Remember:

-When a wave front falls over an aperture, there is always a "cut " of the wave front and this produces light ***diffraction*** at the output of aperture. Huygens principle explains the diffraction by introducing the set of *secondary sources* that emit *light wavelets*.

- When calculating the effects of slit diffraction on a screen beyond the slit, one refers to the principles of interference. This sentence shows that diffraction and interference are closely related.
In general, one prefers to label as ***diffraction pattern*** the pattern that correspond to a single aperture on a screen and ***interference pattern*** the pattern that correspond to a set of apertures on a screen.

OPTICAL SPECTROSCOPY

-The dispersion of sunlight into a spectrum of different colors by using a glass prism (fig.1) brought to the idea that one may gather information about the source of light by analyzing the light that it emits. This observation was the first step versus the development of optical spectroscopy.

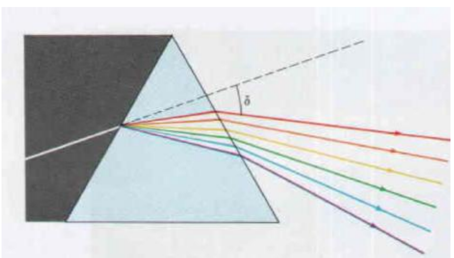


Fig.1 White light dispersion by a prism

-In the following chapters, we will see that, when a source of light contains only one type of atoms (or molecules), it emits a light which contains a *specific finite set of wavelengths* (λ_1, λ_2, ...λ_f) and this set is a kind of "*signature for this type of atom or molecule"*. So, one may identify the elements inside the source of light (i.e. get the constituency of *a sample*) by analyzing the set of wavelengths in *spectra* recorded by a light spectrometer.

-For a long time, all optical spectroscopes were based on dispersion features of prisms. These devices have moderate *spectral resolution* (1-2nm in visible spectrum, i.e. do not allow to distinguish wavelengths "*closer than*" 1nm). When studying the interference from a system of multiple slits (i.e. *optical grating*), one figured out that this system could help to improve the spectral resolution. Nowadays, the optical grating is the ***dispersion tool*** for the majority of light spectrometers.

- The capacity to distinguish two close wavelengths λ_1, λ_2 is a main characteristic for a spectrometer. The *spectral resolution* is $\Delta\lambda_{min} = \lambda_2 - \lambda_1$ between two wavelengths that can be distinguished by a spectrometer; the *spectral resolving* **power** is inverse proportional to *spectral resolution* $R \sim 1/\Delta\lambda_{min}$. One has defined for the **spectral resolving power** of a spectrometer as

$$\boldsymbol{R = \lambda\_av/\Delta\lambda_{min}} \tag{1}$$

where $\Delta\lambda_{min} = \lambda_2 - \lambda_1$ *and* $\lambda_{av} = (\lambda_2 + \lambda_1)/2$

One may get an idea about the advantage of gratings in optical spectroscopy, by referring to a comparison in the middle range of visible spectrum where the resolving power of a common prism spectrometer is R= 550nm/2nm=275. At the same region, the spectral resolving power of a grating spectrometer (as we will see in next section) is easily larger than 1000. This means that for R=1000, the grating offers a *spectral resolution* $\Delta\lambda_{min}$ = 550/1000 = 0.55nm or ~4 times smaller than with a prism.

GRATINGS

-A conventional optical grating contains parallel lines grooved onto a reflecting or transparent surface. The gap between scratched grooves (known as ***rulings*** or ***lines*)** acts as a *single diffracting slit*. In case of a transparent grating (fig.2a), the output waves travel on the other side of grating. In the case of a reflection grating (fig.2b), the incident and output waves propagate on the same side of grating. The distance between the centers of two adjacent slits ("***d*** " parameter) is labeled as ***grating spacing***.

