SUMMARY

1. Conditions for interference: a) ***same frequency*** (coherent waves);
b) same(or a common component) direction for "displacement" vectors;

Note: ***Equal or comparable*** amplitude values produce ***contrasted fringes***. The interference fringes are produced even for different amplitudes but in this case their **contrast** is **low** and they may be ***invisibles***.

2. The "*interference status*" at a given point in space is defined from the "**total phase shift**" at this point

$$\Delta\Phi_{tot} = \Delta\varphi_s + \Delta\varphi_r + \Delta\varphi_p$$

$\Delta\varphi_s$ is the phase shift ***between the sources*** of waves that interfere;
$\Delta\varphi_r$ is the phase shift due to possible reflections in the path of each of waves that interfere;
$\Delta\varphi_p$ is the phase shift due the "***optical paths length difference***" between the two waves at this point .

$\Delta\varphi_p = \frac{2\pi}{\lambda_0}\delta_{op}$ where $\delta_{opt} = n_2 r_2 - n_1 r_1$; if $n_1 = n_2 \equiv n$ then $\delta_{opt} = n(r_2 - r_1) = n\delta$

λ_0 is the wavelength in vacuum (or air);
n is the refractive index of the medium where the "*optical paths length* ***difference***" is build up;
r_2 is the "*length of light path*" from source 2 to the considered point;
r_1 is the "*length of light path*" from source 1 to the considered point.

There is an:
interference maximum if $\Delta\Phi_{tot} = M*2\pi$ M= 0, +/-1, +/-2, +/-3… M-***order*** of maximum
interference minimum if $\Delta\Phi_{tot} = (2m+1)*\pi$ m= 0, +/-1, +/-2, +/-3… m-***order*** of minimum

3. The *diffraction becomes visible* if the *dimension of slit is "comparable" to λ of light wave*.

4. At Young's experiment, the fringes are placed along direction (angles) given by expressions

for ***maxima*** $\sin\theta_{max} = M\frac{\lambda}{d}$; for ***minima*** $\sin\theta_{min} = (2m+1)\frac{\lambda}{2d}$ M, m = 0,±1,±2,..

Note: The production of fringes at ***Young's experiment proves the wave nature of light***.
Also, ***the light waves are TW waves*** (*we accept this without going into details in this course*).

5. The simple expression for light intensity $I = A^2$ is good enough to get out conclusions for *relative values* of light intensity in an interference pattern. For light waves, $A = E_0$ means that the *amplitude* of the wave " ***displacement*** " is the *magnitude* of the ***electric field vector*** $\vec{E}$ of E.M. wave associated to light. Therefore the maximum of light wave intensity is $I_0 = E_0^2$ and the relative intensity for Young's experiment fringes comes out as

$$I = 4I_0\cos^2(\phi/2) \Rightarrow\Rightarrow \frac{I}{I_0} = 4\cos^2(\phi/2) \qquad \text{where } \phi = \Delta\phi_{tot}$$

Remember that, if $\Delta\varphi_s = 0$ and $\Delta\varphi_r = 0$, the interference of two light waves produces :

- a maximum for $\delta_{op} = M\lambda_0$_because_$\Delta\phi_{tot} = \Delta\varphi_p = \frac{2\pi}{\lambda_0}\delta_{op} = \frac{2\pi}{\lambda_0}M\lambda_0 = M*2\pi$
- a minimum for $\delta_{op} = (2m+1)\frac{\lambda_0}{2}$_because_$\Delta\phi_{tot} = \Delta\varphi_p = \frac{2\pi}{\lambda_0}\delta_{op} = \frac{2\pi}{\lambda_0}(2m+1)\lambda_0/2 = (2m+1)\pi$

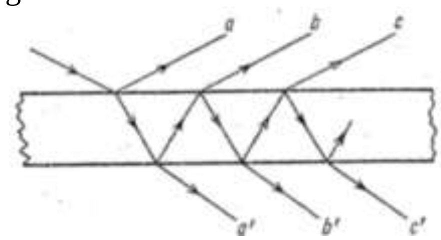
THIN FILMS

- Often, one may see coloured fringes on the surface of thin transparent materials like soap bubbles, oil patches on water or on road surfaces in summer. These fringes are seen in the reflected light and are due to the interference between the reflected rays at the two interfaces that limit a transparent thin film.

