

GENERAL

- The geometrical optics model of "light rays" cannot explain the two following experimental situations:
  - a) If there is **small openings** or **small obstacles** on its path, the light reaches inaccessible regions for optical ray model. It seems that the light "**bends around borders**" ... this effect is known as **DIFFRACTION**;
  - b) If two or more light beams superpose in "some special conditions", a set of fringes of **maxima / minima** brightness (i.e. **light intensity**) come out (on a screen).... this effect is known as **INTERFERENCE**.

**Diffraction** and **interference** are **typical wave phenomena**. One can explain them by using the "same tools" used for the study of mechanical waves. In this chapter, we introduce a general model valid for all diffraction and interference wave phenomena. We will show how to apply it for mechanical, sound and **light waves**.

- Theoretical research and many experiments confirm that **light waves** are **electro-magnetic (E&M) waves**. Radio & TV waves, x &  $\gamma$  rays are part of **E.M.** spectrum, too ( see figures 1.a-b). Those are **TW waves**, i.e. their "**displacement**" is a vector perpendicular to the direction of wave propagation. All the **E&M** waves are characterized by two vectors  $\vec{E}$  (electric) and  $\vec{B}$  (magnetic), but one ties the "**displacement**" to the electric field vector  $\vec{E}$  because majority of detectors used for measurements on E&M waves react versus electric action.

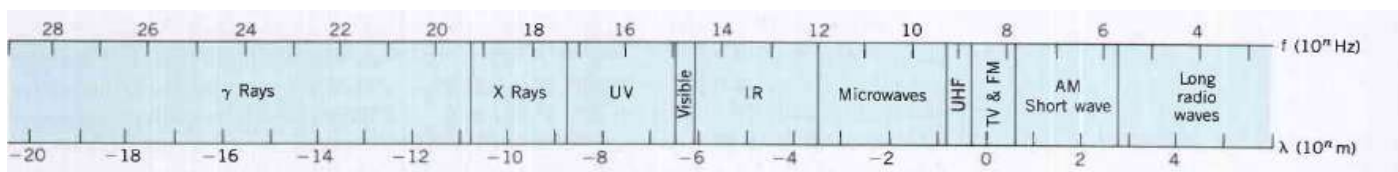


Fig.1a The electromagnetic spectrum

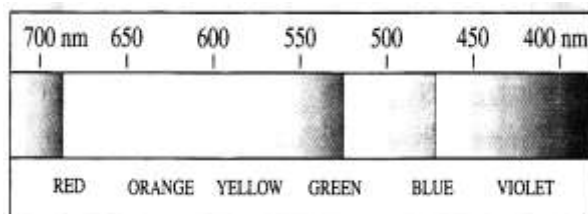
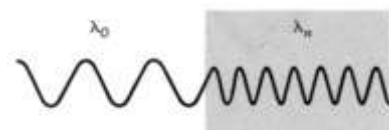


Fig.1.b Visible Light (colour vs. wavelength)

Fig.1.c **The light frequency remains the same but its wavelength decreases inside the matter.**

- The speed of all **E.M.** waves (light included) in vacuum is  $c \approx 3 \cdot 10^8 \text{ m/s}$ . When light pass through a transparent material medium, its speed *decreases* to  $v = c / n$  (1) where  $n$  is the **refractive index** of the medium ( $n > 1$ ).

If the wavelength of light wave in vacuum is  $\lambda_0 = T \cdot c$ ,

inside a transparent medium it becomes  $\lambda_n = T \cdot v$ .

The period  $T$  and the frequency ( $f = 1/T$ ) of light wave remain constant; the wavelength decreases  $\lambda_n / \lambda_0 = v / c = 1/n < 1$ . Remember that :  $\lambda_n < \lambda_0$  and  $\lambda_n = \lambda_0 / n$  (2)

DIFFRACTION

- In geometrical optics, the **rays** (i.e. *directed straight lines*) show the path followed by "light particles or energy droplets of light" i.e. the **direction of light energy flow**. In a **wave model**, the **energy propagates** on directions **perpendicular to the wave fronts**. So, it comes out that the optical rays are **perpendicular** to the **wave fronts** of light waves. As a general rule, for a *point source* emitting waves inside a homogenous medium, the wave fronts are spherical and rays would follow the radial directions. As mentioned previously(at sound waves), at a

big distance from the source, a portion of wave front can be fitted by a plane (see fig.2). Note that, at any point on a wave front, the optical ray is **perpendicular** to the **wave front** independently of its geometrical form.

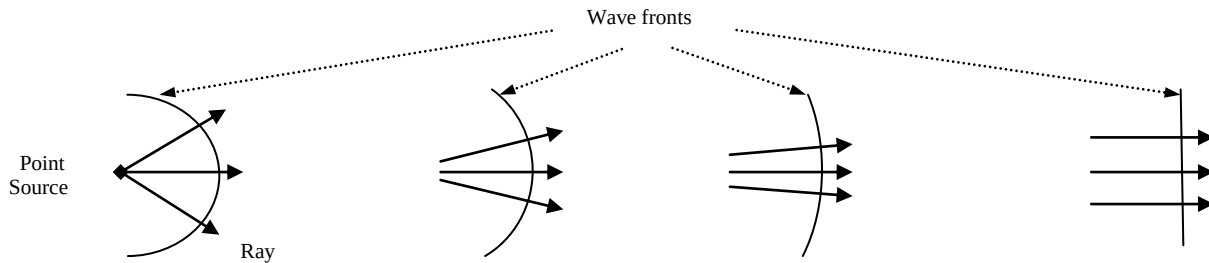


Fig. 2 The **rays** show the **direction of energy flow** and they are **perpendicular** to the **wave fronts**.

-If a TW **plane wave** on water hits on a large opening (or obstacle), there is *no visible wave motion* out the space region covered by rays (Fig.3.a). But, if the opening (or obstacle) **dimension** are smaller than a certain limit ( $\sim 10\lambda$ ), a wave motion appears inside zones that cannot be reached by rays (Fig. 3.b). These observations are related to **diffraction** and can be explained by the **Huygens' Principle** and the concept of **wave fronts**.

