versus object* orientation. One assumes that upside objects / images are "positive" and downside directed are "negative".

Note: Assume that a set of rays emitted by a point on object falls on a lens.
The image of this point is located at the cross point of corresponding rays that leave the lens.
If the paths of outgoing rays cross to each other, there is a real image.
If only the virtual extension of outgoing rays can cross to each other, there is a virtual image.

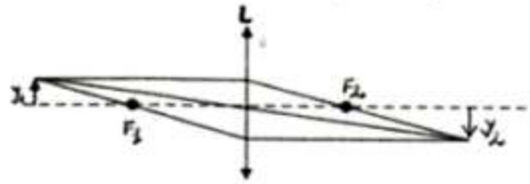
USING THE THREE PRINCIPAL RAYS TO FIND THE IMAGE BUILT BY AN OPTICAL LENS

Rule: Draw the real object on the left of the lens. Then:

- If the **object** is on the <u>**left**</u> of the lens it is a real object; **take $p > 0$**
- If the **object** is on the <u>**right**</u> of the lens it is a virtual object; **take $p < 0$**

I/ *Real object (on the left of lens).*

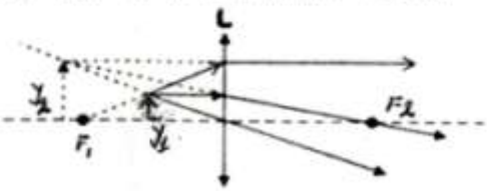
a) CONVERGENT LENS
Object ***outside focal distance***
Real image



Ex: $f = 20cm$; $p = 40cm$; $y_1 = 4cm$; $q = ?$; $y_2 = ?$ $\quad 1/40 + 1/q = 1/20 \quad 1/q = 1/20 - 1/40 = 1/40$

$q = 40cm$ **(real image)** $\quad y_2/y_1 = -q/p \quad y_2 = -(40/40)*(4) \quad y_2 = -4cm$**(down)**

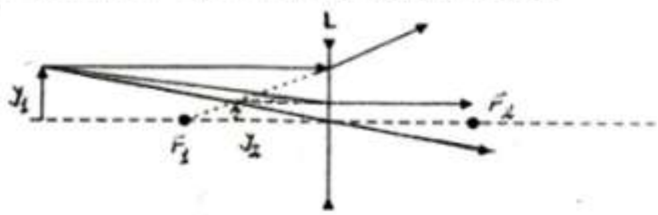
b) CONVERGENT LENS
Object ***inside focal distance***
Virtual image



Ex: $f = 20cm$; $p = 10cm$; $y_1 = 4cm$; $q = ?$; $y_2 = ?$ $\quad 1/10 + 1/q = 1/20 \quad 1/q = 1/20 - 1/10 = -1/20$

$q = -20cm$ **(virtual image)** $\quad y_2/y_1 = -q/p \quad y_2 = -(-20/10)*(4) \quad y_2 = 8cm$**(up)**

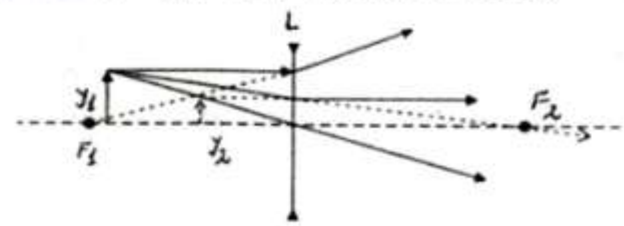
c) DIVERGENT LENS
Object outside focal distance
Virtual image



Ex: $f = -10cm$; $p = 20cm$; $y_1 = 4cm$; $q = ?$; $y_2 = ?$ $\quad 1/20 + 1/q = 1/-10 \quad 1/q = -1/20 - 1/10 = -3/20$