- Before starting with the modelling of these phenomena *it is important to mention that when the light wave falls* ***from*** *a medium with* ***smaller*** *refraction index "****n****"* ***onto*** *a medium with a* ***larger*** *index "****n****" , a* ***phase shift by π***[1] *is added to the displacement phasor of* ***reflected light;*** *there is* ***no phase change*** *of phasor when the reflected light falls* ***from*** *a medium with* ***larger*** *"****n****" onto a medium with* ***lower*** *"****n****"* index.

- Now, let's consider a homogeneous and uniform thickness transparent film limited by two media with *smaller "****n****" on both sides* (like soap bulb in air, oil film on water,..). When a light beam falls on the upper interface of transparent thin film, it generates a series of reflected and refracted light beams (see fig.1).

Figure 1



- The ***reflection coefficient*** (defined as $R = I_{reflected} / I_{incident}$) of non silvered surfaces is very low. A quick calculation based on R = 0.05 gives the values of table No.1 for the intensities of beams, a, b, c, a', b', c'. ($\mathbf{I_{reflected} = R*I_{incident}}$; $\mathbf{I_{refracted}} = I_{incident} - I_{reflected} = I_{incident} - R*I_{incident} = \mathbf{(1-R)*I_{incident}}$; $I_{incident} \equiv$ Io)

Table No1

$I_{reflected}$	***Ia = 0.05Io***	***Ib = 0.045Io***	Ic = 0.001Io
$I_{refracted}$	Ia' = 0.9Io	Ib' = 0.002Io	Ic' = 0.0000001Io

Remember: two interfering waves produce a ***visible interference pattern*** only if their ***amplitudes*** are ***comparable*** (i.e. comparable intensities). This basic criterion and the data in table 1, show that ***only the interference between rays "a and b" may produce observable fringes.*** So, in the following one considers ***only*** the ***interference*** produced by the ***two waves reflected*** along ***"a*** *and* ***b"*** paths (i.e. two first reflections).

- As they are "*emitted* " by the same source ($\Delta\phi_S = 0$), the interference status of those waves is defined from the phase shifts due to reflections ($\Delta\phi_R$) and the difference in length of their optical paths ($\Delta\phi_p$).
Remember: *If the* ***geometrical path length*** *of a ray inside the medium with refraction index "****n****" is "****r****", its* ***optical path length*** *is "****n*r****"* **and** if two rays pass through the same medium with refraction index "***n***", the difference of their *geometrical paths length* "$\boldsymbol{\delta = r_2 - r_1}$ " produces the difference of optical path lengths

$$\boldsymbol{\delta_{opt.} = n*\delta} .$$

When calculating the phase difference in a medium with n ≠1 one must refer to $\boldsymbol{\delta_{opt}}$ (*not to δ*).

From the fig. 2 one may see that the ***optical path length difference*** between two first reflected waves is

$$\boldsymbol{\delta_{opt.}} = n(AB + BC) - AD \qquad (1)$$

Assuming that the first and third medium is air ($\boldsymbol{n_1, n_3 = 1}$) and the incidence angle is $\boldsymbol{\theta_1}$, from Snell's law

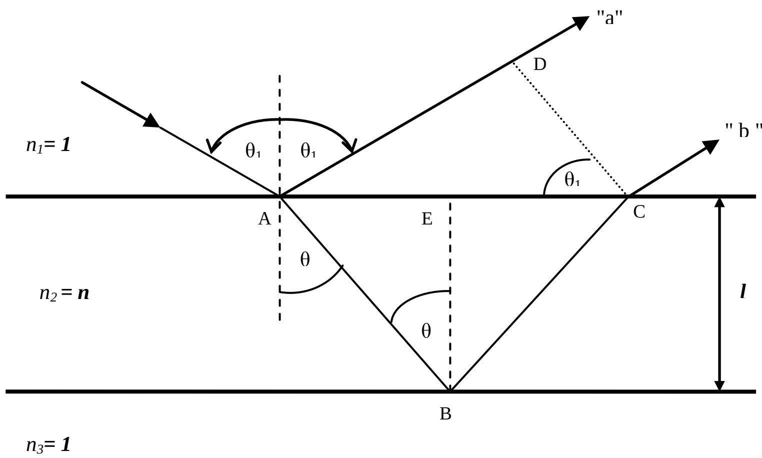
[1] We will take a positive shift **+ π**.