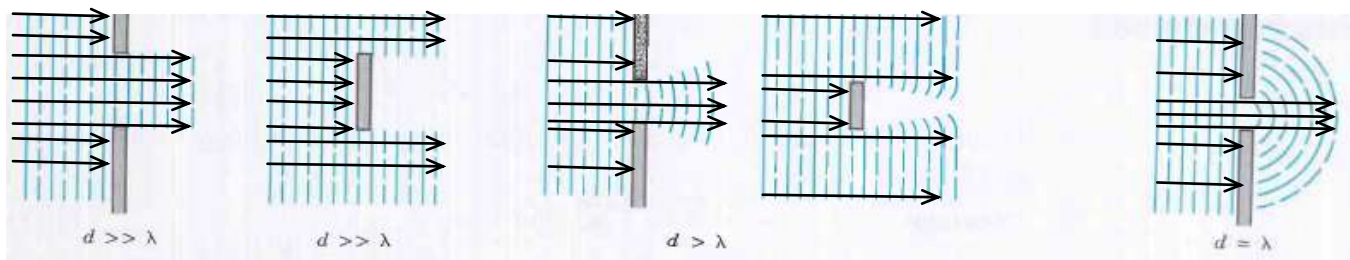


Fig 3. a) Ray "normal situation" ( $d \gg \lambda$ ) b) Ray "abnormal" situation" ( $d \sim$  or  $< \lambda$ ) (see the [video1](#) and [video 2](#) )

**Huygens principle:** *Any point on a wave front acts as source of secondary waves (wavelets). The new wave front is the envelope of wave fronts of these wavelets.*

This principle allows to explain the effect of diffraction in all circumstances;

a) **Large** openings (or obstacles). There are "too many" points on the wave front that contribute into building the new wave front by wavelets that they emit. The large central part of common envelope "hides" the curving effect due to a few wavelets originated from the borders and the result is a new **almost plane wave front** (fig. 4)

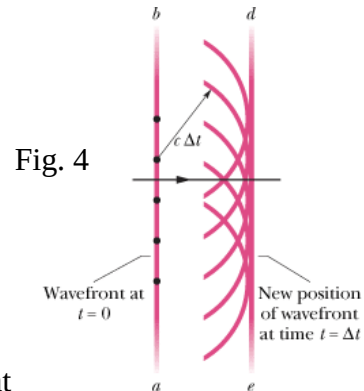


Fig. 4

b) **Small** openings (or obstacles). There is a limited number of points on the wave front that contribute into building the new wave front by their wavelets. The central plane part of envelope "cannot hide" the curved parts due to the wavelets emitted at borders; the **wave front** is **curved** (fig.5).

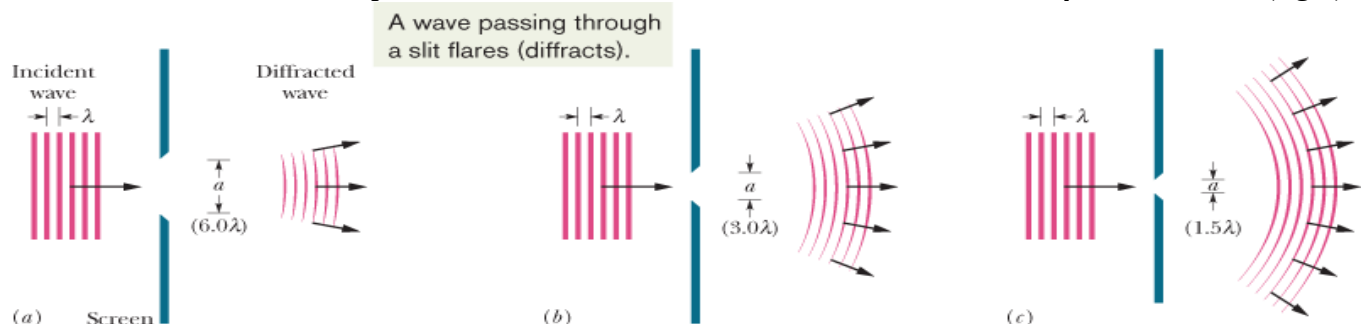


Fig. 5 a- many wavelets ( $d > \lambda$ )

b- several wavelet ( $d \approx \lambda$ )

c- only a few wavelet

- As mentioned previously, one can explain all the diffraction phenomena by use of *wave fronts* and *Huygens principle*. One may observe easy way the diffraction of **sound waves** because their wavelength is comparable to dimensions of many common opening (doors) or obstacles(objects). Ex.: for  $f=150\text{Hz}$  (low frequencies) the wavelength is  $\lambda \approx 340\text{m s}^{-1}/150\text{ s}^{-1} = 2.2\text{m}$  which is comparable to widths of corridors or door.

- In the case of light, the **diffraction** effects appear only when the light wave falls on **very small openings** or **objects** because  $\lambda$  -values for light are **very small**. Nevertheless the mechanism of diffraction is the same and the *Huygens principle applies the same way because it does not depend on the physical nature of a wave*.

## POLARISATION OF A "VECTOR WAVE " ; LIGHT WAVE POLARIZATION

As mentioned previously, if the propagating "**disturbance**" is a scalar parameter one deals with *scalar waves*; if it is a vector parameter one deals with *vector waves*. The "**displacement**" of vector waves has a direction in space. In a *LW wave*, the displacement vector is directed along the direction of the wave propagation. In a *TW wave* the "**displacement**" vector lies in a plane perpendicular to the direction of wave propagation; it may have any direction in that plane. If a TW wave has the **displacement aligned along a given space direction all the time**, one says that it is a **linearly polarized** wave. In figure 6, two TW waves travel on a string aligned along Ox axe. The wave Wy is *linearly polarized along Oy direction* and the wave Wz is *linearly polarized along Oz direction*. If one wants to allow only Oy polarized waves to propagate on the string, one may use a screen with a "*thin window*" aligned along Oy axe; this window is functioning as a **polarizing filter** (or simply *polarizer*). One may select the direction of wave polarization after the polarizer by rotating the "*polarizing window*" in the plane Oyz.

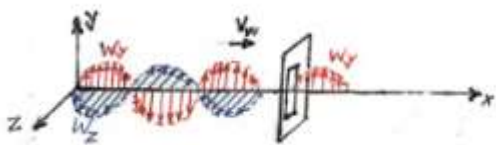


Figure 6. Polarisation of TW waves and the polarizing filter

Electromagnetic waves are characterized by two vectors  $\vec{E}$  and  $\vec{B}$  which are perpendicular to each other and lie in a plane perpendicular to direction of wave propagation. For example, if the *EM wave* propagates in Ox direction of fig.6 and  $\vec{E}$  direction is aligned to Oy axe, then  $\vec{B}$  vector would be aligned along Oz direction.

As one ties the "**displacement**" to the electric field vector  $\vec{E}$ , if it remains all time along Oy, one would say that the E&M wave is *linearly polarized* in y-direction.

If a radio broadcast antenna is aligned vertically, the emitted EM waves are *linearly polarized* vertically all time (i.e.  $\vec{E}$  vector of the emitted radio waves is aligned to the direction of electric dipole of antenna).

