WAVE OPTICS

GENERAL

- The model of "*straight rays*" for light does not fit to results of the two following experimental situations:

a) If there is **small** *openings* or **small** *obstacles* on its path, the light reaches inaccessible regions for optical ray model. It seems that the light " ***bends around borders*** " ... this effect is known as DIFFRACTION;

b) If two or more light beams superpose in "*some special conditions"*, there are fringes of ***maxima / minima*** brightness (i.e. *light intensity*) produced on a screen…. this effect is known as INTERFERENCE.

Diffraction and ***interference*** are ***typical wave phenomena***. One can explain them by "*same tools*" used at the ***wave model*** of 1D mechanical waves. So, we will start by an introduction of 2D model applied for diffraction and interference of wave phenomena. Next, we will apply it for mechanical, sound and ***light waves.***

- Theoretical research and many experiments confirm that ***light waves*** are ***electro-magnetic (E&M) waves.*** Radio & TV waves, x & γ rays are part of **E.M**. spectrum, too *(see figures 1.a-b*). They are **TW waves**, i.e. their "*displacement*" is a vector perpendicular to the direction of their propagation. Actually, ***E&M*** waves are characterized by two vectors $\vec{E}$ (electric) and $\vec{B}$ (magnetic), both perpendicular to direction of wave propagation.

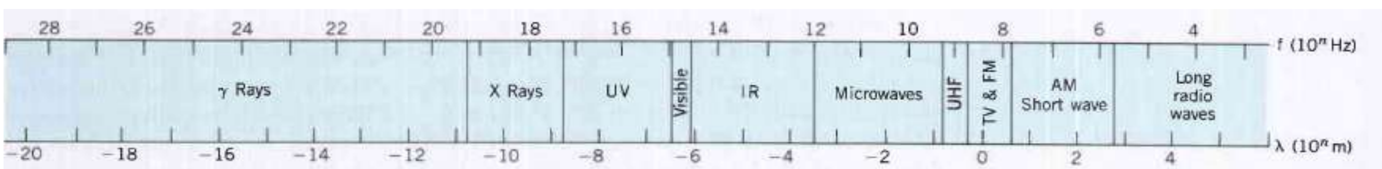


Fig.1a The electromagnetic spectrum

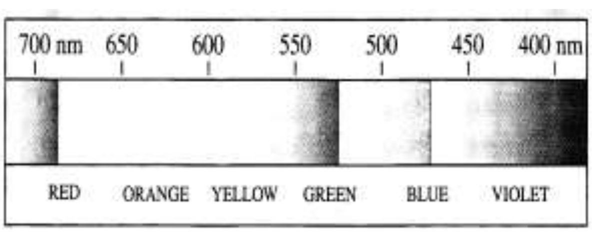
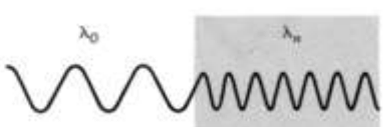


Fig.1.b Visible Light (colour vs. wavelength)



- The speed of all ***E.M.*** waves (light included) in vacuum is **c ≈ 3*10^{8}m/s**. When light travels through a transparent material medium, its speed *decreases* to $\upsilon = c / n$ (1)

where ***n*** is the **refractive index** of the medium (***n >1***).

If the wavelength of light in vacuum is $\lambda_0 = T * c$, when propagating in a medium its wavelength becomes $\lambda_n = T * \upsilon$.

The period ***T*** and the frequency (***f =1/ T***) of light wave remain constant; the wavelength decreases $\lambda_n/\lambda_0 = v/c = 1/n < 1$

Remember that : $\lambda_n < \lambda_0$ **and** $\lambda_n = \lambda_0/n$ (2)

Fig.1.c ***The light frequency remains the same but its wavelength decreases inside the matter.***

DIFFRACTION

- In geometrical optics, the ***rays*** (*i.e. directed straight lines*) show the path followed by *"light particles or energy droplets of light"* i.e. the *direction* ***of light energy*** *flow*. In a ***wave model***, the *energy propagates* on directions ***perpendicular to the wave fronts***. So, it comes out that ***the optical rays** are **perpendicular** to the **wave fronts*** of light waves. As a general rule, for a *point source* emitting waves inside a homogenous medium, the wave fronts are spherical and rays would follow the radial directions. As mentioned previously(at sound waves), at a big distance from the source, a portion of wave front can be fitted by a plane (see fig.2). Note that, at any point on a wave front, the optical ***ray*** *is* ***perpendicular*** *to* the ***wave front*** independently of its geometrical form.

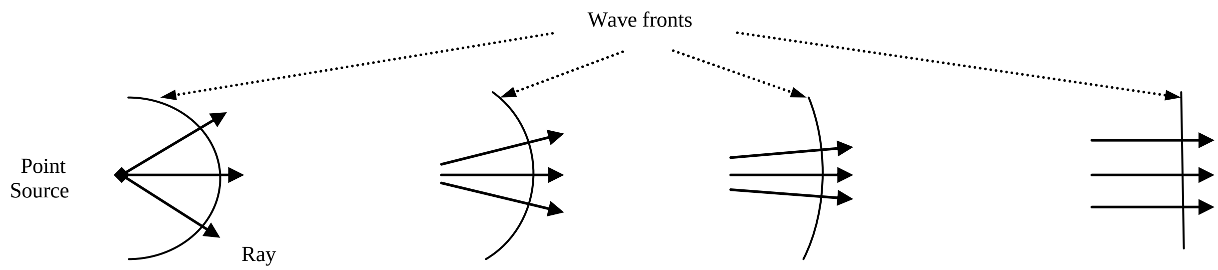


Fig. 2 The **rays** show the ***direction of energy flow*** and they are ***perpendicular*** to the ***wave fronts***.