$$\sin \theta_1 = n^* \sin \theta \qquad (2)$$

Then, $AE = AB \sin \theta$; $AC = 2AE = 2AB \sin \theta$ (3)

and $AD = AC \sin \theta_1 = 2AB \sin \theta \sin \theta_1 = 2nAB \sin^2\theta$ (4)

Figure 2



As AB = BC and, inside thin film, the total optical path length of wave in direction ray " *b* " is

$$n(AB + BC) = 2nAB \qquad (5)$$

one finds out that the **optical path difference** between the waves along "a" and "b" rays is

$$\delta_{opt} = n(AB + BC) - AD = 2nAB - 2nAB \sin^2\theta = 2nAB(1 - \sin^2\theta) = 2nAB\cos^2\theta \qquad (6)$$

From figure 2 is easily seen that $\cos\theta = l / AB$ or $AB = l / \cos\theta$ (7)

By substituting AB as $(l / \cos\theta)$ at relation (6) one gets $\boldsymbol{\delta opt = 2n^*l^*\cos\theta}$ (8)

- The corresponding **phase shift** (advance of "b-phasor") due to this ***optical path length difference*** is

$$\Delta\varphi_p = \frac{2\pi}{\lambda_0}\delta_{op} = \frac{2\pi}{\lambda_0}(2nlcos\theta) \qquad (9)$$

By adding the phase change due to reflections at the two interfaces $\Delta\varphi_R = (0 - \pi) = -\pi$

one finds out that $\Delta\varphi_{tot} = \Delta\varphi_p + \Delta\varphi_R = \frac{2\pi}{\lambda_0}(2nlcos\theta) - \pi$ (10)

Note that beyond the line DC the phase difference between the two waves remains unchanged.

Important remark: ***One might figure out that the interference produced by those two waves is seen only if they pass both through the eye iris (diameter 3-5mm).*** This means that the distance DC must be smaller than 5mm and smaller it is, easier the interference pattern is observed. From fig.2 is clear that DC is smaller for incident ***rays close to the normal ($\theta_1 \approx 0$)***. In these circumstances, as $\sin\theta_1 \approx \theta_1$ from the relation (2) one can get $\theta \approx \theta_1/n \approx 0$ which brings to $\cos\theta \approx 1$ at expression (8). This way, the length of ***optical path difference*** for observation close to the normal to thin film surface is

$$\boldsymbol{\delta_{opt} = 2n^*l} \qquad (11)$$

and the **total phase difference** is $\Delta\varphi_{tot} = \frac{2\pi}{\lambda_0}\delta_{op} - \pi = \frac{2\pi}{\lambda_0}(2nl) - \pi$ (12)

-Also, from fig.2, one may see that the DC decreases as film thickness *"l"* decreases. As DC must be small, the interference is seen only for ***small film thickness "l"***. If the films thickness is *not small enough*, one may use optical systems to make the "*two first reflected* waves *a and b*" fall inside the iris.

-Let's consider now that a monochromatic light (***one*** $\boldsymbol{\lambda_0}$) falls *from air* on this *thin film* and one observes the reflected light close to the perpendicular ($\boldsymbol{\theta \sim 0}$) to films' surface. One will see a ***maximum*** if

$$\Delta\varphi_{tot} = \frac{2\pi}{\lambda_0}\delta_{op} - \pi = \frac{2\pi}{\lambda_0}(2nl) - \pi = M * 2\pi\_or\_ \frac{2\pi}{\lambda_0}\delta_{op} = (2M+1)\pi \dots .i.e.\delta_{op} = (2M+1)\lambda_0/2$$

So, if the used λ_0 fulfills the condition $\delta_{op} = 2nl = (2M+1)\lambda_0/2$ (13)
one will see a bright light because a **maximum** is produced in the reflected light.

