THE AETHER ISSUE

-The function of an harmonic travelling wave (*no matter what is the physics parameter "y"*) that propagates along positive direction of Ox axis depends on two variables (***x, t***) and has the form:

$$y(x,t) = A\sin(kx - \omega t + \varphi) \qquad (1)$$

One may take the *partial derivatives* of this function and combine them as follows:

$$\frac{\partial y}{\partial x} = k * A\cos(kx - \omega t + \varphi)\,; \qquad \frac{\partial^2 y}{\partial x^2} = -k^2 * A\sin(kx - \omega t + \varphi)\,; \qquad \text{So,} \quad \frac{\partial^2 y}{\partial x^2} = -k^2 y \qquad (2)$$

$$\frac{\partial y}{\partial t} = -\omega * A\cos(kx - \omega t + \varphi)\,; \qquad \frac{\partial^2 y}{\partial t^2} = -\omega^2 * A\sin(kx - \omega t + \varphi)\,; \qquad \text{So,} \quad \frac{\partial^2 y}{\partial t^2} = -\omega^2 y \qquad (3)$$

By isolating $y = -\frac{1}{\omega^2}\frac{\partial^2 y}{\partial t^2}$ from (3) and substituting it at (2), one gets $\frac{\partial^2 y}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2 y}{\partial t^2}$

Next, by substituting $\frac{k}{\omega} = \frac{1}{\upsilon}$ this relation transforms to the **wave equation** $\frac{\partial^2 y}{\partial x^2} = \frac{1}{\upsilon^2}\frac{\partial^2 y}{\partial t^2}$ (4)

-The equation (4) relates the second derivative of "**displacement**" in *space domain* to the second derivative of "**displacement**" in *time domain*. It is **valid for all types of waves**; i.e. it does not depend on the physical nature of parameter "*y*" which "*displacement*" propagates as a wave phenomena.
The parameter "***υ***" stands for the ***propagation speed*** of the considered ***wave*** in a physical medium. Remember that the *speed* of propagation of a wave *depends on medium characteristics*.
(Example: For a mechanical wave travelling along a string the speed is $\upsilon = \sqrt{T/\mu}$).

-By the end of 19th century, the scientists had proven the ***wave model of light*** (*via diffraction and interference experiments),* had identified it as a ***transverse wave*** (*via polarization effects*) and classified it as part of *electromagnetic spectrum.* Like all wave phenomena, the *light waves* obey to the equation (4). One refers to ***electric field*** vector as propagating "***displacement"(i.e. y = E)*** for the *electromagnetic waves* and after noting their ***speed in vacuum*** as *"**c**"* writes the wave equation in form

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \qquad (5)$$

This wave equation describes the propagation of light waves, too. James Maxwell assumed that there is a ***medium that fills the whole universe*** (*even the "empty space" between the stars*) which ***propagates*** the light waves or any other E&M wave. He found a theoretical relation between the speed *"**c**"* and the parameters of this medium, too. The physicists called that medium *"**aether**"* and, for a while, there was an intensive research activity (*theoretical and experimental*) related with the definition of its properties.

-The scientific community welcomed the *aether* because it would be "*the absolute reference frame for all physics' laws*"; any other reference frame would be at rest or in relative motion with respect to this medium. The theoretical study of its model predicted that the *aether* should *penetrate all matter object*, be *weightless,* be *extremely elastic,... i.e.* some strange characteristics. Anyway, to accept its existence the physics had to prove it experimentally. *H**ow to prove experimentally the "aether" existence?***

THE EXPERIMENT MICHELSON-MORLEY

-This experiment remained in the history as a major case which demonstrates that *no theoretical development is accepted without experimental proof in physics*. To verify if *aether exist*, A. Michelson and E. Morley assumed that : **as the earth moves around the sun at a speed $\upsilon \approx 30Km/s$, it should be moving with respect to the aether**(i.e. *a frame tied to it*), ***at least*** at ***30Km/s***. They used the Michelson interferometer (*which can measure very small changes of $\Delta\varphi_p$*) to check the existence of this motion.

- One starts by aligning the arm PM_1 (fig.1) along the direction of earth motion around the sun and one assumes that the interferometer slides through the *still aether* at υ = ***30Km/s*** along the direction PM_1. Two light wavelets "born initially" at beam splitter P travel *through aether* towards the mirrors M_1, M_2 *at same speed **c***. As explained at Michelson interferometer section, the *total phase shift $\Delta\varphi_{tot}$* between those wavelets, when they interfere at telescope eyepiece, depends only on their *path length difference*.