The visible light is emitted by molecules or atoms (*inside the light source*) when they fall from an excited state to a lower energy state (*we will explain this in quantum mechanics*). This process happens in a random way and the light emitted by each of them has an  $\vec{E}$  vector directed randomly in space. Then, because of its TW nature, the part of light that propagates as a beam along a given space direction (say Ox) will have all those vectors orientated randomly in a plane perpendicular to beam direction (i.e. Oyz). Due to their randomness at origin, all those  $\vec{E}$  vector have equal magnitudes(see fig.7). Such light is labelled as *natural light* or *unpolarized light*.

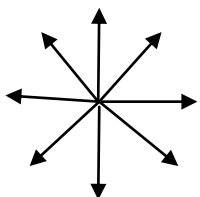


Figure 7. Random directions of  $\vec{E}$  vectors in a beam of natural light directed perpendicular to the page. Due to their random original emission, the set of  $\vec{E}$ -vectors have equal magnitude along each direction in the plane of page.

One must use a light polarizer (or polarizing filter) to get a linearly polarized light beam from natural light. A light polarizing filter allows to pass through it only the light polarized along a single direction which is called **polarizing axis** of the **polarizer** (Oy in fig. 8). As each vector  $\vec{E}_i$  of natural light can be decomposed into two components ( $\vec{E}_i = \vec{E}_{i-x} + \vec{E}_{i-y}$ ), one may gather together those components and present the  $\vec{E}$  vector of the "net" natural light as  $\vec{E}_{nat} = \vec{E}_x + \vec{E}_y$ . Note that, due to their random nature, both light wave components have **equal amplitudes**; i.e.  $E_{x_0} = E_{y_0}$ . As wave intensity ( $I = \frac{Power}{area}$ ;  $Power = \frac{Energy}{time}$ ) is proportional to the amplitude ( $E \sim A^2$ ), it comes out both components have equal intensity:  $I_x \sim [E_{x_0}]^2 = [E_{y_0}]^2 = I_y$ . Then, the intensity of natural light can be expressed as  $I_{nat} = I_x + I_y = 2I_y$ . As only the part  $I_y$  pass through Oy polarizer, the intensity of polarized light is  $I_y = 0.5I_{nat}$ . (3)

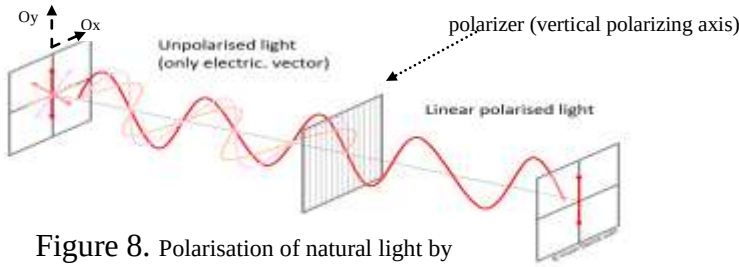


Figure 8. Polarisation of natural light by a vertical polarizing filter

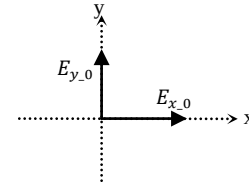
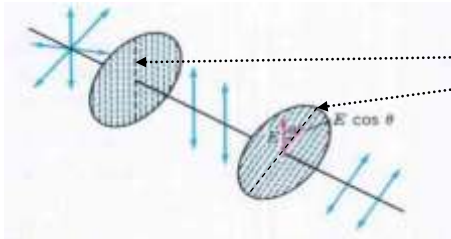


Fig.9 The natural light is the sum of two light linearly polarized waves  $\vec{E}_x$ ,  $\vec{E}_y$  at equal amplitudes  $E_{x_0} = E_{y_0}$  and equal intensities  $I_x = I_y$

If **linearly polarized** light " $\vec{E}$ " pass through a polarization filter (called **analyzer**) with polarization axis at angle " $\theta$ " versus direction of  $\vec{E}$ , only the component of  $\vec{E}$  along its polarization direction, i.e.  $\vec{E}_{an} = \vec{E} * \cos\theta$  (see fig. 10) gets transmitted. The transmitted light is linearly polarized along the polarization direction of analyzer and its intensity is (Malus's law)  $I_{an} = E_{an-0}^2 = (E_0 * \cos\theta)^2 = E_0^2 * (\cos\theta)^2 = I * \cos^2\theta$  (4)



The direction of **transmission axis** is "vertical" at the polarizer and "along angle  $\theta$ " at the analyzer shown in figure.

Fig.10 The intensity of light decreases after the polarizer ( $I_{pol} = I_{nat} / 2$ ) and after the analyzer ( $I_{an} = I_{pol} * \cos^2\theta$ )

The scheme presented in figure 10 corresponds to the use of two **Polaroid** sheets as **polarizer** and **analyzer**. The **Polaroid** is a plastic sheet that contains long chains of hydrocarbon molecules aligned in same direction. As a chain of such molecules interacts with an electric field  $\vec{E}$  aligned the same way, it absorbs efficiently the energy of incident EM wave and transforms it into internal energy of plastic sheet. By contrary, it does not interact at all with  $\vec{E}$  vectors aligned perpendicular to chain. So, if a natural light falls on a **Polaroid** sheet only the light with  $\vec{E}$  vectors aligned perpendicular to molecular chains get transmitted. The direction perpendicular to that of molecular chains is labelled as "**transmission axis**". In general, the transmission axis is shown on a Polaroid sheet. The light that pass through a Polaroid is polarised along the direction of its transmission axis. The action of a Polaroid sheet is called "**Polarization by spatial direction absorption**". E.H.Land was the first to set up a procedure for the alignment of a long chains of hydrocarbon molecules inside a plastic sheet by stretching and simultaneously heating it.

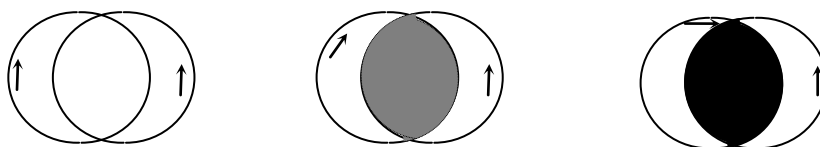


Fig.11 The intensity of light transmitted through a polarizer followed by an analyzer depends on the angle between their transmission axes (shown by arrows).

## Light polarization by reflection

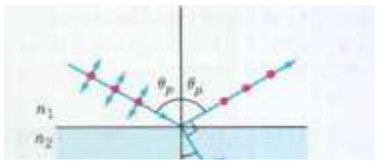


Fig.12 Red dots show the  $\vec{E}$  vector parallel to the surface

When the natural light falls on the plane surface of a transparent medium, due to the interaction of light wave  $\vec{E}$  vector with the molecules in layers near surface of medium, the reflected beam gets "*partially polarized*". This means that a larger part of reflected light wave has the vector  $\vec{E}$  parallel to the surface. But, for a particular angle of incidence  $\theta_{\text{incid.}} = \theta_p$  ( known as "*polarizing angle*" ) *all the reflected light is linearly polarized parallel to surface* (fig.12).