If the incident λ_0 fulfills the condition

$$\Delta\varphi_{tot} = \frac{2\pi}{\lambda_0}\delta_{op} - \pi = (2m+1)\pi\_\_or\_\_\frac{2\pi}{\lambda_0}\delta_{op} = 2m\pi + 2\pi = 2(m+1)\pi = 2m'\pi;\ m' = m+1$$

i.e. $\frac{2\pi}{\lambda_0}\delta_{op} = 2m'\pi\_\_\delta_{op} = 2nl = m'\lambda_0$ or $2nl = m'\lambda_0$ m' = 1,2,3 (14)

one cannot see light because there is a **minimum** in reflection. So, for a given λ_0 value, depending on its thickness, a ***transparent thin film in air*** produces **in reflected light**

a) a maximum if $2nl = (2M+1)\lambda_0/2$ M = 0, 1,2,3.. (13)
b) a minimum if $2nl = m\lambda_0$ m = 1,2,3,… (14)

-Assume now that one illuminates such a thin film by a white light with equal intensity(simplified model) for each color and observes reflected light perpendicularly to its surface. If the condition (13) is fulfilled for several colours, one will observe an increase of their intensity in reflected light while several other colours will be missing in reflected light because their *λ-values* fulfill the condition (14). As a result, the ***relative intensities*** of different colours are modified in reflected light and one will see coloured light in reflection (*ex; from oil patches on the road*) even though the film is illuminated by white light(*sun light*). *Actually, the colours seen are the complementary ones of the missing colours in reflected light.*

-Here it´s worth to mention that the complementary colour of a ***primary colour*** (***red, green, blue***) is the colour one get by mixing the two remaining others.

Missing	***Mixture***	***Seen Colour*** (complementary)
Red	blue + green	cyan
Green	red + blue	purple
Blue	red +green	yellow

LENS COATING

-The reflection coefficient for normal incidence of light on the interface between two mediums is

$$R_{\perp} = \left(\frac{(n_2/n_1)-1}{(n_2/n_1)+1}\right)^2 \qquad (15)$$

n_1, n_2 are the refraction indexes for the first and second medium.
For normal incidence of light from air ($\boldsymbol{n_1=1}$) on a glass with $\boldsymbol{n_2=1.5}$ or vice-versa, one finds $R_{\perp} = 0.04$.

-When designing an optical system, one has to maximize output light because this affects the sensitivity of the whole system. The following example shows that a considerable amount of light collected at input can be lost due to reflections onto the surfaces of lenses that constitute the optical system.
Ex: *A high quality optical system with 5 lenses has 10 " reflecting" interfaces. Only 66% of the light collected at camera input [$I_{trans} = (1-0.04)^{10} I_0 = 0.96^{10} I_0 = 0.66 I_0$] falls on the light detector(at output).*

-One can minimize the reflection effect by coating the surface of glass lenses with such thin films that build up ***destructive interference*** <u>on the **reflected light**</u>. Here, one might remember that a destructive interference for a wavelength in the reflected light is associated with a maximum for this wavelength in transmitted light because the <u>***interference only redistributes the light***</u> .

-This effect requests ***equal or comparable*** values of ***intensity*** for the ***two first reflected rays*** (*a*, *b*) and one may find out that this condition is met only if the refraction index " n_{film} " of thin film fulfils the following relation (*Homework- Hint: Use (15) ,* $I_{reflected} = R*I_{incident}$ *;* $I_{refracted} = (1-R)*I_{incident}$ *and look for equal* I_a *and* I_b)