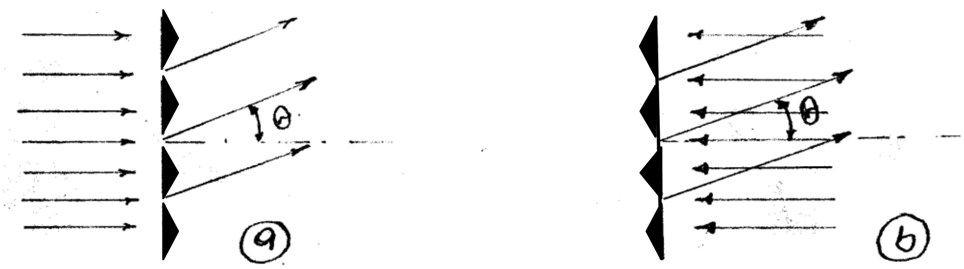


Fig. 2

-An optical grating operates as a dense set of multiple parallel slits at distance "***d*** " from each other. The slits width ***a*** < ***d*** and the corresponding central maximum of diffraction is very large. Also, one places the screen " *far enough* " from grating so that one can use the model of "*plane wave fronts* ".

*Example. One grating has **500 rulings/mm**. Find the maximum distance "D" where should be placed a screen with width 30cm (2R=30cm in figure 3) so that it can be entirely illuminated by the central diffraction maximum of a single slit pattern. Consider a light wave around the middle of visible spectrum (ex. take λ = 500nm).*

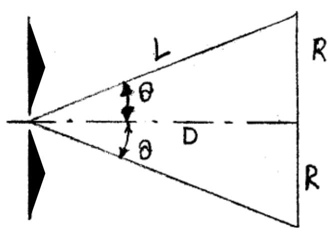


Figure 3

*Each slit produces a diffraction pattern on the screen with light concentrated inside the central maximum. We want the extension of this maximum to cover the screen width. The distance " D " is such that the **first order minimum of single slit diffraction falls on screen border**. The first minimum direction is defined*

by relation $a sin\theta_{s=1} = \lambda \longrightarrow sin\theta_{s=1} = {}^{\lambda}/_{a}$ *(*)*
*where "**a**" is the "slit width".*

*From our data, d =1mm/500 = 2*10^{-3} mm= 2*10^{-6} m = 2*10^{3} nm.*
*With a reasonable conservative assumption, let's assume **a = d/2 =1000nm**.*
From the figure 3, one may see that R=30/2=15cm and it comes out that

$$sin\theta_1 = \frac{R}{L} = \frac{\lambda}{a} = \frac{500}{1000} = 0.5 \quad and \quad sin^2\theta_1 = \frac{R^2}{L^2} = \frac{R^2}{D^2+R^2} = 0.5^2 = 0.25 \rightarrow R^2 = 0.25(R^2 + D^2)$$

$$\text{and } (1-0.25)R^2 = 0.25D^2 \rightarrow 0.75 * R^2 = 0.25D^2 \longrightarrow D = \sqrt{\frac{0.75}{0.25}} * R = 0.1732 * 15 \cong 26cm$$

- Assume that a grating (*i.e.* ***many slits per mm***) is placed parallel to a screen at a distance where the central maximum of diffraction of "all single slits" covers the screen width. (*Note that, due to the shift of centers of different slits, the width of common central maximum of diffraction, is a bit larger than that due to one single slit)*. Now, consider that a **monochromatic plane** wave falls perpendicularly on the grating plane. Each single slit produces a wave that would illuminate "*almost uniformly*" the area of screen. Also, the ***waves from different slits*** superpose on the screen. They ***are all coherent*** waves with ***same wavelength.*** In these circumstances, an ***interference pattern*** will be produced on the screen. One may find out the position of interference fringes on the screen by calculating the **path length difference** and the corresponding **phase shift** between ***waves emitted from different slits*** that superpose along the same direction and consequently fall on the same location on the screen.