$q = -20/3 = -6.7cm$ **(virtual image)** $\quad y_2/y_1 = -q/p \quad y_2 = -(-6.7/20)*(4) \quad y_2 = 1.34cm$**(up)**

d) DIVERGENT LENS
Object inside focal distance
Virtual image

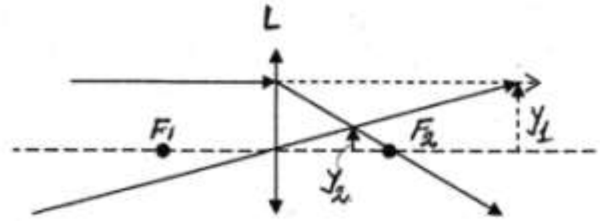


Ex: $f = -10cm$; $p = 5cm$; $y_1 = 4cm$; $q = ?$; $y_2 = ?$ $\quad 1/5 + 1/q = 1/-10 \quad 1/q = -1/5 - 1/10 = -3/10$

$q = -10/3 = -3.33cm$ **(virtual image)** $\quad y_2/y_1 = -q/p \quad y_2 = -(-3.33/5)*(4) \quad y_2 = 2.7cm$**(up)**

2] ***Virtual object (on the right of lens).*** *In an optical system containing two or more lenses, the image built by a preceding lens becomes the object for the next lens. If the location of this object is on the right side of the considered lens, this object is a virtual object and its* $p < 0$.

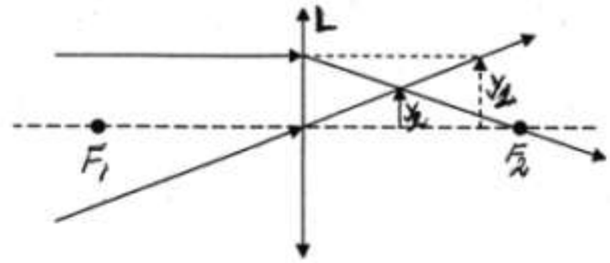
a) CONVERGENT LENS
Virtual object ***outside focal distance***
Real image



Ex: f=10cm; p = -16cm; y_1 = 4cm ; q =? ; y_2=? $-1/16 + 1/q_2 = 1/10$ $1/q_2 = 1/10 + 1/16 = 26/160$

$q_2 = 160/26 = 6.15$cm **(real image)** $y_2/y_1 = -q/p$ $y_2 = (-6.15/-16)*(4)$ $y_2 = 1.54$cm**(up)**

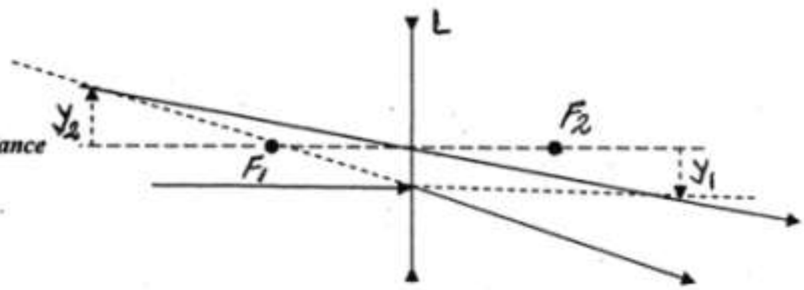
b) CONVERGENT LENS
Virtual object inside ***focal distance***
Real image



Ex: f=10cm; p = -8cm; y_1 = 4cm ; q =? ; y_2=? $-1/8 + 1/q_2 = 1/10$ $1/q_2 = 1/10 + 1/8 = 18/80$

$q_2 = 80/18 = 4.4$cm **(real image)** $y_2/y_1 = -q/p$ $y_2 = (-4.4/-8)*(4)$ $y_2 = 2.2$cm(up)