David Brewster found that for  $\theta_{\text{incid.}} = \theta_p$ , the *angle between the reflected and refracted rays is  $90^\circ$* . Breswster's law (5) allows to find the value of " $\theta_p$ "

$$\tan \theta_p = \frac{n_2}{n_1} \quad (5)$$

## Light polarization by double refraction

The speed of light wave in a gas or liquid ( $v_{\text{light-fluid}} = c / n$ ) does not depend on its direction of propagation because fluids are **isotropic** substances ( *their physical properties do not depend on the space direction*). This is not true for all solids. Depending on their internal structural order, one distinguishes *amorphous* solids and *crystalline* solids. The atoms are distributed randomly inside a transparent **amorphous solid** ( *like glass or the plastic materials*) and it has the same refraction coefficient " n-value " for all space directions. In a **crystalline solid** ( *like quartz, calcite,.*), there is organization of atoms in space which makes them **anisotropic** ( *i.e. their physical properties depend on the space direction*). In particular, the speed of light (and n-value) does depend on the direction of beam inside them and its polarization (direction of  $\vec{E}$  versus axes of order axes in crystal). In the group of birefringent (double refracting) crystalline solids there is two n-values ( "*ordinary  $n_o$* " and the "*extraordinary  $n_e$* " ) for each direction of a refracted light beam. The refraction index  $n_o$  is the same along all direction inside the crystal while the  $n_e$  value changes with direction. So, when an *unpolarized beam* of light falls on a birefringent material, there are two refracted beams getting out of it ; the ordinary beam " O " and the extraordinary beam " E " which propagate along different directions. Each beam has linearly polarized light and  $\vec{E}_E$  is perpendicular to  $\vec{E}_O$  (see fig. 13). However, in any birefringent material, there is a specific direction, labelled as " **the optic axis** ", along which  $n_e = n_o$  and both refracted beams follow the same direction. Also, for this direction, both polarization components travel at same speed and the outgoing beam keeps the same polarization status as that of incident beam.

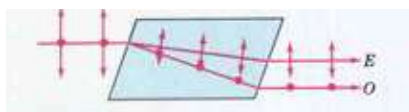


Fig.13 The natural light in incident beam contains two reciprocally perpendicular  $\vec{E}$  vectors. Each beam inside the birefringent crystal is polarized and keeps the same polarization when get out of it. Red dots show the  $\vec{E}_O$  vector perpendicular to page.

## Light polarization by scattering

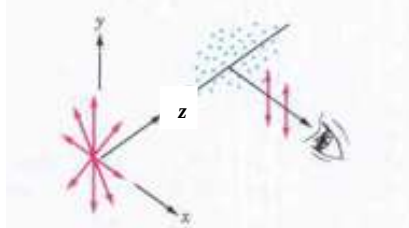


Figure 14

The sunlight is the primary source of light for our day vision. Even though we cannot see in sun direction by naked eye, we can see most of the objects due to the scattering of sunlight from their surface; even the blue sky vision is due to the sunlight scattered by air molecules. The light emitted by sun is unpolarized, but due to scattering , the light that falls on our eyes is partially polarized. Figure 13 shows a beam of unpolarized light propagating along Oz direction; as we know, all its  $\vec{E}$  vectors lie in Oxy plane. (why?)

We may present this light as composed by two  $E$  vectors directed along Ox and Oy. When this beam falls on the air molecules(blue dots in fig.14) they become sources of scattered light with  $\vec{E}$  vector in Oyz plane and

normal to the direction of its propagation. So, if one observes the scattered light parallel to Ox direction ( see fig.14), the scattered light will be polarized along Oy axe, because there was no Oz-component of incident unpolarized light and there is no Ox component of E-vector in a beam directed parallel to Ox. Note that in this schematics, the observed light is 100% polarized along Oy because the angles between the incident beam and the observed beam are 90°. But, in everyday situations, when we observe the scattered light coming from an object, there are many different values of this angle (not only 90°); this makes it **partially polarized light**.

## WAVE INTERFERENCE

- *The principle of linear superposition*: If the paths of two (or more) waves that propagate in the same medium cross to each other, then the "**displacement**" at **crossing point** is equal to **the sum of displacements** " $y_i$ " due to each wave **at this point** of medium.

$$Y_{\text{superposition}} = \sum_{i=1}^n y_i \quad i = 1, 2, \dots \quad (3)$$

This principle is **valid no matter what is the physical nature of "displacement"**.

- Remember: If two "vector waves" pass by the same point of a medium, there is **always a superposition** but **not always interference**. There is interference only if the two waves have "**displacements polarized**" along the **same space direction** (fig 6.a,b) or "**at least**" **not perpendicular** to each other.

This condition is not met always for TW waves. (Ex. Two crossed pulses on a rope; fig 6.c)

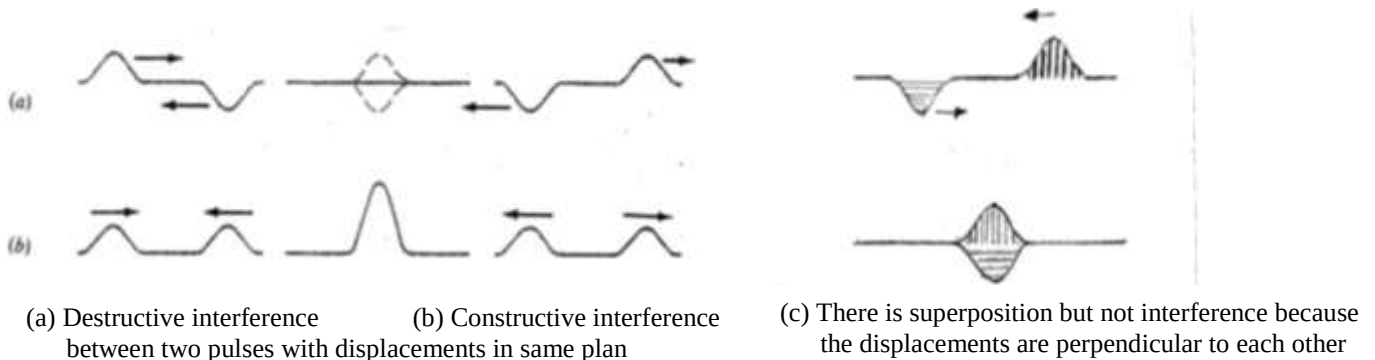


Fig. 6

**Important:** The interference can build a standing wave only if the two waves have the **same frequency**; i.e. same  $\omega$ ,  $k$  values. This requirement is **essential for a standing wave**. For observing a phenomena of **stable pattern** with **maxima** and **minima**, one should refer to a **standing wave** interference situation. So, in the following we will refer to the interference of two waves with the same  $\omega$ ,  $k$  ( $k = \omega / v$ ) values.