-If a TW mechanical **plane wave** hits on a large opening (or obstacle), there *is no visible* ***wave motion*** *out the space region covered by rays* (Fig.3.a). But, if the opening (or obstacle) ***dimension*** are smaller than a certain limit *(~ 10λ*), a wave motion appears inside zones that cannot be reached by rays(fig. 3.b). This phenomenon is due to ***diffraction*** and can be *explained by* the concept of ***wave fronts*** *and the* ***Huygens' Principle***.

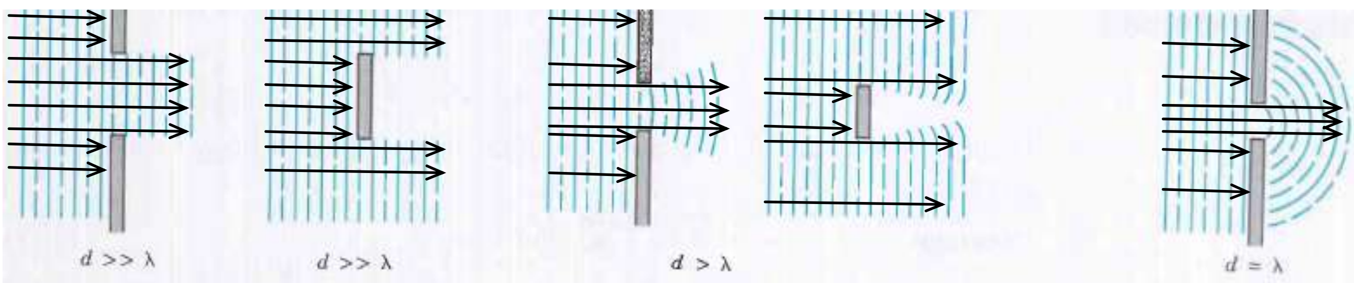
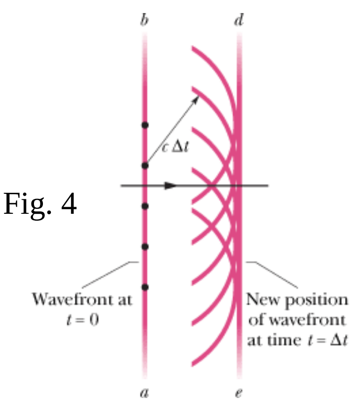


Fig 3. a) Ray "normal situation" (d >>λ) b) Ray "abnormal" situation" (d $\sim$ *or* $<$ λ)
(see the video1 and video 2)

Huygens principle: ***Any point on the wave front is a source of secondary waves (wavelets). The new wave front is the envelope of wave fronts of these wavelets***.

This principle allows to explain the effect of diffraction in all circumstances;

Fig. 4



a) ***Large*** openings (or obstacles). There are "*too many*" points on the wave front that contribute into building the new wave front by wavelets that they emit. The large central part of common envelope "*hides*" the curving effect due to a *few* wavelets originated from the borders and the result is a new *almost* ***plane wave front*** (fig. 4)

b) ***Small*** openings (or obstacles). There is a limited number of points on the wave front that contribute into building the new wave front by their wavelets. The central plane part of envelope "*cannot hide"* the curved parts due to the wavelets originated at borders; the result is a ***curved wave front*** **(**fig.5).