Along arm PM_1;When the light wavelet arrives at M_1 (*time t_{for}*), this mirror has advanced by $\upsilon * t_{for}$ in *aether* frame with respect to its initial position (*length $M_{1...}M_1$ in fig.1.a*). So, the length of this path is:

$$c*t_{for} = L_0 + \upsilon t_{for} \rightarrow t_{for}(c-\upsilon) = L_0 \_and\_ t_{for} = L_0/(c-\upsilon) \tag{6}$$

The *length* of wavelet *path* when returning from M_1 to P is shorter " $L_0 - \upsilon * t_{back}$ ", (see 1.b)**.** So, one gets

$$c*t_{back} = L_0 - \upsilon t_{back} \rightarrow t_{back}(c+\upsilon) = L_0 \_and\_ t_{back} = L_0/(c+\upsilon) \tag{7}$$

Therefore, the total travelling time spent by this wavelet along the arm PM_1 is

$$t_1 = t_{for} + t_{back} = \frac{L_0}{c-\upsilon} + \frac{L_0}{c+\upsilon} = \frac{L_0(c+\upsilon) + L_0(c-\upsilon)}{c^2-\upsilon^2} = \frac{2L_0 c}{c^2(1-\upsilon^2/c^2)} = \frac{2*L_0/c}{1-\upsilon^2/c^2} \tag{8}$$

and the corresponding total (*forward-backward*) travelled path length "S_{M1}" is

$$S_{M1} = c * t_1 = \frac{2L_o}{1-v^2/c^2} \tag{9}$$

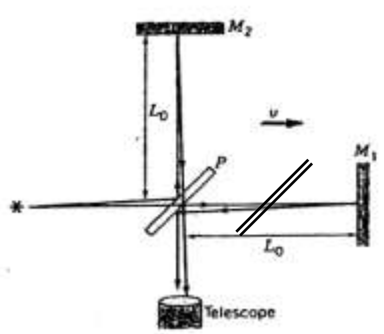
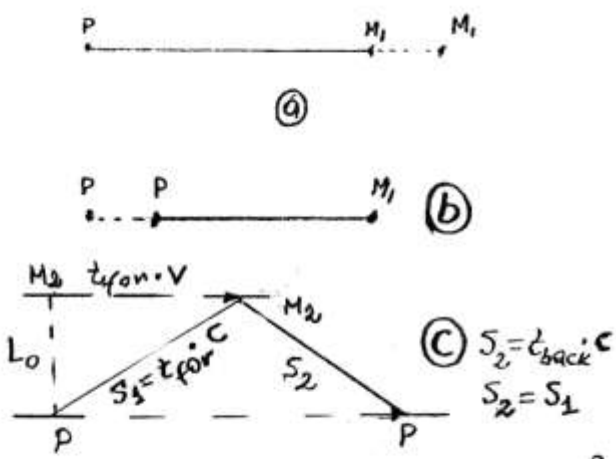


Fig.1



- For the arm PM_2; When the reflected wavelet arrives at M_2 (*after time t_{for}*), this mirror has advanced by $\upsilon * t_{for}$ to the right (see fig 1.c). So, for the forward path traveling time, one gets

$$(c*t_{for})^2 = L_0^2 + (\upsilon t_{for})^2 \rightarrow t_{for}^2(c^2-\upsilon^2) = L_0^2\_and\_t_{for} = \frac{L_0}{\sqrt{c^2-\upsilon^2}} = \frac{L_0/c}{\sqrt{1-\upsilon^2/c^2}}$$

When returning from M_2 to P, the wavelet travels the same path length (see fig.1.c). So, its total travelling time is $t_2 = t_{for} + t_{back} = \frac{2*L_0/c}{\sqrt{1-\upsilon^2/c^2}} = \frac{2*L_0/c}{(1-\upsilon^2/c^2)^{1/2}}$ (10)

and the corresponding total travelled path length "S_{M2}" is $S_{M2} = c*t_2 = \frac{2L_o}{(1-v^2/c^2)^{1/2}}$ (11)

-When returning at beam splitter P (*and till eyepiece*) the wavelets would have a ***path length difference***

$$\delta = S_{M1} - S_{M2} = \frac{2L_o}{1-v^2/c^2} - \frac{2L_o}{(1-v^2/c^2)^{\frac{1}{2}}} = 2L_0\left[\frac{1-(1-v^2/c^2)^{\frac{1}{2}}}{1-v^2/c^2}\right] \cong$$

$$2L_0\left[\frac{1-(1-0.5*v^2/c^2)}{1-v^2/c^2}\right] = 2L_0\left[\frac{0.5*v^2/c^2}{1-v^2/c^2}\right] = L_0\frac{v^2/c^2}{1-v^2/c^2} \cong L_0\left(\frac{v}{c}\right)^2\_as\_v^2/c^2 \ll 1$$

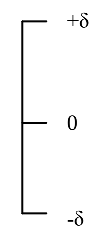
So, one gets $\delta = L_0\left(\frac{v}{c}\right)^2$ (12)

Note: One has used $\boldsymbol{v/c} = 30*10^3/3*10^8 = 10^{-5} \equiv \boldsymbol{\varepsilon}$; $(1-\varepsilon)^{1/2} \approx 1 - 1/2*\varepsilon$ and $1-\varepsilon^2 \approx 1$ to get expression (12).