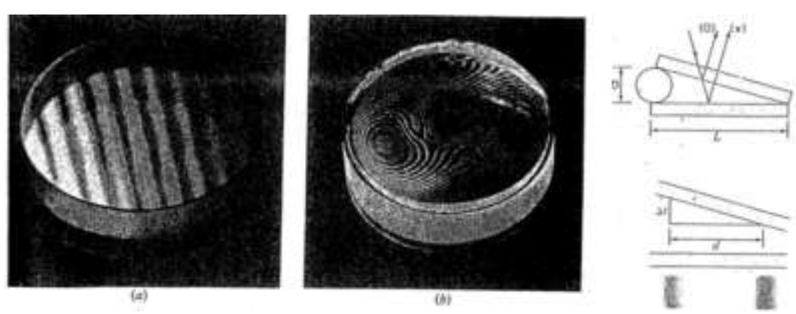
$$n_{Film} = \sqrt{n_{air} * n_{glass}} = \sqrt{1.5} = 1.225 \qquad (16)$$

In general practice, the lens designers use MgF_2 (***n = 1.38 at 550nm***) because of its high durability that is another important requirement in relation to thin film coatings. One The selects the ***coatings thickness*** so that an ***interference minimum*** in reflection corresponds to the middle of visible spectrum (λ ~550nm). When a white light falls upon such a thin film, the green colour (λ = 550nm) will be ***missing in reflected light*** and the complementary colour appears on the whole lens surface in reflection. The purple colour (*green missing*) of cameras' lens is due to this effect. (Homework, Benson Ex. 37.3)

<u>FRINGES OF EQUAL THICKNESS</u>

-When the white light falls on a homogeneous and <u>*uniform thickness*</u> film, the condition for ***minima in reflected light*** depends only on λ-value (*it produces the same apparent colour on all its surface*). But, if the film *thickness* is *not uniform*, the apparent colour of reflected light will depend on thickness, too. At positions with <u>*thickness "* t_1*"*</u> the minima in reflection corresponds to wavelength λ_1 and one sees a fringe of apparent colour "<u>*complementary_1*</u>" while at a location with <u>*thickness "* t_2*"*</u> where the minima in reflection corresponds to wavelength λ_2 , one sees one fringe of apparent colour "<u>*complementary_2"*</u> and so on. This explains the ***multiple colours*** seen on different locations over ***soap*** and ***oil thin films***.

Fig.3

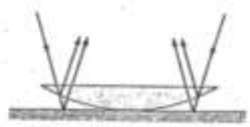


-One uses thin films interference to *control the flatness of transparent plates*. At first, one builds up a *wedge-shaped air film* between the two plates by inserting a fine wire at one end. Then, one illuminates them by <u>*a monochromatic*</u> wide beam of light (ex. Na lamp beam with λ≈589nm.). If the surface of plates is flat, a clear system of fringes (fig 3.a) parallel to the wire direction appears in reflected light. One might figure out that the condition for a given maximum (or minimum) in reflected light is fulfilled at the points over the same thickness of air film and these points are aligned parallel to the wire direction. If one of surfaces is not flat, the thickness of (*air*) thin film changes in an irregular way and fringes are not any more parallels to the wire direction(fig 3.b). By using this criterion and a standard flat plate, one may control the flatness of other plates during a production process.(Homework, Benson Ex. 37.4)

Note: When solving thin film problems, one must *pay attention to phase change due to reflections at interfaces. This parameter depends on the values of refraction coefficients on both sides of interface.*

NEWTONS' RINGS

- A particular case of ***air thin films*** with ***variation of thickness*** is built by placing a plane – convex lens over a flat glass plate. In this situation, the equal thickness of thin film corresponds to the points on a circle centered to the contact point. When illuminating this system by a monochromatic light, one may see (*by using a low magnifying power microscope*) in reflected light a set of circular bright/dark fringes and a **dark spot** at their **center** (fig4). Newton, who was the first to observe these fringes, tried for a long time to explain the origin of the central dark spot but he was unsuccessful.