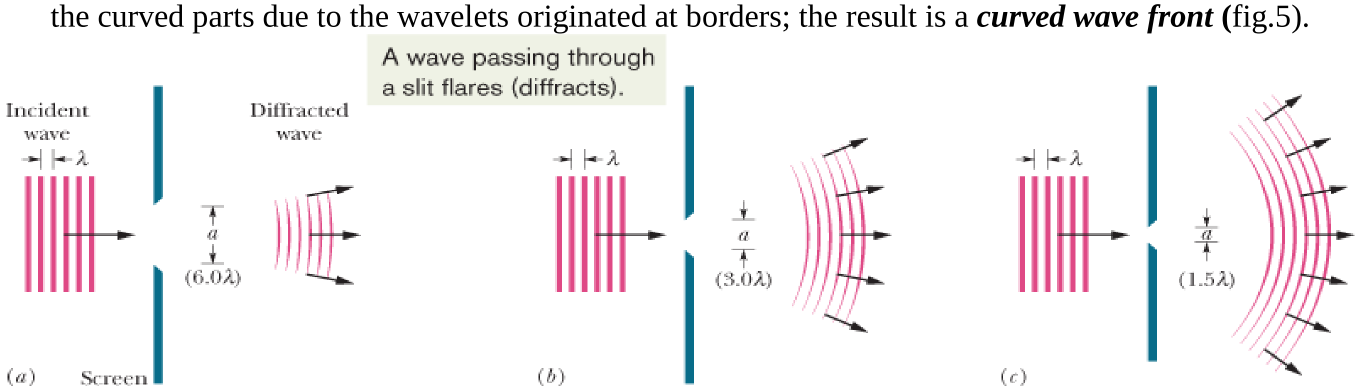


Fig. 5 a- many wavelets (d >λ) b- several wavelet (d ≈ λ) c- only a few wavelet

- One can explain all the diffraction phenomena by use of *wave fronts* and *Huygens principle*. Note that one may observe easy way the diffraction of ***sound waves*** because their wavelength is comparable to dimensions of many common opening (doors) or obstacles (objects). *Exemple:* for $f = 150Hz$ (low sound frequencies) the wavelength is $\lambda \approx 340m\ s^{-1}/150\ s^{-1} = 2.2m$ which is comparable to widths of corridors or door.

- In the case of light, the ***diffraction*** effects appear only when the light wave falls on ***very small openings*** or ***objects*** because λ -values for light are ***very small***. Note that the mechanism of diffraction is the same and the <u>*Huygens principle applies the same way because it does not depend on the physical nature of a wave*</u>.

WAVE INTERFERENCE

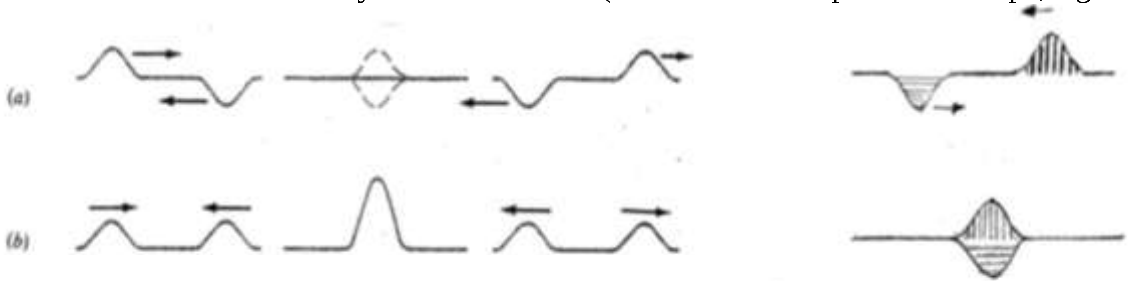
- *The principle of linear superposition:* If the paths of two (or more) waves that propagate in the same medium cross to each other, then the "***displacement*** " at <u>crossing point</u> is equal to ***the sum of displacements*** "y_i" due to each wave ***at this point*** of medium.

$$y_T = \sum_{i=1}^{n} y_i ;\ \_\_ i = 1,2,... \qquad (3)$$

This principle is ***valid*** *no matter what is* ***the physical nature of "displacement "***.

- Remember: When two " *vector waves*" pass by the same point of a medium, there is ***always a superposition*** but ***not always interference***.

There is interference only if the waves have " ***displacements*** " along the ***same space direction***(fig 6.a,b). This condition is not met always for TW waves. (*Ex.* Two crossed pulses on a rope; fig 6.c)



(a) Destructive interference (b) Constructive interference between two pulses with displacements in same plan

(c) There is superposition but not interference because the displacements are perpendicular to each other

Fig. 6

Important: The interference can built a standing wave only if the two waves have the ***same frequency;*** i.e. same ω, k ($k=\omega/v$) values. This requirement is ***essential for a standing wave.*** For observing a phenomena of ***stable pattern*** with ***maxima*** and ***minima***, one should refer to a ***standing wave*** interference. In the following we will refer to the interference of two waves with the same ω, k ($k=\omega / v$) values.

- Assume that a harmonic wave (TW, LW, mechanical or not) propagates through a medium. The **phase** of this travelling wave, i.e. $\Phi(x,t) = kx \pm \omega t + \varphi$, can be calculated at any location x and any time t, provided one has defined an axe Ox and knows k and ω. The ***wave front*** is defined (in 2D or 3D) as the locus of space ***points*** (locations) in ***medium*** at which the wave function has the ***same phase (Φ- value) at any moment.*** This means that, at a moment of time, the ***displacement*** is the same at all points on a given wave front, too. One shows a 2D travelling wave by sketching a set of ***crests&troughs*** (wave fronts at **maxima displacements**) at a given moment of time. For an harmonic wave, at any time, there is a phase change by $\pm 2\pi$ between any two consecutive "*similar*" wave fronts and there is the ***same displacement*** on any of them**.** One says that the points on two consecutive crest (or trough) wave fronts "***oscillate*** in ***phase***" (*or **are in phase)***. One may easily show that the ***distance*** between any two *similar(ex. crests) consecutive* (i.e. $\boldsymbol{\Phi_2 = \Phi_1 +/- 2\pi}$) *wave fronts* is equal to *the* ***wavelength***. Just choose an axe Ox (as shown in figure 7a), use the expression of phase $\boldsymbol{\Phi = kx - \omega t + \varphi}$ (*+Ox direction of propagation*) and find out that, at any given moment of time "***t*** " :

$$\Phi_1 = kx_1 - \omega t + \varphi; \_ \Phi_2 = kx_2 - \omega t + \varphi;$$
$$\Phi_2 - \Phi_1 = k(x_2 - x_1) = \frac{2\pi}{\lambda}\Delta = 2\pi \rightarrow \Delta = \lambda \qquad (4)$$

Remember**:** A*ny* ***two similar*** (say crests) ***consecutive wave fronts are at a distance λ to each other (fig.7a)***.