This path length difference produces a given value of *phase shift* $\Delta\varphi_{tot} = \Delta\varphi_p = \boldsymbol{2\pi/\lambda*\delta}$, which places a *bright* (or *dark)* fringe at the telescope crosshair. Next, one rotates the interferometer arms by 90^0 CW so that PM_2 substitutes PM_1. This operation that makes $S'_{M1} = S_{M2}$, $S'_{M2} = S_{M1}$ would produce the new ***path length difference*** $\boldsymbol{\delta'} \equiv (S'_{M1} - S'_{M2}) = (S_{M2} - S_{M1}) = \boldsymbol{-}(S_{M1} - S_{M2}) = \boldsymbol{-\delta}$ (13)

So, after rotation, the ***path length difference*** between the interfering wavelets **changes by**

$\Delta = 2\delta = 2L_0\frac{v^2}{c^2}$ (14) In their set-up, $L_0=11m$, $\Delta \cong 2*11m\frac{(30*10^3 m/s)^2}{(3*10^8 m/s)^2} \cong 220nm$



This path length *change* would produce a phase *change* by $\Delta\varphi_{tot} = \frac{2\pi}{550nm}220nm = 0.4*2\pi = 0.8\pi$ at each location on the view field of telescope. Based on this result, one would expect to observe *almost a complete fringe shift* on the telescope crosshair (*a change by* $\Delta\varphi = \pi$ *corresponds to a complete shift from bright to dark or vice-versa)* during ***the rotation of its arms***. Actually, their device could detect a phase change as small as $\Delta\varphi_{min} = 0.02\,\pi$, but they could never get any observable fringe shift.
All the measurements confirmed a "phase shift 0" inside the experiment uncertainty.

- The two following reasons could justify the negative result of experiment:
  a) The upper calculation based on *motion through aether* is wrong because ***the aether does not exist***.
  b) The *arm* of interferometer (PM_1) *aligned on motion direction* **shrinks** from $\boldsymbol{L_0}$ to a value $\boldsymbol{L_1}$ such that a **path length difference δ = 0** is produced at (12). In this case, the expression (9) becomes $S_{M1} = c*t_1 = \frac{2L_1}{1-v^2/c^2}$ and $\delta = \frac{2L_1}{1-v^2/c^2} - \frac{2L_o}{(1-v^2/c^2)^{1/2}} = 0$ if the shrink is such that

$$\boldsymbol{L_1 = L_0(1-v^2/c^2)^{1/2}} \quad (15)$$

COVARIANCE

- The theory of electrodynamics developed by James Maxwell was the major achievement in physics since the time of Newton. This theory which contains a compact system of equations based on an *"absolute reference"* frame tied to the *aether* allows to explain all E&M phenomena. For this reason, even many years after the publication of the negative result of Michelson-Morley experiment, the physicists performed experimental efforts for " *proving the existence of aether* ", but without success.

-The theory of Maxwell had another limitation**; it was not covariant**. Meanwhile all the mechanics' laws are **covariant**; this means that **they keep the same form and give the same result when applied to any inertial frame**. How to understand this? Here it is a simple example of covariance; If one studies the translational accelerated motion of an object with mass ***m*** versus an ***inertial frame***, after defining *an axe Ox along its motion,* one can write the second law of Newton, in scalar form, as

$$F = ma \tag{16}$$

The fact that mechanics' laws are ***covariant means that if one studies the movement of the same object versus another inertial frame Ox₁(*** *i.e. moving at* ***constant velocity*** *υ with respect to the first frame*)***, the law of Newton does not change form and one would get the same acceleration and force; i.e., one gets in Ox_1 frame*** $a_1 = a$ ***and*** $F_1 = ma_1 = ma = F$ (17)