- Assume that a harmonic wave (TW, LW, mechanical or not) propagates through a medium. The **phase** of this travelling wave, i.e.  $\Phi(x,t) = kx \pm \omega t + \phi$ , can be calculated at any location " $x$ " and time " $t$ ", provided one has defined an axe Ox and knows  $k$  and  $\omega$ . The **wave front** is defined (in 2D or 3D) as the locus of space **points** (locations) in **medium** at which the wave function has the **same phase ( $\Phi$ - value) at any moment**. This means that, at a moment of time, the **displacement** is the same at all points on a given wave front, too. One shows a 2D travelling wave by sketching a set of **crests&troughs** (wave fronts at **maxima displacements**) at a given moment of time. For an harmonic wave, at any time, there is a phase change by  $\pm 2\pi$  between any two consecutive "**similar**" wave fronts and there is the **same displacement** on any of them. One says that the points on two consecutive crest (or trough) wave fronts "**oscillate in phase**" (or **are in phase**). One may easily show that the **distance** between any two **similar(ex. crests) consecutive** (i.e.  $\Phi_2 = \Phi_1 \pm 2\pi$ ) wave fronts

is equal to the **wavelength**. Just choose an axe Ox (as shown in figure 7a), use the expression of phase  $\Phi = kx - \omega t + \phi$  (+Ox direction of propagation) and find out that, at any given moment of time "t" :

$$\begin{aligned}\Phi_1 &= kx_1 - \omega t + \phi; \quad \Phi_2 = kx_2 - \omega t + \phi; \\ \Phi_2 - \Phi_1 &= k(x_2 - x_1) = \frac{2\pi}{\lambda} \Delta = 2\pi \rightarrow \Delta = \lambda\end{aligned}\quad (4)$$

Remember: Any **two similar** (say crests) **consecutive wave fronts** are at a distance  $\lambda$  to each other (fig.7a).

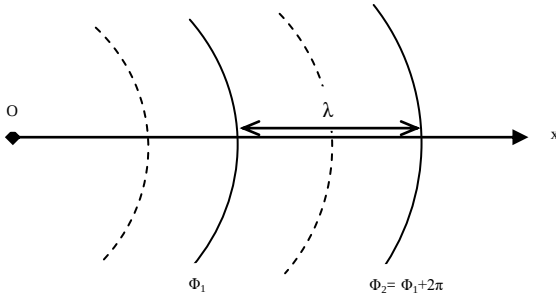


Fig. 7a

2D sketch of spherical wave fronts

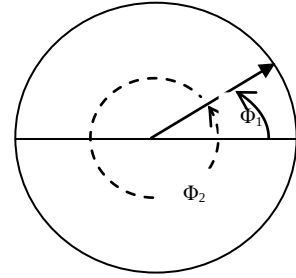


Fig. 7b

Phase shift  $2\pi$  between phasors of two consecutive wave fronts

## THE STATUS OF INTERFERENCE PRODUCED BY TWO INTERFERING HARMONIC WAVES

-The interference status at P-point is defined by **phase shift** between the **waves that interfere at this location**.

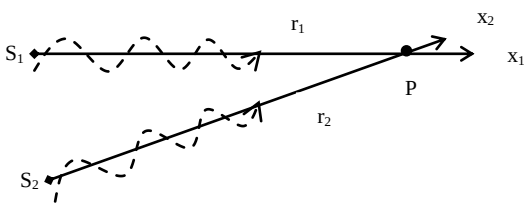


Figure 8

If this point P is at **distance  $r_1$**  from source  $S_1$  and  **$r_2$**  from source  $S_2$  (fig.8), the phases of the first and second wave at P location are  $\Phi_1 = kr_1 - \omega t + \phi_1$  and  $\Phi_2 = kr_2 - \omega t + \phi_2$ .

Then, the **phase difference** between the two waves at P point is  $\Delta\Phi = \Phi_2 - \Phi_1 = k(r_2 - r_1) + (\phi_2 - \phi_1) = \Delta\Phi_p + \Delta\Phi_s$  (5)

$\Delta\Phi_p = k(r_2 - r_1) = k\delta = (2\pi/\lambda)\delta$  phase shift due to path lengths difference  
 $\Delta\Phi_s = \phi_2 - \phi_1$  the phase shift between the two sources

As  $k = 2\pi/\lambda$  is a constant, the phase shift  $\Delta\Phi_p$  depends only on the paths **length difference  $\delta$**  between the two waves at P point.

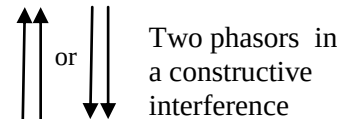
$$\delta = r_2 - r_1 \quad (6)$$

$r_1$  is the distance from the first source;  $r_2$  is the distance from the second source;

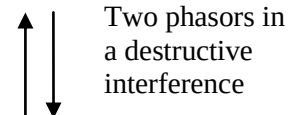
If there are reflections in the path of those two waves, there is a phase shift  $\Delta\Phi_r$  due to reflections and

$$\Delta\Phi_{\text{tot}} = \Delta\Phi_s + \Delta\Phi_r + \Delta\Phi_p \quad (7)$$

- There is a **constructive interference** if  $\Delta\Phi_{\text{tot}} = \pm M \cdot 2\pi$  ( $M=0,1,2,3,\dots$ )  
 In a constructive interference the two phasors are **parallel**, all time.



There is a **destructive interference** if  $\Delta\Phi_{\text{tot}} = \pm (2m+1) \cdot \pi$  ( $m=0,1,2,3,\dots$ )  
 In a destructive interference the two phasors are **inverse parallel**, all time.



-Assume now that both sources  $S_1, S_2$  oscillate *in phase* (i.e.  $\Delta\Phi_s = 0$ ) and there is *no wave reflections* ( $\Delta\Phi_r = 0$ ) till P point. In this case  $\Delta\Phi_{tot} = \Delta\Phi_p$  and one finds out that there is a **constructive interference** at P point if

$$\begin{aligned} \text{a) } \Delta\Phi_p &= 0 & \text{which happens for } \delta &= 0, \text{ i.e. } r_1 = r_2; \\ \text{b) or } \Delta\Phi_p &= \pm M * 2\pi & \text{which happens for } \delta &= r_2 - r_1 = \pm M\lambda \end{aligned} \quad (8.a)$$

There is **destructive interference** if :

$$\Delta\Phi_p = \pm (2m+1) * \pi \quad \text{which happens for } \delta = r_2 - r_1 = \pm (2m+1) \lambda/2 \quad (8.b)$$

**Remember:** At *locations* of **constructive interference** there is *oscillations at maximum amplitude of displacement* while at *locations* of **destructive interference** there is *zero amplitude of displacement* (for two travelling waves with equal amplitude).