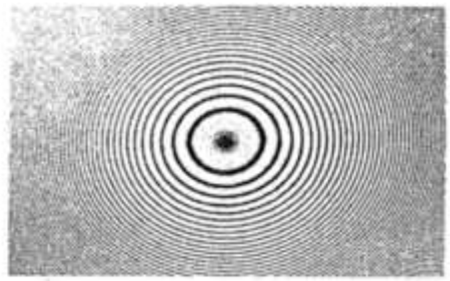


Fig 4

- Thomas Young used this experiment to ***prove*** that there is a ***phase shift by π*** during reflection of a light wave from a medium with lower "***n***" index onto a medium with higher "***n***" index.
He assumed the ***presence*** of an ***infinitely thin (δ~0) film of air*** between surfaces "*in contact at center point* " such that $\Delta\varphi_p \sim 0$ and the total phase shift *between two waves that interfere in reflection* is due only to $\Delta\varphi_r = \pi$ related to reflection ***from air on glass at the lower interface***. With this assumption, he explained the presence of a destructive interference(*black point*) in the reflected light at central position. To confirm this explanation he modified the set up by:

1. Using two *different glasses* for the lens and flat plate such that $n_{lens} < n_{plate}$;
2. *Substituting* the air film between them by oil with refraction coefficient $n_{lens} < n_{oil} < n_{plate}$;

This combination adds a π-phase shift to the wave reflected on the upper boundary of oil thin film, too. Now, as the total phase difference due to the reflections between the two waves becomes $\Delta\varphi_r = 0$ (π-π), a bright central spot should appear in center. Young showed that this does happen in experiment.

MICHELSON INTERFEROMETER

-Michelson interferometer is an optical device used for high precision optical measurements. Its function is based on thin films interference, too. The figure 5.a presents its principal scheme.

-A wide beam of ***monochromatic*** light falls on the **beam splitter C** (glass plate coated by a thin film of silver over its right side). Half of incident light intensity (**wave 1**) passes through lightly silvered surface of **C**, through the compensator plate **D.** After reflection from mirror M_1, it passes through **D** plate and goes versus the observer after another reflection at the silvered surface of **C**. The other half of incident light wave intensity is reflected (**wave 2**) from silvered surface at point **P** and further reflected back from mirror M_2 ; next, it goes versus the observer after passing through **C** plate. The compensator **D** has the same thickness as **C** and ensures that waves 1 and 2 pass through the same path lengths inside glass. M_2 is a movable mirror.

-One starts by fixing $L_1 = L_2$ and mirrors M_1, M_2 perpendicular to "*rays*" 1, 2. With this geometry, the *virtual image* of mirror M_1 (formed by reflection on C) superposes to mirror M_2. As $\Delta\varphi_s = 0$, $\Delta\varphi_p = 0$ and $\Delta\varphi_R = (2\pi - 2\pi) = 0$ there is a maximum in reflection. The observer sees a bright field (*in the telescope*).