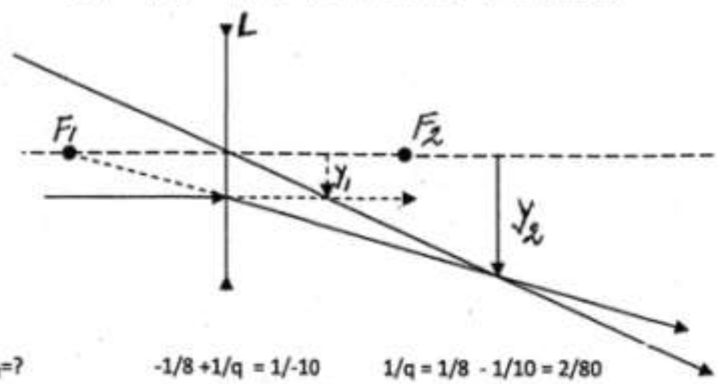
c) DIVERGENT LENS
Virtual object ***outside focal distance***
Virtual image



Ex: f=-10cm; p = -16cm; y_1 = -4cm ; q =? ; y_2=? $-1/16 + 1/q = 1/-10$ $1/q = 1/16 - 1/10 = -6/160$

$q = -160/6 = -26.7$cm **(virtual image)** $y_2/y_1 = -q/p$ $y_2 = -(-26.7/-16)*(-4)$ $y_2 = 6.7$cm**(up)**

d) DIVERGENT LENS
Virtual object inside ***focal distance***
Real image

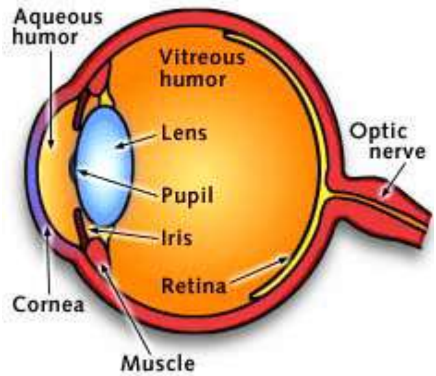


Ex: f=-10cm; p = -8cm; y_1 = -4cm ; q =? ; y_2=? $-1/8 + 1/q = 1/-10$ $1/q = 1/8 - 1/10 = 2/80$

$q = 40$cm **(real image)** $y_2/y_1 = -q/p$ $y_2 = -(40/-8)*(-4)$ $y_2 = -20$cm**(down)**

THE EYE and problems of vision

- The "optical parts" of the eye are: **cornea** (*produces an initial light refraction*), **crystalline** (*a controlled focus lens*) and **pupil** (*adaptable diameter diaphragm*). The eye operates as a ***converging optical system that can modify its focal distance*** in such a way that, for objects at different distances, it can produce a ***real image*** *onto* the "*Sensor part*": **retina** (a set of light-sensitive "*rods* and *cones*"). The optic nerve carries out image information to the brain.



- A normal eye produces sharp images of objects located far away (**p =** ∞) on the retina ***without accommodation***. The infinity distance is known as "***far point*** " of normal eye. Eye makes use of *accommodation* (**decrease of the focal distance of crystalline lens**) *to produce sharp images on retina for objects at closer location. A normal eye can produce sharp images for objects at a minimum distance* ***p = 25cm,*** *known as the "* ***near point*** *". A standard normal eye cannot see sharp images for objects closer than 25cm.*

EYE PROBLEMS

a) The ***focal point*** of **non-accommodated eye** is ***in front of retina***. The person cannot ***see*** a sharp image for an ***object far away*** because its image falls at focal plane, i.e. *in front of retina* (only a *blurry image* is produced on *retina)*. This problem is known *as* ***eye Myopia*** and often the person is called ***nearsighted.*** Note that eye *accommodation* (*activated by the eye muscle*) *can only decrease the focal distance* of eye; so, the eye accommodation cannot correct myopia. One uses ***diverging*** lenses ***to correct Myopia***.

b) The person cannot ***see*** clearly an ***object because its image*** **falls** ***behind the retina*** *even with maximum eye accommodation.* If one has this problem **only for objects nearby** but can see clearly the objects far away one says that the person has **farsighted eyes**.