One may prove this statement easily. Since *x-coordinates and the time* in the two inertial frames in relative motion along x-direction are tied to each other by Galilean transformations

$$x_1 = x - \upsilon * t; \_and\_ t_1 = t; \tag{18}$$

one gets $$a_1 = \frac{d^2x_1}{dt_1^2} = \frac{d^2x_1}{dt^2} = \frac{d^2(x-vt)}{dt^2} = \frac{d^2x}{dt^2} = a \tag{19}$$

Since the mass of object does not depend on the reference frame, one gets the same value of the force in the new frame, $F_1 = ma_1 = ma = F$ and this confirms the *covariance of second law in two inertial frames*. Without entering in details, we note that ***all mechanic laws and conservation principles*** (energy, linear momentum ...) are ***covariant*** with respect to Galilean transformations. The mechanics' laws do apply equally in any "***inertial frame***", i.e. the *mechanics motions in nature can be expressed equally in all inertial frames.* Physicists think that ***the covariance is an essential propriety for all nature laws***.

- But, the E&M theory of Maxwell was not **covariant**. The following example clarifies this issue.

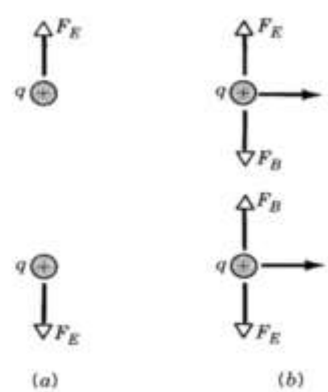


Fig 2

Two equal positive charges move at same velocity versus the lab frame. If studied with respect to an inertial frame that moves at the same velocity, the two charges are at rest (fig2.a). In this frame, they just push each other by the electrostatic force $\boldsymbol{F_E}$.
If studied with respect to the laboratory frame, each moving charge creates a current and a related magnetic field which gives rise to an attractive force ($\boldsymbol{F_B}$) on the other charge. In this frame (fig 2b) each charge is submitted to the resultant force

$$\vec{F} = \vec{F}_E + \vec{F}_B$$

So, in two different inertial frames (*related by Galilean transformations*), one gets two different results for the same physical situation and this does not make sense.

- The missing of covariance would make Maxwell theory to be true *only in one specific frame (tied to aether, which existence was not proved)* and this would make it not functional. Meanwhile this theory provided clear explanation for all experimental results related to electricity and magnetism phenomena. To make it covariant, one *should correct* either *the Maxwell equations or the Galilean transformations*.

- As E&M waves are "*objects that travel at very high speed* ", Albert Einstein decided ***to update the Galilean transformations*** so that they can allow covariance for ***" high speed objects "*** like electrodynamics ***waves***, too**.** He initiated these corrections by introducing the following two **postulates**.

TWO POSTULATES OF SPECIAL RELATIVITY

-This theory is known as the theory of ***special relativity*** because it **is referred to inertial frames** (*like Newton's laws*). The ***first postulate*** is a restriction and, in the same time, it is a guide line for the formulation of any physical theory. It requires " ***covariance " for all physics laws***.
1st Postulate: The laws of physics are the same (or covariant) in all inertial frames.

This means that the study of any physics phenomena in *different inertial frames* must provide the same result. Since all inertial frames must give the same result, there is *no need for an absolute frame tied to "aether"* and Maxwell equations must be covariant versus all inertial frames like all other physics' laws. Einstein ***updated the Galilean transformations*** *so that* the equations of Maxwell theory be covariant versus all inertial frames.

- The ***second postulate*** deals with the speed of light:
2nd Postulate: The speed of light in vacuum "c " is the same versus any inertial frame of reference.

By referring to our everyday experiences, one would say that this does not make sense. Note that even Einstein *did not pretend* to explain *why it is like that*. He just formulated the result of multiple experiments as a postulate. So, one has to accept this principle with two justifications: a) *our routine experience does not deal with so high speed values;* b) *the experimental results show that this is true*.

IMPORTANT CONSEQUENCE OF SECOND POSTULATE: No ***observer (or a material object) can travel at speed "c". The speed of light in vacuum is a maximum limiting speed which cannot be attained by any object with mass.*** One may confirm this by showing that an *object with mass* traveling at speed "**c**" would produce a *logical contradiction*. Let's refer to the following " *thought experiment* ": Assume that a spacecraft moves at speed "**c**" relative to the earth, i.e. versus an **observer O at rest on the earth**. Let's suppose next that the spacecraft pilot turns on shortly a headlight and emits a light pulse. The observer O measures the speed of light pulse and finds its value " **c** ". So, *for the observer O on the earth, the* headlight *and the "bubble of light" move together and will be all time at the same positions*. *Meanwhile**, the second postulate** tells that the light travels at speed **"c" versus an observer in spacecraft frame,** too; consequently**, after emission, the** "bubble of light" **cannot be at*** headlight ***position.*** So***, one gets** two different predictions for the same experiment and this means a **logical contradiction and missing covariance!*** One can avoid this contradiction ***only*** by accepting that motion with speed equal "**c**" is ***impossible*** for ***objects with mass*** (***spacecraft, man, particle, ..***)