**-IMPORTANT :** The derivation of conditions (7-8) is based only on **phase difference** considerations. So, they are valid for the interference **of any type of waves; sound waves, light waves, radio waves, TV waves...** These same rules explain *bad* or *good sound hearing* at different positions inside the same concert hall and also the low or high quality reception of radio /TV signals for the different positions of antenna in a room .

### OBSERVATION OF INTERFERENCE ON A TWO DIMENSIONAL SPACE (see [video 3](#))

The standing waves on a string are a **1D space** example. One can see a **2D space** standing wave interference on the still surface of a water tank(fig.9) by using two plungers that vibrate at the **same frequency** and **phase**.

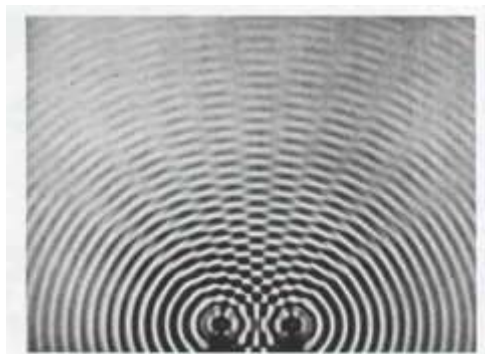


Fig. 9 Interference observed on water surface.

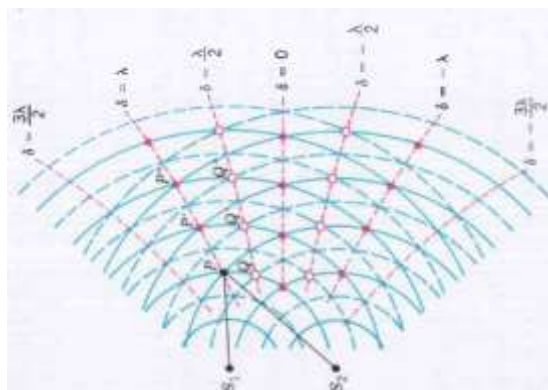


Fig. 10 Directions of constructive and destructive interference

- The **wave model** (fig.10) explains the structure produced on water surface by the interference of two TW mechanical waves originated at two point sources ( $S_1, S_2$ ) that oscillate *in phase* ( $\Delta\Phi_s = 0$ ) at **same frequency**, **same amplitude** and the same space direction of " **displacements** ". As there is *no reflections* ( $\Delta\Phi_r = 0$ ), the expressions 8.a,b apply. So, one can easily figure out that there is :

- a **CONSTRUCTIVE (double amplitude)** interference at any point where  $\delta = r_2 - r_1 = \pm M\lambda$  ( $M=1$  at P points) (a **crest** wave front meets another **crest** or a **trough** wave front meets another **trough** wave front).
- a **DESTRUCTIVE interference (zero amplitude)** at any point where  $\delta = r_2 - r_1 = \pm (2m+1) \lambda/2$  ( $m=0$  at Q points) (a **crest** wave front meets **trough** wave fronts or vice versa).
- Between the *central line of constructive interference* and closest *wing of constructive interference* there is a *line of destructive interference* at points (empty circles Q) where  $\delta = r_2 - r_1 = \pm \lambda/2$  or  $\Delta\Phi_p = +/\pi$ . One can show that those P&Q points are located on lines that have a hyperbola shape. So, based on the *phase shift* of the two interfering waves, one can explain the whole picture of observed interference fringes.

## YOUNG'S EXPERIMENT

- Thomas Young showed the wave nature of light by proving that it diffracts and interferes like any other wave phenomena. He used an experimental set up similar to that presented in figure 11. In this scheme:

- Two **sources** of "in phase" oscillations are two sources of "secondary wavelets"; i.e. the narrow slits  $S_1, S_2$  equidistant from the primary source (a pinhole receiving sun light).
- The "large distance" (compared to  $\lambda$  of light) from the pinhole gives **plane waves at slits input**.
- The two narrow slits  $S_1, S_2$  produce **cylindrical wave fronts** on the other side of their plane.
- The model of a screen "far from" the slits simplifies expression for **path length difference**.
- By using a coloured glass in front of pinhole one can get a **monochromatic light** (one  $\lambda$ ).  
*Nowadays, one gets easily the monochromatic light by using a laser source.*

Coloured glass

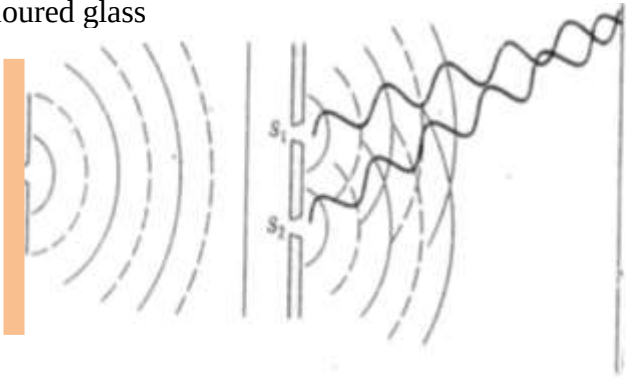


Fig. 11

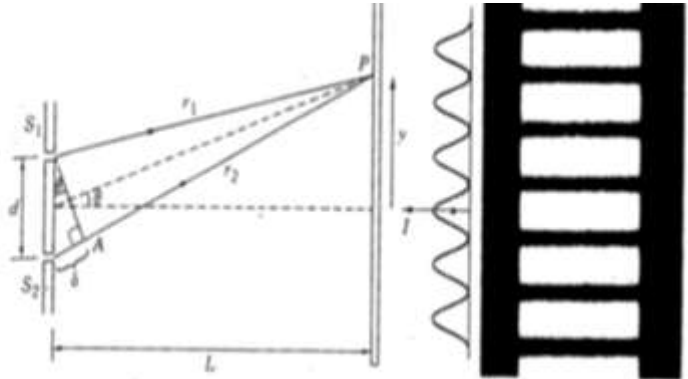


Fig. 12

-The **geometrical path length difference** for a given point P on the screen along direction " $\theta$ " (see fig 12) is  $\delta = r_2 - r_1$ . As  $L \gg d$ , for points near "screen center", the angle  $\theta$  is very small and the two paths are *almost parallel*. So, by referring to the drawing in fig.13, one gets

$$\delta \cong d \sin \theta \quad (9)$$



$d$ - distance between the two narrow slits;

$\theta$ - the angle that correspond to the location of point P on screen.