b-1) **Presbyopia.** Even for *maximal available accommodation* the sharp image of ***nearby objects*** falls beyond the retina. Due to the ***blurry image*** formed on retina one cannot see sharp images for ***nearby objects***. This is a common age related problem but it may be a born problem, too. One uses ***converging*** lenses ***to correct Presbyopia***.

b-2) ***Hyperopia.*** The ***focal point of eye falls beyond the retina*** for *non accommodated* eye. The person cannot observe *without accommodation* even ***distant objects.*** In many cases, he (she) does not feel a problem because the eye accommodation makes the necessary correction to bring the focus on retina. As the *accommodation power* of eyes decreases with age, one needs to use ***converging lenses*** even to see objects far away.

- One measures the *correction effect* of an eyeglass by its ***lens power*** $P[D] = \frac{1}{f[m]}$ (7)
where " D " stand for *dioptre*. A lens with focal distance f = 0.5m has +2 D lens power.

THE SIMPLE MAGNIFIER

- A normal eye may see without effort an object far away but it misses its details. One can see only the shape of an object (ex. Moon) if its image falls on a few sensible elements of retina or even only one point (ex. a star) when *the image* falls on a *single sensible element* of retina. For the same reason (*image falling on a few sensor elements of retina*) one can miss the details of a small object even if it is not located that far from the eye(ex. a sand particle).

-An object ***appears small*** if the eye forms a ***small size*** image on the retina. The ***apparent size*** of an object over retina depends on the angle that it subtends to the eye (see fig.4a-up). One can increase this angle by approaching (*if possible*) the object (*or the eye*) until p = 25cm. The ***angle***[r] that corresponds to the object size at distance 25cm from eye is known as "α_{25} ".

$$\alpha_{25} = \frac{y_0[m]}{0.25} = \frac{y_o[cm]}{25cm} \qquad y_0 \text{ is the object size (or height)} \qquad (8)$$

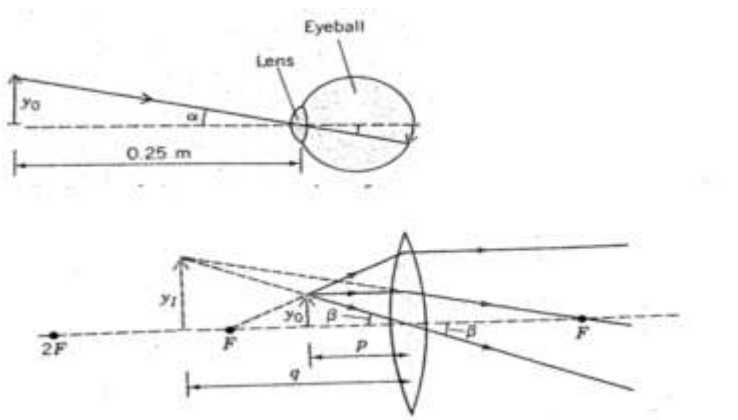


Fig.4a Object closer than ***f*** to a converging lens

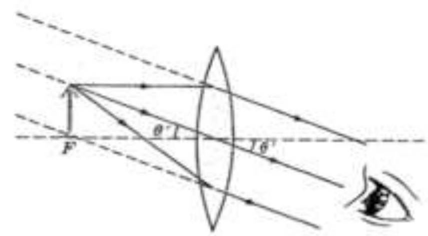


Fig.4b Larger angle to eye

-In many cases one can see "*just a point image even at 25cm distance*". One may increase the subtended angle by bringing object closer than 25 cm but the image falls behind retina and becomes blurry. In those situations, one can use *a simple magnifier* (*a convergent lens with focal distance f < 25 cm; generally 2-10cm*), place the object inside lens focal distance and get a magnified *virtual* image "***y_1***" at a bigger distance (see fig.4a-down). Now, the eye accommodation produces a sharp final image on retina at the corresponding angle (***β** at fig.4a; θ' at 4b*) which is larger than α_{25}. From fig.4a one can see that $\beta = \frac{y_0}{p}$ and this angle is equal to that subtended on retina (*at 4.b,* θ' = β) as the eye is behind the magnifier. .