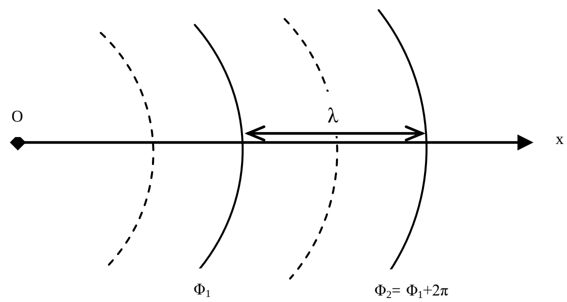


Fig. 7a
2D sketch of spherical wave fronts

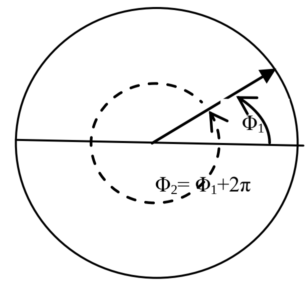


Fig. 7b
Phase shift 2π between phasors of two consecutive wave fronts

THE STATUS OF INTERFERENCE PRODUCED BY TWO HARMONIC WAVES

-The interference status at P-point is defined by ***phase shift*** between the ***waves that interfere at this location***.

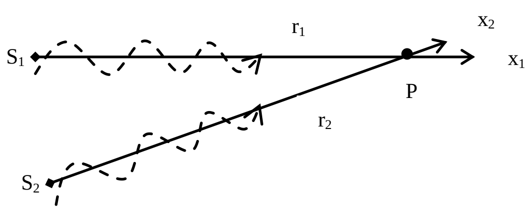


Figure 8

If this point P is at ***distance*** $\boldsymbol{r_1}$ from source S_1 and $\boldsymbol{r_2}$ from source S_2 (fig.8), the phases of the first and second wave at P location are
$\Phi_1 = kr_1 - \omega t + \varphi_1 \_ and \_ \Phi_2 = kr_2 - \omega t + \varphi_2$.
Then, the ***phase difference*** between the two waves at P point is

$$\Delta\Phi = \Phi_2 - \Phi_1 = k(r_2 - r_1) + (\varphi_2 - \varphi_1) = \Delta\Phi_p + \Delta\Phi_s \qquad (5)$$

$\Delta\Phi_p = \boldsymbol{k(r_2-r_1) = k\delta = (2\pi/\lambda)\delta}$ phase shift due to path lengths difference
$\Delta\Phi_s = \boldsymbol{\varphi_2 - \varphi_1}$ the phase shift between the two sources

As $\boldsymbol{k = 2\pi/\lambda}$ is a constant, the phase shift $\Delta\Phi_p$ depends only on the paths <u>***length difference***</u> $\boldsymbol{\delta}$ between the two waves *at P point*.

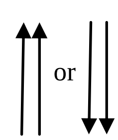
$$\delta = r_2 - r_1 \qquad (6)$$

r_1 is the distance from the first source; r_2 is the distance from the second source;

If there are reflections in the path of these two waves, there is a phase shift $\boldsymbol{\Delta\Phi_r}$ due to reflections and

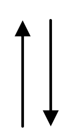
$$\boldsymbol{\Delta\Phi_{tot} = \Delta\Phi_s + \Delta\Phi_r + \Delta\Phi_p} \qquad (7)$$

- There is a <u>**constructive interference**</u> if $\boldsymbol{\Delta\Phi_{tot} = \pm M^*2\pi}$ (M=0,1,2,3,...)
In a constructive interference the two phasors are <u>**parallel**</u>, all time.



Two phasors
in constructive
interference

There is a <u>**destructive interference**</u> if $\boldsymbol{\Delta\Phi_{tot} = \pm (2m+1)^*\pi}$ (m=0,1,2,3,...)
In a destructive interference the two phasors are <u>**inverse parallel**</u>, all time.



Two phasors
in destructive
interference

-Assume that *sources* S_1,S_2 oscillate *in phase* ($\Delta\Phi_s$= 0) and there is *no* wave *reflections* ($\Delta\Phi_r$= 0) till P point.
In this case $\mathbf{\Delta\Phi_{tot} = \Delta\Phi_p}$ and one finds out that there is a **constructive interference** at P point if

a) $\mathbf{\Delta\Phi_p = 0}$ which happens for $\boldsymbol{\delta = 0}$, i.e. $\mathbf{r_1 = r_2}$;

b) or $\mathbf{\Delta\Phi_p = \pm M * 2\pi}$ which happens for $\boldsymbol{\delta = r_2 - r_1 = \pm M\lambda}$ (8.a)

There is **destructive interference** if :

$\Delta\Phi_p = \pm \mathbf{(2m+1) * \pi}$ which happens for $\boldsymbol{\delta = r_2 - r_1 = \pm *(2m+1)\, \lambda/2}$ (8.b)

Remember: At ***locations*** of ***constructive interference*** there is *all time oscillations at* ***maximum amplitude of displacement*** while at ***locations*** of ***destructive interference*** there is ***zero amplitude of displacement*** (*for two travelling waves with equal amplitude*)**.**

-IMPORTANT : The derivation of conditions (7-8) is based only on **phase difference** considerations. So, they are valid for the interference ***of any type of waves; sound waves, light waves, radio waves, TV waves...***. These same rules explain *bad* or *good* ***sound*** **hearing** at different positions inside the same concert hall and also the low or high quality reception of radio signal and TV signals for the different positions of an antenna.