Figure 13

- After noting that, in this case  $\Delta\Phi_{S=0}$ ,  $\Delta\Phi_R = 0$  and  $\Delta\Phi_{tot} = \Delta\Phi_p$ , one might remember that there is:

constructive interference for  $\Delta\Phi_p = k\delta = \pm M * 2\pi \rightarrow \rightarrow \rightarrow \rightarrow \delta_M = \pm M * \lambda$

destructive interference for  $\Delta\Phi_p = k\delta = \pm(2m + 1) * \pi \rightarrow \rightarrow \delta_m = \pm(2m + 1) * \lambda/2$

So, one can derive the corresponding directions ( i.e. the corresponding **angles  $\theta$**  ) for **fringes** of a **maximum** intensity by the expression (10) and a **minimum** intensity by the expression (11)

$$d \sin \theta_M = M\lambda \quad M = 0, \pm 1, \pm 2.. \quad (10) \quad d \sin \theta_m = (2m + 1) \frac{\lambda}{2} \quad m = 0, \pm 1, \pm 2.. \quad (11)$$

**Notes:** a) For  $M = 0$  one gets the **central maximum** ( located in the middle of interference pattern).

b) The **first minimum** "upside ( $\theta > 0$ )" the central maximum corresponds to  **$m = 0$** .

-Those **interference fringes** are parallel to slits. The position "y" (see fig 12) of low order **maxima fringes** (located close to the center of interference pattern where  $\theta$  is small and  $\tan\theta \cong \sin\theta$ ) can be found as follows:

$$\frac{y_M}{L} = \tan \theta_M \cong \sin \theta_M \Rightarrow \Rightarrow \frac{y_M}{L} = M \frac{\lambda}{d} \Rightarrow \Rightarrow y_M = M \frac{\lambda}{d} L \quad (12)$$

- One uses widely the term “**coherence**” in **interference** studies. If the wave is a **pure harmonic function** (one frequency and infinite wave train) one says that it is a “**coherent wave**” emitted by a “**coherent source**”. Two **sources** (or two **waves**) that oscillate at the **same frequency** ( $\omega_1 = \omega_2$ ) are said to be **coherent between them** in the sense that there is a **constant phase shift** (not necessarily 0) between their **phasors all the time**

$$\Delta\Phi_{1,2} = [(\omega_2 t + \phi_2) - (\omega_1 t + \phi_1)] = \phi_2 - \phi_1. \quad (13)$$

Two **coherent sources** (or two **waves**) are said to oscillate **in phase** if  $\Delta\Phi_{1,2} = M*2\pi$  where  $M=0, \pm 1, \pm 2, \dots$ . In the Young's experiment,  $S_1, S_2$  are two **coherent sources** ( $\omega_1 = \omega_2$ ); they are **in phase** because  $\Delta\Phi_{1,2} = 0$ . If the phase shift is constant but  $\Delta\Phi_{1,2} \neq M*2\pi$ , two **coherent sources** are considered to be **out of phase**. Very often this nomination means two coherent sources in **opposite phase**;  $\Delta\Phi_{1,2} = (2m+1)\pi$  where  $m = 0, \pm 1, \pm 2, \dots$

- **Remember:** The phase shift due to difference in path lengths is  $\Delta\Phi_p = k\delta$ . If the two interfering waves propagate in vacuum (or air), the refractive index is  $n=1$  and  $k \equiv k_0 = 2\pi/\lambda_0$ . If the two waves propagate inside another transparent medium “ $n > 1$ ” and  $k \equiv k_n = 2\pi/\lambda_n = 2\pi/(\lambda_0/n) = n*(2\pi/\lambda_0) = n*k_0$

$$\text{So, inside a medium} \quad \Delta\Phi_p = k_n \delta = n k_0 \delta = k_0 n \delta = k_0 \delta_n \quad \text{where} \quad \delta_n = n \delta \quad (14)$$

$$\text{When two waves propagate inside the same transparent medium,} \\ \text{the optical path length difference } \delta_n \text{ is} \quad \delta_n = n * \delta \quad (15)$$

$$\text{When they propagate in two different mediums,} \quad \delta_n = n_2 * r_2 - n_1 * r_1 \quad (16)$$

$$\text{The general expression for the phase shift due to path length difference is} \quad \Delta\Phi_{path} = k_0 \delta_n \quad (17)$$

## THE INTENSITY OF INTERFERENCE PATTERNS

- In Young's experiment, one uses a small pinhole at "big distance" from two slits  $S_1, S_2$  to get a plane wave fronts at their input. Essentially, this produces **equal frequency, equal phase and equal amplitude for the two waves at sources** (slits  $S_1, S_2$ ). As the path lengths  $r_1, r_2$  have comparable values and the emitted powers " $P_0$ " are equal, at any point close to the "screen center", the light waves produce **similar intensity** ( $I \sim P_0/r^2$ ) which means **similar magnitude of displacement amplitude** ( $I \sim A^2$ ) which for light displacement is written as  $I \sim E_0^2$ .

- Therefore, one can express the **light wave "displacement E"** at a point in central area of observation screen, produced by the source  $S_1$  as

$$E_1(t) = E_0 \sin \omega t \quad (18)$$

produced by the source  $S_2$  as

$$E_2(t) = E_0 \sin(\omega t + \phi) \quad (19)$$

$E_0$  - same amplitude of **light wave displacement** on the screen (at distance "r" from source)

$\omega$  - coherent waves i.e. same frequency (first requirement for interference).

$\phi$  - ( $\Delta\phi_{p-1,2} \equiv \phi$ ) phase shift due to different lengths **between** their paths is **constant all time**.

The *phase shift*  $\phi$  at (19) is defined by the **position** of considered point on the screen and does not change in time because the two waves are **coherent**. Actually, as  $\phi = (2\pi/\lambda) * \delta = \Delta\phi_p$ , this *phase shift* is fixed by the *path length difference*  $\delta$  from two slits to that point (i.e. along direction  $\theta$  at expression "9").

-The principle of linear superposition gives the total "displacement  $E_T$ " at this point as

$$E_T = E_1 + E_2 = E_2 + E_1 = E_0 \sin(\omega t + \phi) + E_0 \sin(\omega t) = 2E_0 \cos \frac{\phi}{2} \sin(\omega t + \frac{\phi}{2}) \quad (20)$$

( by applying expression  $\sin \alpha + \sin \beta = 2\sin[(\alpha + \beta)/2] * \cos[(\alpha - \beta)/2]$  )

This is the expression of a standing wave with amplitude 
$$A_T = 2E_0 \cos \frac{\phi}{2} \quad (21)$$

- As the intensity  $I = \text{Power/Area} = \text{Energy}/(\text{Time*Area}) \sim \text{Energy} \sim A^2$ , the **evolution of intensity** for interfering **light** over different points on the screen is the same as **the evolution of  $A_T^2$  through these point**. So, ( by taking  $I = A^2$  for simplicity), it comes out that, if a single source sends a light wave at a point P on the screen, the *light intensity* at this point is  $I_0 = E_0^2$ . If the light waves from two coherent sources illuminate simultaneously this point, the *light intensity* at point P becomes

$$I = A_T^2 = 4E_0^2 \cos^2 \frac{\phi}{2} = 4I_0 \cos^2 \frac{\phi}{2} \quad (22)$$

Figure 14 presents the graph of this function for different locations on screen ( *different phase shifts*  $\phi = \Delta\phi_p$ ). As expected, its maxima and minima are placed along the space directions given by expressions (10,11).