Exercise: A high-speed spacecraft moving at speed **c/2** versus an *observer O on earth*, sends out a pulse of light in all directions. An *observer O', inside spacecraft*, observes a spherical wave front of light *centered at spacecraft* (*it spreads away at same speed " **c** " in all directions versus a frame tied to spacecraft*). The *observer O* makes his measurements, too. What is the propagation speed of light for him?["***c"***]; Does the light ***wave front*** travel at the same speed in all directions for him?[***yes***] ; What is the shape of the *wave front* for him? [**sphere]**. Is the wave front centered on the spacecraft at any moment? [***No***]

PRECISIONS ON THE MEASUREMENT PROCESS

- A. Einstein introduced some essential conceptual precisions concerning the measurement process. At first, he ***considered the concept of simultaneity***, a concept that *depends essentially* on *measurement procedure*. He underlined that a *position* ***measurement*** is an **event** that occurs at a **single point** of space (***x, y, z***) at an **instant** (***t***) of time in a **given frame of reference**. So, he labelled a physical measurement as an ***event*** with *coordinates* (***x,y,z,t***) in an ***inertial frame*** "**S**". The ***same event*** has another set of *coordinates* (***x',y',z',t'***) in another ***inertial frame*** " S' " (*moving at constant velocity versus S*).

- Next, Einstein noted that any measurement is made by an **observer** (*person or device*) **located** at a given point of space. This observer can observe and record only the events that happen near to him but cannot observe events that happen far from his location. So, one needs **a set of observers** to perform measurements that happen at *different locations* of an **inertial reference frame S**. One may observe events that happen in all the space by **distributing uniformly** a set of observers in this frame. If each of them records what happens close by him and shares recordings with all the colleagues, all observers can know and have the ***same information*** about what happens ***anywhere in reference frame S***.

- As a first step, all the observers in an inertial frame have to fix the measurement procedures so that they proceed the same way. This requests that they use identical stick meters and identical watches; also they must start measuring time at *"t=0"* *in synchrony to each other*. How to realize synchronization?

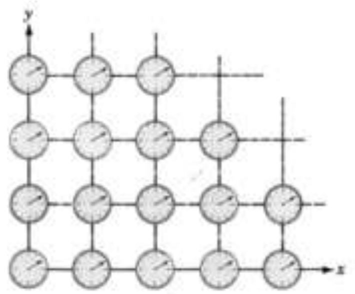


Fig 3

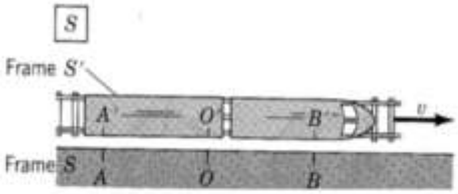
Einstein proposed the following method; Fix the **distance** between the adjacent observers equal to **$3*10^8$m.** As it will take ~**1s** for a light pulse to travel between them, all observers can easily synchronize the same moment **t = 0** of their watch by the following procedure. At t = 0 on his watch, the "*first observer"* (A) sends a light flash; the next one (B) knows that his watch must be set at 1sec past when he receives the light flash; the third (C) at 2sec past and so on. This scheme allows *one to verify if two events that happen in **different locations** of the same **inertial frame** are **simultaneous** or not*.
One labels as "**rest frame**" the inertial frame versus which the ***object of study (or an observer) is at rest***.

RELATIVITY OF SIMULTANEITY

Let's consider now the simultaneity of *same set of two events* in two different **inertial frames S**, **S'** .

- In general, one is able to verify the ***simultaneity*** (happening at the ***same instant)*** of two events that happen *close to the same observer*. Meanwhile, to verify the simultaneity of two ***events that happen at different locations far from each other***, one will need information from the *system of equidistant observers at rest in the same inertial frame as*suming that there is one observer close to the location of each event.
Let's assume that two observers A and B (at ***rest in frame S***) explode two firecrackers (fig. 4.a) and the midway observer O (in ***frame S***) receives two flashes ***simultaneously***. This results means that the **two** explosion **events** happened **simultaneously** in frame S. In science, when one publishes *a measurement result one must clearly explain the method used to get it*. In this case, one would affirm that :

Two events that happen at different locations of an inertial frame S are simultaneous if an observer in midway between their locations receives the information (two light flashes) at the same instant.