OBSERVATION OF INTERFERENCE ON A TWO DIMENSIONNAL SPACE (see video 3)

The standing waves on a string are a ***1D space*** example. One can see a ***2D space*** standing wave interference by using two plungers that vibrate at the ***same frequency*** and ***phase*** on the still surface of a water tank(fig.9).

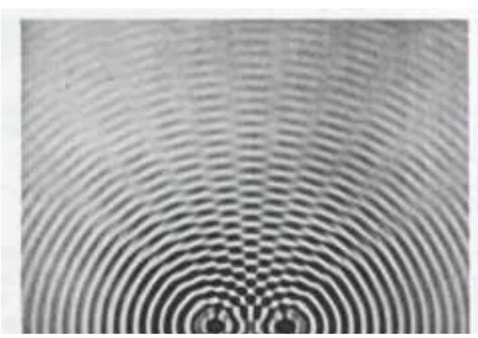


Fig. 9 Interference observed on water surface.

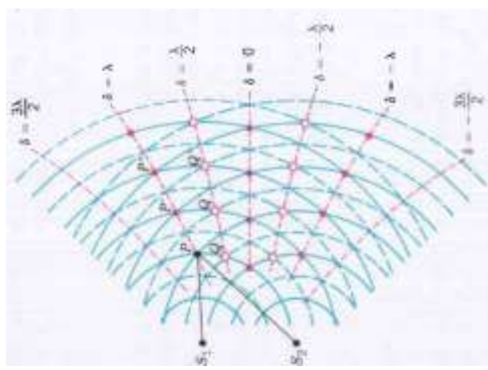


Fig. 10 Directions of constructive and destructive interference

- The ***wave model*** (fig.10) explains the structure produced on water surface by the interference of two TW mechanical waves originated at two point sources (S_1, S_2) that oscillate *in phase* ($\Delta\Phi_s$= 0) at ***same frequency***, ***same amplitude*** *and* the *same space direction* of " ***displacements*** ". As there is *no reflections* ($\Delta\Phi_r$ = 0), the expressions 8.a,b apply. So, one can easily figure out that:

There is CONSTRUCTIVE (***double amplitude***) interference at any point where $\mathbf{\Delta\Phi_p = 0}$ or $\mathbf{\pm M *2\pi}$ (a **crest** *wave front* meets another **crest** or a **trough** *wave front* meets another **trough** *wave front)*.

There is DESTRUCTIVE interference (***zero amplitude***) at any point where $\mathbf{\Delta\Phi_p = \pm (2m+1) * \pi}$ (a **crest** wave front meets **trough** wave fronts or vice versa).

-As $\boldsymbol{\Delta\Phi_p = k * \delta = k*(r_2 - r_1)}$ ***the interference status at a point on water surface depends only on its location***.
From the figure 10, one may see that there is a **constructive interference** with the ***phase difference*** :

a) $\boldsymbol{\Delta\Phi_p = 0}$ for each point located on central line ($\mathbf{r_1 = r_2}$, $\boldsymbol{\delta = r_2 - r_1 = 0}$).

b) $\boldsymbol{\Delta\Phi_p = \pm M*2\pi}$ at all points (*red dots*) where $\boldsymbol{\delta = r_2 - r_1 = \pm M*\lambda}$ **.**

c) Between the *central constructive line* and closest *wing* of *constructive interference* there is a *line of* ***destructive interference*** at points (*empty circles*) where **$\Delta\Phi_p$ = +/-π** or **$\delta = r_2 - r_1 = \pm \lambda/2$**. Those points are located on lines that have a hyperbola shape, too. So, based on the *phase shift* of the two interfering waves, one may explain the whole picture of interference fringes observed on the surface of the water tank.

YOUNG'S EXPERIMENT

- Thomas Young showed the wave nature of light by proving that it diffracts and interferes like any other wave phenomena. He used an experimental set up similar to that presented in figure 11. In this scheme:

- Two ***sources*** of *" in phase "* oscillations are two sources of *"secondary wavelets";i.e. the narrow slits* ***S_1, S_2*** equidistant from the primary source (*a pinhole receiving sun light*).
- The "large distance" (*compared to λ of light*) from the pinhole gives ***plane waves at slits input***.
- The two narrow slits ***S_1, S_2*** produce ***cylindrical wave fronts*** on the other side of their plane.
- The model of a screen "*far from*" the slits simplifies expression for ***path length difference.***
- By using a coloured glass in front of pinhole one can get a ***monochromatic light*** (***one λ***) . ***Nowadays, one gets easily the monochromatic light by using a laser source.***