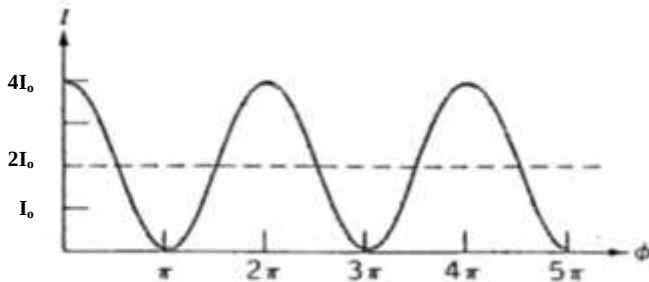


Figure 14

- a) **increases twice** " $2*(2 I_0) = 4*I_0$ " if the considered point is at a **maximum location**;
- b) is reduced to **zero** if the considered point is at a **minimum location**.

-If the two waves had different frequency ( $\omega_2 \neq \omega_1$ ) they would **superpose on the screen but** they would not produce standing waves. In this case the **intensity** of light at a considered point on the screen would be the simple sum  $I_0 + I_0 = 2I_0$ . Because of standing wave interference, the **light intensity**:

**Important Notes:** a) The **interference produces only energy redistribution** but the average (see fig.14) **intensity ( $2*I_0$ )** and **the total energy on the screen remain unchanged**.

b) One must use two waves with *similar amplitudes* to observe a well *contrasted pattern of interference*. If the amplitudes of two interfering waves are not equal, the interference fringes are not well seen. Even, the fringes may become invisible if  $E_{01} \gg E_{02}$  or vice versa. This happens because the *intensity of minima is not zero* and it may become comparable to that of maxima. The figure 15 explains what happens in these situation.

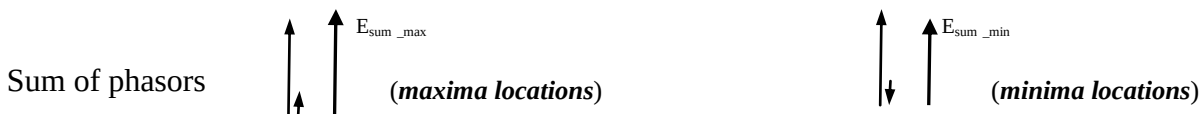


Figure 15 The amplitudes of resultant phasors are comparable; the intensities become comparable, too.

**Example:** A plane monochromatic wave with  $\lambda_0 = 600\text{nm}$  falls on two narrow slits and produces a set of maxima(bright) and minima(dark) fringes on a screen located on the other side of slits. Assume that one covers the upper slit ( as shown in figure ) by a plastic transparent strip with refractive index  $n = 1.5$ .

a) Show that all the interference pattern **shifts upward**.

b) Find the thickness " $I$ " of the strip such that the pattern shifts up exactly by one inter fringe distance.

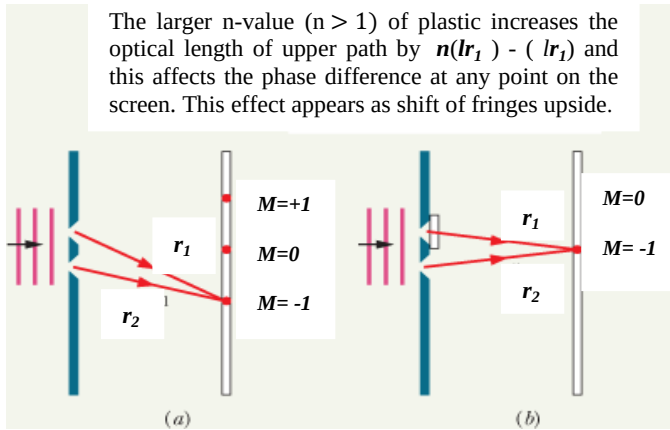


Figure 16

**a)** Without the plastic strip, the maximum of "order 0" on screen is located over a line passing through the midway of two slits. So, initially,  $\delta_{op} = 0$  and  $\Delta\phi = 0$  at this location; i.e. the maximum of order  $M=0$  is located at center (fig.16.a).

**In presence of the plastic strip on upper slit**, the optical path length of the first wave to the center " $r_1$ " increases. As the length of optical path for second wave to center " $r_2$ " does not change, it comes out that the introduction of plastic strip makes  $\delta'_{op} = r_2 - r_1'$  and  $\Delta\phi' = k \delta'_{op} < 0$  at center of patten; the location with  $\Delta\phi = 0$  is shifted upside the center (together with all the set of fringes)

**b)** To shift the whole pattern up by one inter fringe distance, the thickness " $I$ " of plastic strip should be such that the phase shift between wave<sub>2</sub> and wave<sub>1</sub> at center becomes exactly " $-2\pi$ " (or  $M = -1$ ).

In presence of the strip with thickness  $I$ , the path length of the first wave till center point changes from  $r_1$  to  $r_1' = (r_1 - I) + nI$  and the difference of optical lengths for the two waves at center ( $r_2$  does not change) becomes

$$\delta' = (r_2 - r_1') = [r_2 - ((r_1 - I) + nI)] = [r_2 - r_1 + I - nI] = [(r_2 - r_1) - (nI - I)]$$

$$\text{As, at the center } (r_2 - r_1) = 0, \text{ it comes out that } \delta' = (r_2 - r_1') = - (nI - I) = -I(n-1)$$

This means that the phase shift between waves at center of screen becomes

$$\Delta\phi' = (2\pi/\lambda_0) \delta' = - (2\pi/\lambda_0) * I(n-1)$$

As one should get  $\Delta\phi_1 = -2\pi$  at center, it comes out that  $-2\pi = - (2\pi/\lambda_0) * I(n-1)$  or  $(I/\lambda_0) * (n-1) = 1$

$$\text{So, } I = \lambda_0 / (n-1) = 600 / (1.5-1) = 600 / 0.5 = 1200\text{nm}$$

$$\text{and one gets } I = 1200\text{nm} = 1200 * 10^{-9}\text{m} = 1.2 * 10^{-6}\text{m} = 1.2\mu\text{m}$$