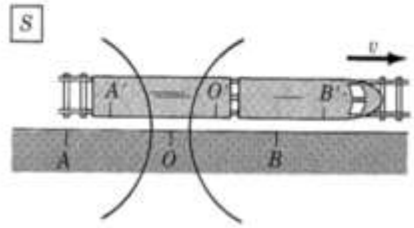


Fig 4 (a) (b)

- Now, let's consider the same events (*explosions at A,B*) in an ***inertial frame S'*** tied to a "*long train*" travelling right side at very high constant speed ***v*** (*say 0.8c*) with respect to frame S. Here one refers to light wave fronts leaving **A'**, **B'** observers (*in front of A, B at explosions events moment*) and travelling versus **O'** observer. As seen from fig. 4.b, by the time the wave fronts have reached observer **O** in frame **S**, the observer **O'** in frame **S'** has moved on the right. Therefore the wave front *from point B'* has already passed by O' while the wave *front from A'* has not yet reached O'. For the observer ***O' at rest in frame S'*** the explosion in B' happened before explosion in A' and they are ***not simultaneous events***.
So, while the time interval between two events is $\Delta t = 0$sec in frame S and $\Delta t' \# 0$sec in frame S'.

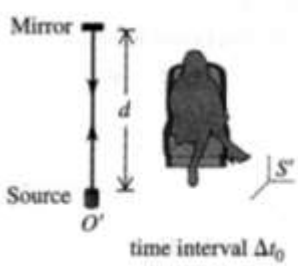
This way, one realizes that the **simultaneity depends on the frame of reference**:
The spatially separated events that are simultaneous in one inertial frame are not simultaneous in any other inertial frame (**moving at a constant velocity relative versus the first one**).

Consequences

1) The *time interval* between the <u>same two events</u> measured *in different frames* is *different* (direct).
2) Even the *lengths* of the same *object* measured in *different* frames are different (indirect).

<u>TIME DILATION</u>

-Assume that a frame of reference S' is tied to a train moving at constant speed "***v***" versus frame S (tied to station, see fig. 5) along a common direction (*Ox-O'x'*). An observer O', at rest in frame S', measures the interval of time Δt_0 between two events that occur at the ***same location for*** *her rest frame* **S'**: A light flash ***emitted*** by a source ER(*emitting-recording*) on floor is reflected at the mirror M on the ceiling and ***captured*** by a receiver incorporated(*same location)* with the source.



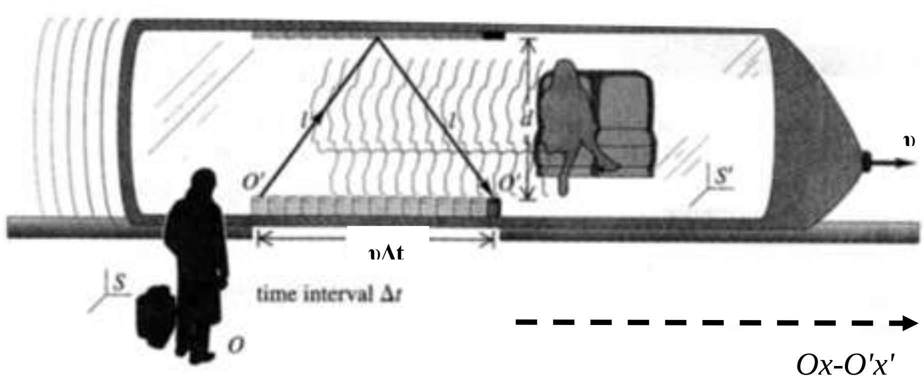


Fig5. (a) (b)

The *interval of time* between these two events (*emission-reception*) in the ***rest frame*** S'(train) **of** observer **O'** (see fig 5.a) is:

$$\Delta t_0 = \frac{2*d}{c} \tag{20}$$

-Another observer O, at rest in the frame S (i.e. *station*), finds out a different interval of time between the same two events (*emission-reception*). As he "sees" (fig. 5.b) these two events to happen at two different locations (*versus frame S*) in space, he observes that the total path of light has a length:

$$2*l = 2*\sqrt{d^2 + (\frac{\upsilon\Delta t}{2})^2} \tag{21}$$

Note that one has to use:

a) the same **speed of light pulse** "*c*" in both frames – in accord to the ***second postulate***.

b) assume the same vertical distance "**d**" for the two observes. This will be clarified in the next section. So, it comes out that, for observer O, the *time interval* between emission and reception is:

$$\Delta t = \frac{2*l}{c} = \frac{2}{c}\sqrt{d^2 + (\frac{\upsilon\Delta t}{2})^2} \tag{22}$$