Coloured glass

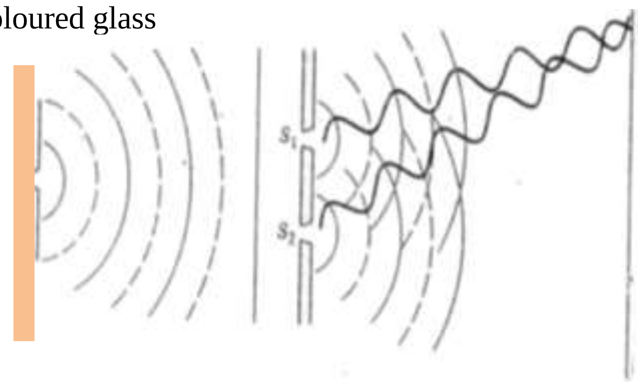


Fig. 11

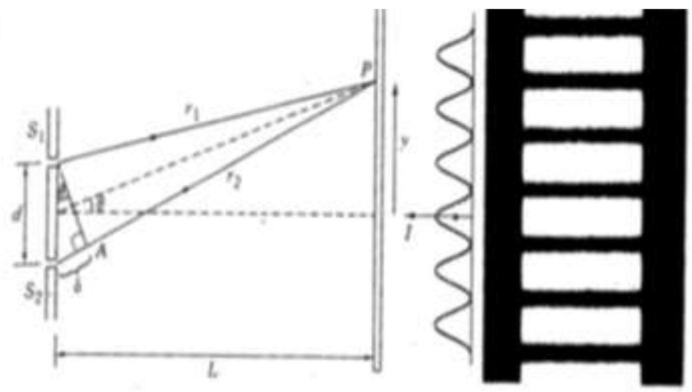


Fig. 12

-The ***geometrical path length difference*** for a *given point* P on the screen along direction **"θ"** (see fig 12) is **$\delta = r_2 - r_1$**. As **L >> d** , for points near "*screen center*", the angle θ is very small and the two paths are *almost parallel*. So, by referring to the drawing in fig.13, one gets

$$\delta \cong d \sin\theta \tag{9}$$

d- distance between the two narrow slits;
θ- the angle that correspond to the location of point P on screen.

Figure 13

- After noting that, in this case $\Delta\Phi_{S=0}$**,** $\Delta\Phi_R = 0$ *and* $\Delta\Phi_{tot} = \Delta\Phi_p$, one might remember that there is*:*

constructive interference for $\Delta\Phi_p = k\delta = \pm M * 2\pi \rightarrow\rightarrow\rightarrow \delta_M = \pm M * \lambda$

destructive interference for $\Delta\Phi_p = k\delta = \pm(2m+1) * \pi \rightarrow\rightarrow \delta_m = \pm(2m+1) * \lambda/2$

So, one can derive the corresponding directions (i.e. the corresponding ***angles θ***) for ***fringes*** of a ***maximum*** intensity by the expression (10) and a ***minimum*** intensity by the expression (11)

$$d\sin\theta_M = M\lambda \quad \text{M = 0, ±1, ±2..} \quad (10) \qquad d\sin\theta_m = (2m+1)\frac{\lambda}{2} \quad \text{m = 0, ±1, ±2..} \quad (11)$$

Notes: a) For M = 0 one gets the **central** **maximum (** located in the middle of interference pattern)**.**
b) The ***first minimum*** ***"upside (θ > 0)"*** *the central maximum corresponds* ***to m = 0.***

-Those ***interference fringes*** are parallel to slits. The position "***y***" (see fig 12) of low order ***maxima fringes*** (*located close to the center of interference pattern where* ***θ is small*** and ***tanθ≅sinθ***) can be found as follows:

$$\frac{y_M}{L} = \tan\theta_M \cong \sin\theta_M \Rightarrow\Rightarrow\Rightarrow \frac{y_M}{L} = M\frac{\lambda}{d} \Rightarrow\Rightarrow y_M = M\frac{\lambda}{d}L \qquad (12)$$

- One uses widely the term **"coherence"** in **interference** studies**.** If the wave is a ***pure harmonic function*** (*one frequency and infinite wave train*) one says that it is a **"coherent wave"** emitted by a **"coherent source".** *Two* ***sources*** *(or two* ***waves****) that oscillate at the* ***same frequency*** *(*$\omega_1 = \omega_2$*) are said to be* ***coherent*** ***between them*** *in the sense that there is a* ***constant phase shift*** *between their* ***phasors*** ***all the time***

$$\Delta\Phi_{1,2} = [(\omega_2 t+\varphi_2) - (\omega_1 t+\varphi_1)] = \varphi_2 - \varphi_1. \qquad (13)$$

Two ***coherent sources*** *(or two* ***waves****)* are said to oscillate ***in phase*** if $\Delta\Phi_{1,2}$ = M*2π where *M=0,±1,±2,..*
In the Young's experiment, ***S₁, S₂*** are two ***coherent sources*** *(*$\omega_1 = \omega_2$*);* they are ***in phase*** because $\Delta\Phi_{1,2}$ = 0.
If the phase shift is *constant* but $\Delta\Phi_{1,2} \neq$ M*2π, two *coherent sources* are considered to be ***out of phase*** . Very often this nomination means two coherent sources in ***opposite phase*** ; $\Delta\Phi_{1,2}$ = (2m+1)π where m = 0,± 1 ,±2,..