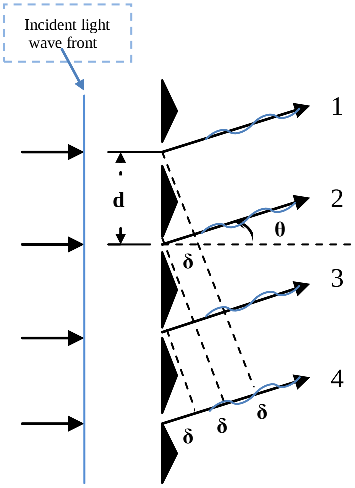


Figure 4

- Let's consider at first a system of ***four slits*** *equally spaced* (distance "**d**") and a monochromatic (***one*** λ) plane wave falling on it. Assume that if angle shown in fig. 4 has a value $\boldsymbol{\theta = \theta_1}$ there is a ***path length difference*** $\boldsymbol{\delta_{1\text{-}2} = \lambda}$ between the wavelets "1-2". This means that for $\theta=\theta_1$ $\qquad \delta_{1-2} = d\sin\theta_1 = \lambda \qquad$ (1)

As the corresponding phase difference is $\qquad \Delta\phi_{1-2} = \frac{2\pi}{\lambda} * \lambda = 2\pi \qquad$ (2)

it comes out that the waves from consecutive slits 1, 2 produce a *maximum of interference of order* **M = 1** along this direction. Along this same direction θ_1, any two consecutive waves (1-2, 2-3, 3-4) produce the phase difference 2π. So, the set of four slits will place a maximum of interference of order **M=1** along the direction θ_1. If along direction $\theta=\theta_2$ would correspond a path length difference $\boldsymbol{\delta_{1\text{-}2} = 2\lambda}$, then ***dsin*** $\boldsymbol{\theta_2 = 2\lambda}$ and along this direction $\Delta\phi_{1-2} = \frac{2\pi}{\lambda} * 2\lambda = 2 * 2\pi$

which means that a *maximum of order* **M = 2** would be placed along direction "θ_2". One may figure out that the same phase difference ($2*2\pi$) is produced by *<u>any two consecutive waves</u>* (1-2, 2-3, 3-4) along this direction "θ_2". So, it comes out that the set of four slits would produce a maximum of interference of order **M = 2** along the direction θ_2. This logic shows that the system of four slits produces maxima of interference along the same directions that correspond to the interference ***maxima*** for ***a two slit*** , i.e.

$$d \sin\theta_M = M\lambda \qquad M = 0, \pm 1, \pm 2.. \qquad (3)$$

and one calls them **PRINCIPAL MAXIMA**.

-What happens along the directions between any two consecutive principal maxima? For simplicity, let's consider maxima of order M = 0 and M = 1. Here we have to note that, for M = 1, the expression (3) gives a path length difference $\delta = d\sin\theta_1 = \lambda$ between any two ***consecutive waves*** (say 1-2). In the meantime, ***<u>along the same direction</u>*** $\boldsymbol{\theta_1}$, there is a path length difference $3\delta = \boldsymbol{3\lambda}$ between waves ***1- 4***. This gives a corresponding ***phase shift*** $\boldsymbol{\Delta\varphi_{1\text{-}4} = (2\pi/\lambda) * 3\lambda = 3*2\pi}$. So**,** the superposition of waves 1- 4 does produce a maximum along direction θ_1, but this maximum is of **order** $\boldsymbol{M'}$**=3**. This means that, due to interference between waves 1- 4 , in the region between the **central maximum** (*direction* $\theta_o = 0^0$) and the **principal maximum of first order** (*direction* θ_1) should be placed ***two additional maxima*** (*of orders M'=1,2*) and consequently **three related minima .** This means that ***the set of 4 slits*** produces ***4 -1 = 3 minima*** between any two principal maxima of interference.

For a set of eight slits, between the **central maximum** and the **principal maximum of order M = 1,** due to interference between waves 1- 8, would appear ***seven minima (8-1 = 7)***. By extending the same logic on any two consecutive principal maxima of interference, it comes out that, for a set of ***eight slits***, there are **seven minima** between ***any two principal maxima*** of interference.