So, the simple magnifier gives an *angular magnification* $M = \frac{\beta}{\alpha_{0.25}} = \frac{y_0/p}{y_0/0.25} = \frac{0.25}{p}$ (9)

-By placing the object at the focal point F of lens (see fig4.b), one gets a *virtual object at infinity (i.e. no need for eye accommodation)* and the ***angular magnification*** *becomes*

$$M_\infty = \frac{0.25}{f} \qquad (10)$$

THE MICROSCOPE

- A microscope achieves ***angular magnifications*** of order 1500-2000 times which is sufficient for majority of applications in biology, geology, metallographic studies, etc.

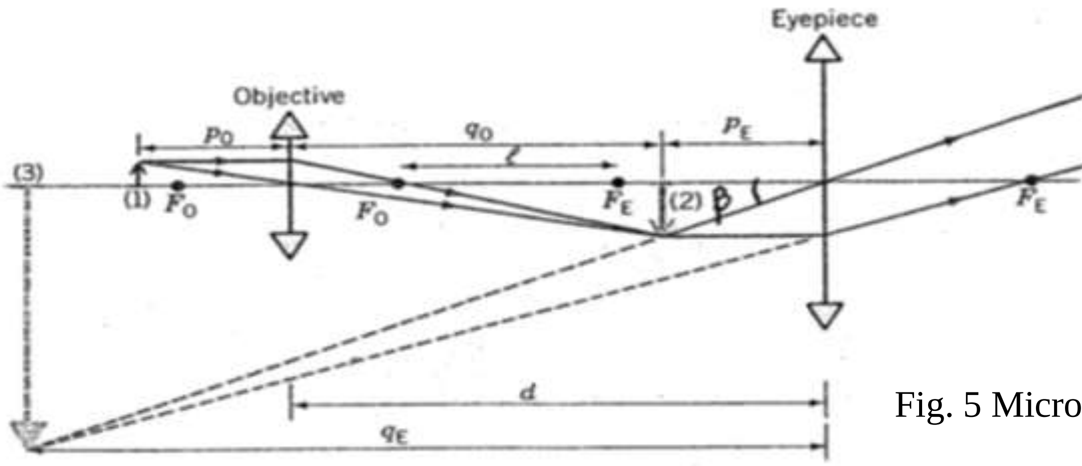


Fig. 5 Microscope scheme

-A microscope is an optical system constituted by ***two convergent lenses***;

a) the ***objective*** (***$f_{objective}$ ≈ 5mm***) which is used to produce an enlarged **real image** of object *inside the focal distance of the eyepiece* and

b) the **eyepiece** (***$f_{eyepiece}$ ≈ 15mm***) which operates as a simple magnifier.

If "***l***" is the distance between the closest focal points of the two lenses, the distance between the objective and eyepiece is (see fig.5) $\boldsymbol{d = l + f_{objective} + f_{eyepiece}}$

- The output angle is $\beta = \frac{y_2}{p_E}$. As seen in fig.5, $\frac{y_2}{q_0} = \frac{y_1}{p_0}$ and $\beta = \frac{y_2}{p_E} = \frac{y_1 * q_0}{p_E * p_0}$ (11)

Therefore, the angular magnification of microscope is $M = \frac{\beta}{\alpha_{0.25}} = \frac{\frac{y_1 * q_0}{p_E * p_0}}{\frac{y_1}{0.25}}$

$$M = \frac{0.25 * q_0}{p_E * p_0} \tag{12}$$

-The optical system is optimised for view by a relaxed eye (image "3" at infinity) which happens when the image " y_2 " from objective is placed on the focal point of eyepiece.