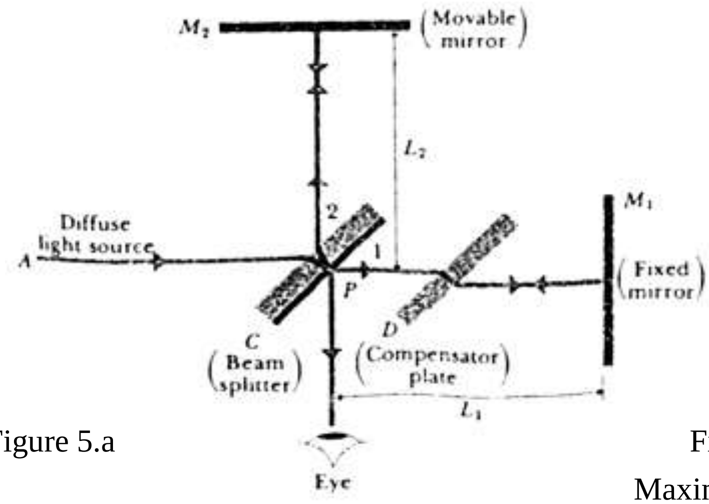


Figure 5.a

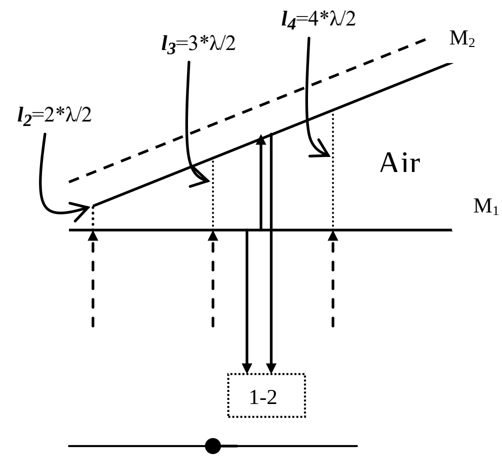


Figure 5.b

Maxima of order $M = 2$ $M = 3$ $M = 4$

- Next, by a small rotation of $\mathbf{M_2}$ around its center (*to the left at* fig 5.b), one creates a "*wedge-shaped thin film of air* " between M_2 and M_1 ***image***. This geometry builds a set of *equal thickness fringes* of interference due to the waves 1, 2 that reflect on mirrors $\mathbf{M_1}$ and $\mathbf{M_2}$. A set of alternating dark-bright fringes appears; a bright fringe will show up at the center of telescope because there is no change of path length difference there. The *path length difference* for a given fringe *is* $\delta_{1-2} = 2(L_2 - L_1) = 2\Delta L$

So, one may calculate the corresponding phase shift as $\Delta\Phi_{tot} = \Delta\varphi_p = \frac{2\pi}{\lambda}\delta_{1-2} = \frac{2\pi}{\lambda}2\Delta L$ (17)

The condition for maxima of interference $\Delta\varphi_{tot} = M * 2\pi$ allows to find the **air thickness** "$l = \Delta L$"

under a bright fringe of order M as follows: $\frac{2\pi}{\lambda}2\Delta L = M * 2\pi$ and $\Delta L \equiv l_M = M * \lambda/2$ (18)

Similarly, one finds out that the **air thickness** under a dark fringe is $\Delta L \equiv l_m = (2m+1) * \lambda/4$ (19)
Expression (18) shows that when shifting *from a bright* fringe *to the next bright* fringe, the thickness of "air wedge" changes by " λ/2 ". As there is a dark fringe between any two consecutive bright fringes, it comes out that there is a change of air thickness by " λ/4 " (see 19) when shifting from a bright fringe to the next dark fringe.

When working with a Michelson interferometer, one uses:

a) An extended monochromatic source of light to light up a good portion of mirror surfaces.
b) A low power telescope to observe several (5-6) fringes simultaneously in view field of eyepiece.

- The operator starts measurements by placing a bright fringe (ex. *max of order "0")* on the crosshair at eyepiece center. Next, one "*moves up*" the mirror $\mathbf{M_2}$. If $\mathbf{M_2}$ is shifted up by "$y = \lambda/4$", L_2 and ΔL are increased *simultaneously* by $\lambda/4$ for "***any spot on air wedge***"; the ***bright*** fringe on crosshair (M = 0) gets substituted by the next ***dark*** fringe(m=0). ***So, the minimum length (****equal to shift* y_{min} ***) measured by this device is*** $\lambda/4$ ***(Ex. if*** $\lambda = 500nm$ $y_{min} = 500/4 = 125nm$***).*** If one follows with another shift by $\lambda/4$, i.e. a total shift $y = \lambda/2$, the next order (ex.*"M=1"*) bright fringe will be placed on the crosshair of the eyepiece.

During a measurement procedure, one ***counts*** the number "***nb***" of ***dark and bright fringes*** that ***pass*** through the ***crosshair*** while the mirror M_2 "*moves up*" along the *length of object* to be measured. As for ***nb = 1*** there is a displacement of mirror M_2 by $y = \lambda/4$, then for a number "***nb***" of passing fringes

(*dark&bright*) the ***displacement*** of M_2 is $$y = nb * \frac{\lambda}{4}$$ (20)

This expression becomes $y = nb * \lambda/2$ if one counts only the number of bright (*or only dark*) fringes. One uses the Michelson interferometer for precise ***measurements*** of ***wavelengths*** and "n" index, too.