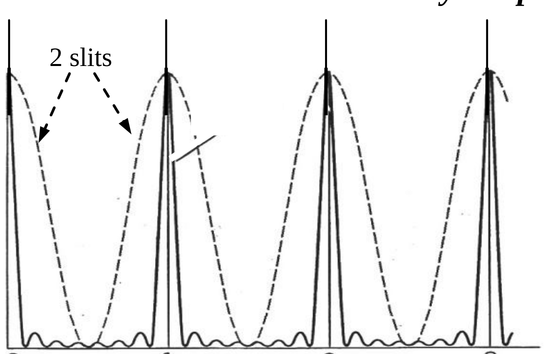


Figure 5 Interference pattern for 8 slits

As shown in figure 5, the presence of *seven secondary minima*, changes the distribution of light between any two principal maxima. It gives rise to ***narrower and more intense principal maxima*** and leaves just a bit of light between them.

-A grating contains ***thousands of slits***. This means ***thousands of secondary minima*** between any two principal maxima. Consequently, a grating produces ***extremely narrow principal maxima*** and no light between them.

The narrow principal maxima are the key feature at the origin of high values of ***spectral resolving power*** (**R**) for gratings.

SOME BASIC RESTRICTIONS FOR USE OF GRATING IN SPECTROSCOPY

-A detailed calculation (see Halliday&Riesnick 9th edition p.1009) shows that the ***spectral resolving power*** of a grating can be expressed as

$$R = \lambda_{Av} / \Delta\lambda_{\min} = N_w * M \qquad (4)$$

λ_{Av} is the average wavelength of two spectral lines that can be barely resolved;
$\Delta\lambda_{min}$ is the ***spectral resolution*** (*minimum wavelength difference* between two lines that can be barely resolved*)*;
N_w is the **total number** of grating **rulings illuminated by the incident beam(*i.e. operating slits*)**;
$N_w = N * w$ if ***N*** (*ruling/mm*) and the incident beam covers a grooved area with width ***w** (mm)*
M is the ***order*** of interference of the maximum used to observe the spectral lines.

Based on this expression, one would attempt to increase the spectral resolving power by working with high orders of interference "M". But, there are several restrictions:

a) The angle of diffraction "θ_M" increases with order "M" of interference but, it cannot go over 90°.
From expression of principal maxima one can get $d\sin\theta_M = M\lambda \longrightarrow M = \frac{d}{\lambda}\sin\theta_M$ (5)
As ***sin90°*** =1, the theoretical maximum possible value for M is $\boldsymbol{M_{max} = d / \lambda}$ (6)

b) The limited *coherence length Lc* of wave trains restricts their superposition for big angles.

c) Like in the case of two slit's pattern, the envelope of central maximum of diffraction (*the result of all single slit central maxima of diffractions)* diminish the intensity of higher orders of principal maxima of interference that fall close to limiting directions of first order minimum of single slit diffraction.

d) When recording *a broad range of wavelengths*, often happens an overlap of different order spectra and the experience shows that this problem is common for interference orders of M = 3, 4,...
If there is an *overlap of different orders* for different λ values at the same position on the screen, one cannot get out any information. For this reason, in general, one works with orders *M =1, 2*.

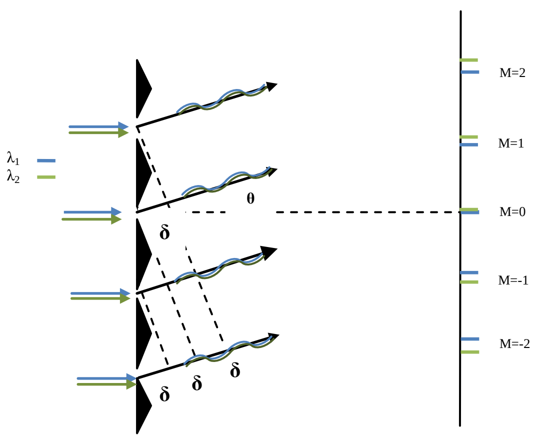


Figure 6 Resolution of two wavelengths in incident beam through different orders of grating.