To find the relation between ***Δt*** and ***Δt₀***, one transforms the expression (22) as follows

$$\Delta t = \frac{2}{c}\sqrt{d^2 + (\frac{\upsilon\Delta t}{2})^2} = \frac{2}{c}\sqrt{\left(\frac{c\Delta t_0}{2}\right)^2 + (\frac{\upsilon\Delta t}{2})^2} \rightarrow\rightarrow \left(\frac{c\Delta t}{2}\right)^2 = \left(\frac{c\Delta t_0}{2}\right)^2 + (\frac{\upsilon\Delta t}{2})^2 \rightarrow\rightarrow$$

$$\left(\frac{c\Delta t}{2}\right)^2 - \left(\frac{v\Delta t}{2}\right)^2 = \left(\frac{c\Delta t_0}{2}\right)^2 \quad \text{or} \quad \left(\frac{c\Delta t}{2}\right)^2 - \left(\frac{c\Delta t*v}{2*c}\right)^2 = \left(\frac{c\Delta t_0}{2}\right)^2 \quad \text{or} \quad \left(\frac{c\Delta t}{2}\right)^2 - \left(\frac{c\Delta t}{2}\right)^2 * \frac{v^2}{c^2} = \left(\frac{c\Delta t_0}{2}\right)^2$$

$$\left(\frac{c\Delta t}{2}\right)^2\left(1-\frac{v^2}{c^2}\right) = \left(\frac{c\Delta t_0}{2}\right)^2 \qquad \Delta t\sqrt{\left(1-\frac{v^2}{c^2}\right)} = \Delta t_o \qquad \text{and} \qquad \text{finally}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{\left(1-\frac{\upsilon^2}{c^2}\right)}} \tag{23}$$

- The observer O' **measures** the interval of time "***Δt₀*** " between *two* ***<u>events</u>*** that occur at the ***<u>same location</u>*** of his ***<u>rest frame S'</u>***. The expression (23) tells that "***Δt₀*** " is the ***shortest*** interval of time between these two events. For the observer O (*his rest frame S is moving at velocity "-* ***υ****" versus frame S'*) the interval of time between the two events "***Δt*** " is measured at ***different locations (i.e. by two different observers)*** and it is ***larger than Δt₀.*** This effect is known as ***<u>time dilation</u>*** in the theory of relativity.

- One prefers to simplify expression (23) and write it in form (26) by introducing parameters (β, γ)

$$\beta = \frac{\upsilon}{c} \quad (24) \qquad \gamma = \frac{1}{\sqrt{\left(1-\frac{\upsilon^2}{c^2}\right)}} = \frac{1}{\sqrt{\left(1-\beta^2\right)}} \quad (25) \qquad \text{and} \qquad \Delta t = \gamma * \Delta t_0 \quad (26)$$

Note that for "*common speed values*" $\upsilon << c$, $\beta \approx 0$ and $\gamma \cong 1$. So, one finds the same interval $\Delta t = \Delta t_0$ for all inertial reference frames (Galilean *model*). For ***high speed*** values, $\gamma > 1$ ***while*** $0 < \beta < 1$

- **Important note** : There is ***<u>only one frame</u>*** of reference in which ***<u>a single clock at rest</u> can measure the interval of time between two events***. It's clear that the time interval measured between two events that occur at **the same location** of a particular frame (*measured by " two ticks of the same watch"*) is a more fundamental quantity than the interval of time between the same events measured (*by two watches*) at different points of any other frame. *One labels as **<u>proper time</u>** the interval of time Δt_0 between **two events** measured **<u>at the same location</u>** of a frame*. In a relativity problem, one identifies the ***<u>proper time as the interval of time measured by a single watch at the same location of an inertial frame.</u>***

<u>LENGTH CONTRACTION</u>

- Consider a "*long*" rod AB *at rest* in a frame S (fig.6). The frame "S" is the ***rest frame*** *for the rod*, and the length of rod measured in this frame is called ***proper length*** L_0. *The **proper length** of an object is the **distance between*** **its ends** ***measured in the <u>rest frame of object</u>***. To avoid any ambiguity, one must define precisely the *procedure of length measurement* and use it the same way for all considered frames.

- Let's consider an observer ***O'*** "*sat in a car* " moving right side at constant speed "υ" versus the frame " S " where the rod AB is at rest (fig.6.a). He measures ***the time*** between ***two events:***

The car is in front of A-end and the car is in front of B -end of the rod.