- *Remember*: The phase shift due to difference in path lengths is $\Delta\Phi_p = k\delta$. If the two interfering waves propagate in vacuum (or air), the refractive index is ***n =1*** and $\mathbf{k \equiv k_0 = 2\pi/\lambda_0}$. When the two waves propagate inside another transparent medium *" **n >1**" and* $\mathbf{k \equiv k_n = 2\pi/\lambda_n = 2\pi/(\lambda_0/n) = n*(2\pi/\lambda_0) = n*k_0}$

So, inside a medium $\Delta\Phi_p = k_n\delta = nk_o\delta = k_o n\delta = k_o\delta_n$ where $\delta_n = n\delta$ (14)

When two waves propagate inside the same transparent medium, the ***optical path length difference*** $\boldsymbol{\delta_n}$ is

$$\boldsymbol{\delta_n = n*\delta} \qquad (15)$$

When *they propagate in two different mediums,* $\boldsymbol{\delta_n = n_2*r_2 - n_1*r_1}$ (16)

The general expression for ***the phase shift due to path length difference is*** $\Delta\Phi_{path} = k_0\delta_n$ (17)

THE INTENSITY OF INTERFERENCE PATTERNS

- In Young's experiment, one uses a small pinhole at "big distance" from two slits S_1, S_2 to get the plane wave fronts at their input. Essentially, this produces ***equal frequency***, ***equal phase*** *and* ***equal amplitude for the two waves at sources*** *(slits* S_1, S_2*).* As the path lengths r_1, r_2 have comparable values and the emitted powers "P_0" are equal, at any point close to the "screen center", the light waves produce *similar intensity* ($\boldsymbol{I \sim P_0/r^2}$) i.e. a *similar amplitude of displacement* ($\boldsymbol{I \sim A^2}$). As for light displacement "$A \equiv E_0$" then $E_{1-0} = E_{2-0} \equiv E_0 \sim I^{1/2}$.

- Therefore, one can express the ***light wave*** "***displacement E*** " at <u>a point</u> *on central area of screen*, produced by the source S_1 as $E_1(t) = E_0 \sin \omega t$ (18)

  produced by the source S_2 as $E_2(t) = E_0 \sin(\omega t + \varphi)$ (19)

  E_0 - same amplitude of ***light wave displacement*** on the screen (*at distance "r" from source*)

  ω - coherent waves i.e. same frequency (first requirement for interference).

  φ - ($\Delta\varphi_{p-1,2} \equiv \varphi$) phase shift ***between*** them is ***constant all time***.

The *phase shift* φ at (19) is defined by the ***position*** of considered point "*P*" on the screen and does not change in time because the two waves are **coherent**. Actually, *as* $\varphi = (2\pi/\lambda)*\delta = \Delta\varphi_p$, this *phase shift* is fixed by the *path length difference* δ *from two slits to that point* (*i.e. direction θ at expression "9"*).

-The principle of linear superposition gives the "total displacement E_T" at this point as

$$E_T = E_1 + E_2 = E_2 + E_1 = E_0 \sin(\omega t + \varphi) + E_0 \sin(\omega t) = 2E_0 \cos\frac{\varphi}{2} \sin(\omega t + \frac{\varphi}{2}) \qquad (20)$$

(by applying expression sinα + sin β = 2sin[(α + β)/2]*cos[(α - β)/2])

This is the expression of an "*interference wave*" with amplitude $$A_T = 2E_0 \cos\frac{\varphi}{2} \qquad (21)$$

- As the intensity ***I = Power/Area = Energy/ (Time*Area) ~ Energy ~ A²***, the ***evolution of intensity*** for interfering ***light*** over different points on the screen is the same as **the evolution of A_T^2 through these point.** So,(*by taking I =* ***A²*** *for simplicity*), it comes out that, if a single source sends a light wave at a point P on the screen, the *light intensity* at this point is $I_0 = E_0^2$. If the light waves from two coherent sources illuminate simultaneously this point, the *light intensity* at point P becomes

$$I = A_T^2 = 4E_0^2 \cos^2\frac{\varphi}{2} = 4I_0 \cos^2\frac{\varphi}{2} \qquad (22)$$

Figure 14 presents the graph of this function for different locations on screen (*different phase shifts* $\varphi = \Delta\varphi_p$). As expected, its maxima and minima are placed along the space directions given by expressions (10,11).

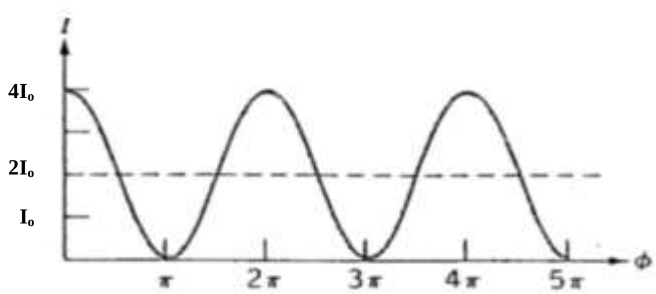


Figure 14

-If the two waves had different frequency ($\omega_2 \neq \omega_1$) they would **superpose on the screen but** they would not interfere. In this case the ***intensity*** of light at a considered point on the screen would be the simple sum ***Io + Io = 2I₀*** .