In a general purpose use of a grating, N ~500grooves/mm and the diameter of light beam is larger than 2mm. So, the number of *working slits* is easily N_w > 1000**.** This means that one can get easily a resolving power R = 2000 by using the spectra at the second order (M=2) of interference. The high " R-values " are the main advantage of a grating spectrometer versus the prism spectrometers. However, the gratings produce several principal maxima (M = 0,±1,±2,±3..) and distribute the energy of the incoming light with the same λ - value through all the principal maxima. Therefore, when analyzing the maxima of different wavelengths in a light beam at a certain order, one has to work with just a portion (~ 1/5 if M_{max}=2) of the available intensity at incident light.
This is a *weak point* for the grating spectrometers.

As an alternative, a spectrograph with prism places all the energy of incident beam of light with same λ value along one direction defined for that wavelength by the prism dispersion. So, if the intensity of light at the input is low and, for the purpose of the study, a spectral solution of order several hundred is good enough, one might prefer to use a prism spectrograph instead of a grating spectrograph.

THIS ADDITIONNAL MATERIAL ON PHASOR MODELLING IS VERY USEFUL FOR THE STUDENTS THAT PLAN TO FOLLOW THEIR EDUCATION IN PHYSICS OR ENGINEERING BUT IT IS NOT A PART OF THE COLLEGE COURSE

THE PHASOR

-We have introduced the trigonometric model for SHO study and we have used it:

a) for the derivation of wave function in SHO;

b) to find the relationship between the phase shift and path difference $\phi = \frac{2\pi}{\lambda}\delta$;

We have used this model to find the interference rules for two coherent waves. Also, we used trigonometric calculations for fringes' intensity in Young's experiment and in these calculations we dealt also we two waves. One may think to use the same method (trigonometric calculations) in case of multiple waves but this is not practical. The calculations become cumbersome even for three-wave interference. It is clear that one has to use another method to calculate the patterns' intensity in case of grating where a big numbers of waves interfere.

-The concept of PHASOR is an extension of our initial trigonometric model and offers full flexibility in the studies of interference from multiple waves.

a) It is a vector. As a consequence all vector operations are valid for phasors. It represents a physical quantity that oscillates sinusoidal in time. In case of light, the physical quantity is the electric field of light wave; $E = E_0 \sin(\omega t)$

b) Its magnitude is equal to wave amplitude (E_0 for light waves).

c) At a given moment 't' it forms the angle $\varphi(t)$ – "phase" with a reference axis "Ox axis" and rotates with constant circular frequency ω.

d) The projection of phasor on to "the vertical axis Oy" represents the variation of physical quantity in time.

e) If several oscillations of the same nature superpose, one may study the result behaviour by use of the sum of respective phasors.

PHASORS IN MULTIPLE SLITS INTERFERENCE

-In the following we perform calculations for a given λ and assume:

a) Actual field vectors lie along the same direction in space.

b) Very narrow slits which, when alone, produce uniform illumination on the screen.

c) Equal separation 'd' between each two adjacent slits.

d) The screen is located in the far-field region. This means that outgoing rays are almost parallel and there is a constant path difference between adjacent slits $\delta = d\sin\theta$.

TWO SLITS SYSTEM

Two phasors with equal magnitude (E_0) rotate all time in phase (same ω).

a) For phase shift φ = 0, 2π,..

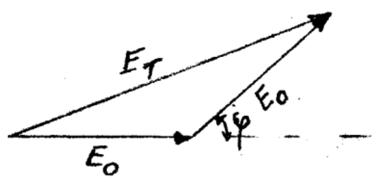
The sum phasor has the magnitude $2E_0$. The corresponding intensity is $4E_0^2 = 4I_0$.

b) For phase shift φ = π, 3π,5 π..

The sum phasor has the magnitude 0. The corresponding intensity is 0.

So, the intensity values vary between the maximum value $4I_0$ and the minimum value 0.

c) For other values of phase difference (see fig.6)



From the theorem of cosines one finds out that

$$\begin{aligned} E_T^2 &= E_0^2 + E_0^2 - 2E_0E_0\cos(\pi-\phi) = \\ &= 2E_0^2 + 2E_0^2\cos\phi = 2E_0^2(1+\cos\phi) = \\ &= 4E_0^2\cos^2(\phi/2) \end{aligned} \tag{12}$$

Fig.6

THREE SLITS SYSTEM

Three phasors with equal magnitude and same phase difference φ. The figure 7 presents the resultant phasor magnitude for different φ- values between [0,2π]. That is between the central and first principle maxima.