The observer ***O'*** inside the car, measures the time by an watch at rest in *his rest frame* **S'**. So, he measures the **proper time** Δt_0.

Two observers located at A and B, at rest in the ***frame S*** measure the interval of time Δt (***<u>not a proper time</u>*** because it is measured by two watches at different locations) and calculate the rod's ***proper length*** L_0 ***(because it is at rest** in frame **S)*** by multiplying the speed of car υ to the time interval between two events (fig 6.a).

$$L_0 = \upsilon * \Delta t \qquad (27)$$

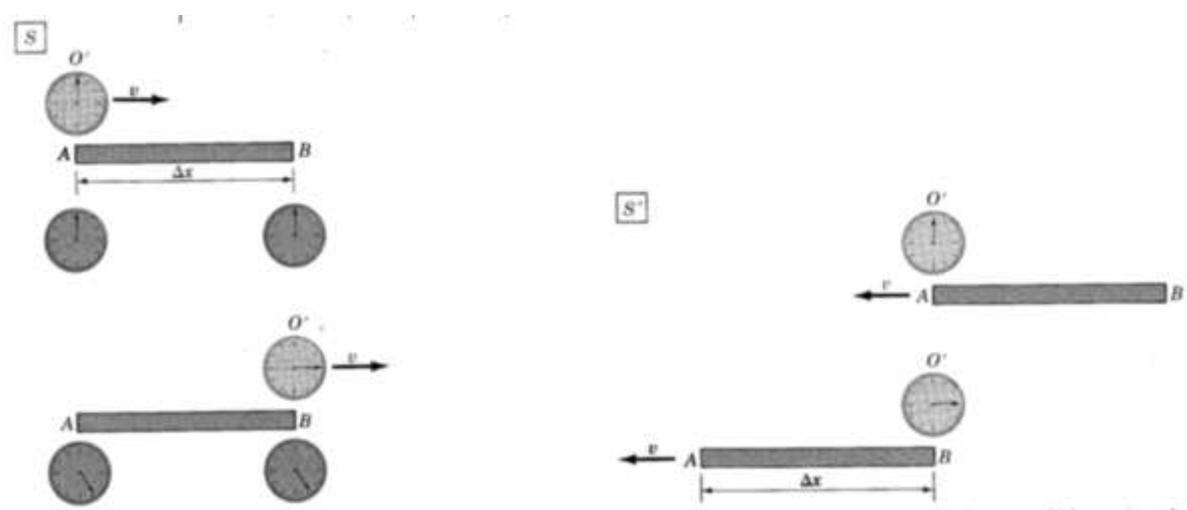


Fig 6 (a) (b)

For the observer ***O'*** inside the car, the rod moves at velocity "- υ " (see fig.6b). So, he measures the *not* **proper length L** because the rod is **not at rest** with respect **to frame S'.**

He calculates length **L** by using the proper time Δt_0 as

$$L = \upsilon * \Delta t_0 \qquad (28)$$

-Next, by isolating the time intervals from relations (27,28) and by using the relation $\Delta t = \gamma * \Delta t_0$ between the ***time Δt*** and the ***proper time Δt₀***, one get relation (29) between L and L_0

$$\frac{L_0}{\upsilon} = \Delta t = \gamma \Delta t_0 = \gamma \frac{L}{\upsilon} \to\to\to\to L = \frac{1}{\gamma} L_0 \qquad (29)$$

If an object is moving at constant speed versus an inertial frame and an observer at rest in this frame measures the length of object, he will find a length "*L*" that is related to the proper length of object as

$$L = \frac{L_0}{\gamma} \qquad (30)$$

As $\gamma \geq 1$, it comes out that $L \leq L_0$. This means that the object length measured in a frame versus which it is moving is always **smaller** than its **proper length**. This effect is called ***length contraction***. The ***proper length*** of an object is its ***largest length*** and it is measured in a frame ***where*** the ***object is at rest***.

Notes:

a) The *length contraction* effect is reciprocal. A similar object at rest in the frame S' will have a contracted length if it is measured by observers in the frame S.

b) *Only lengths parallel to the direction of motion are contracted; those* ***perpendicular*** *to the direction of motion are* ***not affected*** *by the contraction effect*.

c) This effect is ***not an optical illusion***. A *1m ruler is really shorter* when measured in a frame S' that moves at ***very high speed*** with respect to the ***rest frame*** S of ruler.

d) The concept of ***rest frame does not make sense*** for objects ***that propagate at speed "c" like photons*** (or any E&M wave) because the photon should have zero speed versus this frame and this is not allowed by the second postulate which requests light speed "**c**" versus any frame. Note that photons are object "*with zero mass*".