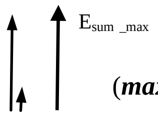
So, because of the interference, the ***light intensity***:

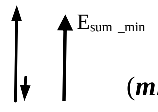
a) increases twice "***2*(2 I₀) = 4*I₀***" if the considered point is at a ***maximum location;***

b) is reduced to ***zero*** if the considered point is at a ***minimum location***.

Important Notes: a) The ***interference produces only energy redistribution*** but the average (see fig.14) ***intensity*** (***2*I₀***) and ***the total light energy on the screen remain unchanged***.

b) One must use two beams with *similar amplitudes* to observe a well *contrasted pattern of interference*. If the amplitudes of two interfering waves are not equal, the interference fringes are not well seen. Even, the fringes may become invisible if $\boldsymbol{E_{1\text{-}0}} >> \boldsymbol{E_{2\text{-}0}}$ or vice versa. This happens because the *intensity of minima is not zero and* it may become comparable to that of maxima. The figure 15 explains what happens in these situation.

Sum of phasors 

(***maxima locations***) 

(***minima locations***)

Figure 15 The amplitudes of resultant phasors are comparable; the intensities become comparable, too.

Exemple: *A plane monochromatic wave with λ_0 = 600nm falls on two narrow slits and produces a set of maxima(bright) and minima(dark) fringes on a screen located on the other side of slits. Assume that one covers the upper slit (as shown in figure) by a plastic transparent strip with refractive index n =1.5 .*

a) Show that all the interference pattern ***shifts upward****.*

*b) Find the thickness "**l**" of the strip such that the pattern shifts up exactly by one inter fringe distance.*

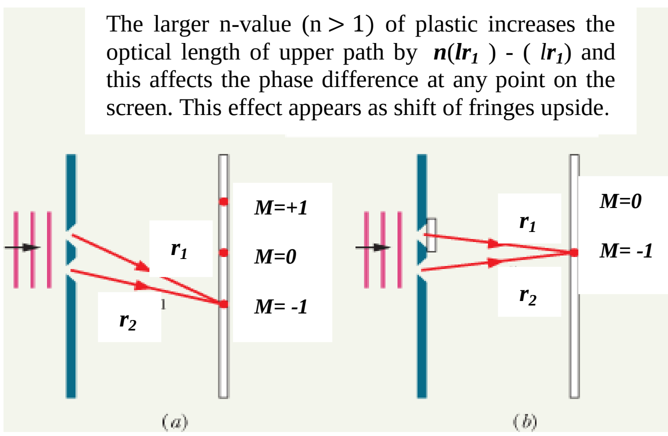


Figure 16

a) Without the plastic strip, the maximum of "order 0" on screen is located over a line passing through the midway of two slits. So, initially, $\delta_{op} = 0$ and $\Delta\varphi = 0$ at this location; i.e. the maximum of order M=0 is located at center (fig.16.a).

In presence of the plastic strip on upper slit, the optical path length of the first wave to the center " r_1' " *increases*. As the length of optical path for second wave to center "r_2" does not change, it comes out that the introduction of plastic strip makes $\delta'_{op} = r_2 - r_1'$ and $\Delta\varphi' = k\,\delta'_{op} < 0$ at center of patter; the location with $\Delta\varphi = 0$ is shifted upside the center (together with all the set of fringes)

b)*To shift the whole pattern up by one inter fringe distance, the thickness "**l**" of plastic strip should be such that the phase shift between wave_2 and wave_1 at center becomes exactly "-2π" (or M= -1).*

<u>*In presence of the strip with thickness* ***l***</u>, the path length of the first wave till center point changes from r_1 to $r'_1 = (r_1 - \boldsymbol{l}) + n\boldsymbol{l}$ and the difference of optical lengths for the two waves *at center* (r_2 does not change) becomes

$$\delta' = (r_2 - r'_1) = [r_2 - ((r_1 - \boldsymbol{l}) + n\boldsymbol{l})] = [r_2 - r_1 + \boldsymbol{l} - n\boldsymbol{l}] = [(r_2 - r_1) - (n\boldsymbol{l} - \boldsymbol{l})]$$

As, at the center $(r_2 - r_1) = 0$, it comes out that $\delta' = (r_2 - r'_1) = -(n\boldsymbol{l} - \boldsymbol{l}) = -\boldsymbol{l}(n-1)$

This means that the phase shift between waves at center of screen becomes

$$\Delta\varphi' = (2\pi/\lambda_0)\,\delta' = -(2\pi/\lambda_0)^* \boldsymbol{l}\,(n-1)$$

As one should get $\Delta\varphi_1 = -2\pi$ *at center*, it comes out that $-2\pi = -(2\pi/\lambda_0)^* \boldsymbol{l}\,(n-1)$ or $(\boldsymbol{l}/\lambda_0)^*(n-1) = 1$

So, $\boldsymbol{l} = \lambda_0/(n-1) = 600/(1.5-1) = 600/0.5 = 1200\text{nm}$

and one gets $\boldsymbol{l} = 1200\text{nm} = 1200^*10^{-9}\text{m} = 1.2^*10^{-6}\text{m} = 1.2\mu\text{m}$