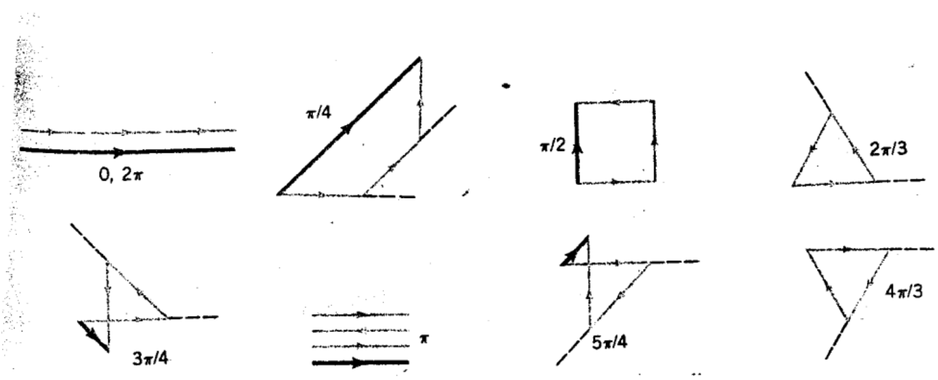


Figure 7

One may get informed about the resultant intensity for different values of phase φ by following the evolution of resultants phasor in figure 7. It is easily seen that it :

a) Is maximum $I_{max} = 9I_0$ for φ = 0 which corresponds to central maxima.
b) Decreases while φ increases to 45^0(π/4) and 90^0 (π/2).
c) Become 0 for φ = 120^0 (2π/3)......... [3*φ = 1*2 π].
d) Increases for φ = 135^0 (3π/4).
e) Get a local (secondary) maximum for φ = 180^0 (π).

f) Decreases for $\varphi = 225^0$ $(5\pi/4)$.
g) Become 0 for $\varphi = 240^0$ $(2*2\pi/3)$……… $[3*\varphi = 2*2\,\pi]$.
h) Increases to $I_{max} = 9I_0$ for $\varphi = 2\pi$ which correspond to first principal maxima.

From this scheme we can get the following basic features of three slits interference:

Principal maxima for $\varphi = m*2\pi$; $m = 0, \pm 1, \pm 2, \ldots$
Minima $\varphi = p*2\pi/3$; $p = \pm 1, \pm 4, \pm 5, \ldots\ldots$
$p \neq 0, \pm 3, \pm 6, \ldots\ldots$
Secondary maxima for $\varphi = (2k+1)\pi$; $k = 0, \pm 1, \pm 2, \ldots$

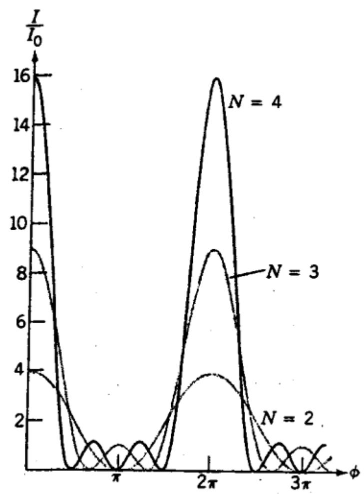


Fig. 8

These features appear in the calculated profiles that are drawn in figure 8. One may calculate these spectra by using N = 2, 3, 4 in the general formulae we will get at the next section.