In this case $p_E = f_E$, $q_O = f_O + l$ and by using the lens formula $\frac{1}{p_O} + \frac{1}{q_O} = \frac{1}{f_O}$ one get

$$\frac{1}{p_O} = \frac{1}{f_O} - \frac{1}{q_O} = \frac{q_O - f_O}{f_O q_O} = \frac{l}{f_O q_O} \tag{13}$$

By replacing $1/p_o$ at expression (12) one gets $M_\infty = \frac{0.25 q_O}{f_E} * \frac{l}{f_O q_O} = \frac{0.25 * l}{f_O * f_E}$ (14)

THE TELESCOPE

-In case of a microscope the object is very close to the objective lens. For a telescope, the object is always at big distance but the principle of function is the same; the objective (*first lens*) produces a *real image inside the focal distance of the* eyepiece (*second lens*), which builds a virtual image and *increases* the angle subtended at eyepiece output from "α to ***β*".**

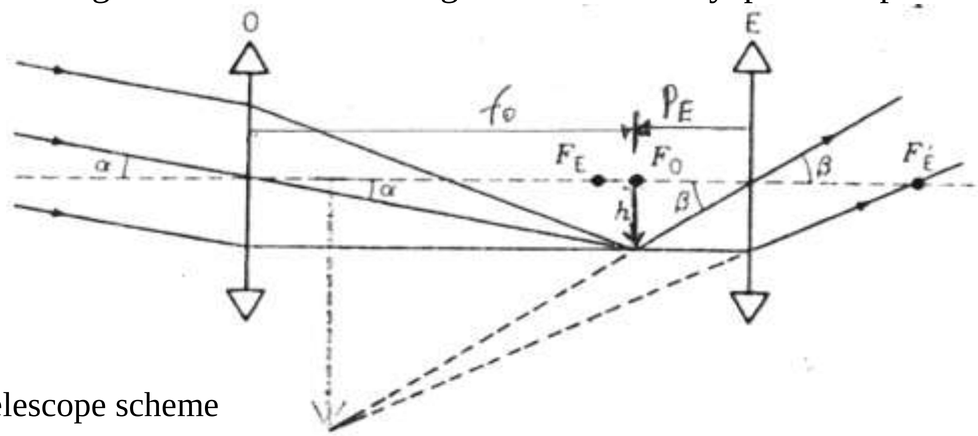


Fig. 6 A telescope scheme

- One object (*not shown in figure*) at infinity sends a parallel beam and subtends a ***very small angle* α** (*it is not* α_{25}) on the free eye. This beam falls by the same angle α on the objective lens (fig. 6), which produces a small real image at its focal plan (F_o). One may calculate the angle "***α***" by referring to the height of image "***h*"** and the focal distance " f_O " of objective;

$$\tan\alpha = \frac{h}{f_O} \_as\_ \tan\alpha \approx \alpha \_ \rightarrow \alpha = \frac{h}{f_O} \tag{15}$$

- The angle "***β*** ", subtended by the virtual image (*built by the eyepiece*) at the telescope output (and the eye of observer) is calculated as:

$$\tan\beta = \frac{h}{p_E} \_as\_ \tan\beta \approx \beta \_ \rightarrow \beta = \frac{h}{p_E} \tag{16}$$

- Then, the *angular magnification* of telescope is:

$$M = \frac{\beta}{\alpha} = \frac{h/p_E}{h/f_O} = \frac{f_O}{p_E} \tag{17}$$

In a telescope the focus of eyepiece is placed at same location as the focal point of the objective. So, the image built by the objective at its focus plane is placed on the eyepiece focus plane, too; i.e. $\boldsymbol{p_E = f_E}$. Consequently, the output beam "*appears as a parallel beam coming from an image at infinity*" for the observer and he has no need for accommodation. Then, the <u>*angular magnification*</u> of the telescope becomes

$$M_\infty = \frac{f_O}{f_E} \tag{18}$$

- To get large magnifications one has to use large values of "$\boldsymbol{f_O}$" and this requires the large length of telescopes(tens of meters); the " minimum ***length*** of device " is $\boldsymbol{f_O + f_E \sim f_O}$.