__THE CALCULATION OF PATTERN INTENSITY FOR A SYSTEM OF N SLITS__

-The phase shift between two adjacent slits in a system of N equally spaced narrow slits with separation 'd' is

$$\delta = d\sin\vartheta \tag{13}$$

Figure 9 presents a phasor diagram for an N slits system. Note that one can get quickly the information about the basic features of spectrum.

a) There is a principal maximum for each angle

$$\phi = m*2\pi;\_m = 0, \pm 1, \pm 2, .. \tag{14}$$

b) There is a minimum for such angles that

$N\phi = p*2\pi;\_p = 0, \pm 1, \pm 2, ..$ or

$$\phi = p*\frac{2\pi}{N};\_p = 0, \pm 1, \pm 2, .. \tag{15}$$

$$\_\_\_\_\_\_\_\_\_\_\_ p \neq N, 2N, 3N..$$

c) The first minimum besides the central maximum (important for resolution issues) occurs for

$$N\phi = 2\pi; \rightarrow \phi = \frac{2\pi}{N}\_and\_\frac{2\pi}{\lambda}d\sin\theta = \frac{2\pi}{N}$$

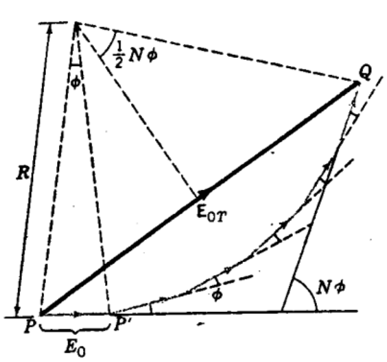


Fig. 9

which defines the angle as $\sin\theta^1_{min} = \dfrac{\lambda}{N*d}$ (16)

-From expression (14) we see that principal maxima positions do not depend on the number of slits. Their location is the same as that of a two slits system. Meanwhile, the expression (16) shows that they are much narrower in the case of big number of slits.

- Based on the fig. 9 we can find easily the expression for intensity as function of phase difference φ. From this drowing one may see that:

$$\sin \phi/2 = \frac{E_0}{2R} \rightarrow E_0 = 2R \sin \phi/2 \tag{17}$$

$$\sin N\phi/2 = \frac{E_{0T}}{2R} \Rightarrow E_{0T} = 2R \sin N\phi/2 \tag{18}$$

Then $$\frac{E_{0T}}{E_0} = \frac{\sin(N\phi/2)}{\sin(\phi/2)} \rightarrow\rightarrow\rightarrow \frac{I}{I_0} = \left(\frac{E_{0T}}{E_0}\right)^2 = \frac{\sin^2(N\phi/2)}{\sin^2(\phi/2)} \tag{19}$$

The expression (19) is very general. It gives the intensity value for each angle and for each number of slits. For example, for two slits we have

$$I = I_0 \frac{\sin^2(2*\phi/2)}{\sin^2(\phi/2)} = I_0 \frac{\sin^2(\phi)}{\sin^2(\phi/2)} = I_0 \frac{[2\sin(\phi/2)\cos(\phi/2)]^2}{\sin^2(\phi/2)} = 4I_0 \cos^2(\phi/2)$$

So, we find the known results for pattern intensity in two slits interference

$$I = 4I_0 \cos^2(\phi/2) \tag{20}$$

N-SOURCES INTERFERENCE IN OTHER REGIONS OF E.M. SPECTRUM

- Note that light waves are electromagnetic waves and we did not do any special restriction on the wavelength of interfering waves. So, the principles we used and the derived conclusions might apply in other regions of E.M. spectrum, too.

- The model of N equally distant source played a decisive role in the discovery of physical nature of x-rays. At the beginning of the last century, the scientists knew that a space dimension ~ 0.1nm was characteristic for these rays but didn't know if they were dealing with a particle or a wave. It was the diffraction of x-rays from the atom arrays in a crystal that proved their wave nature at 1922. As the distance between the atoms is of the same order as x-rays, a system of identical equidistant atoms acts as a system of identical equidistant slits. Nowadays, the x-ray diffraction is the main technique that infers information about the arrangement of atoms in crystal structures.

- The principles of interference from equally distant identical sources are applied to amplify the low intensity signals. This is the case of a system of similar telescopes that receive small signal from one direction in space. All signal are superposed at a center. A special electronic treatment builds the precise phase shift that would correspond to a principal maximum of interference for the λ used by the telescopes.