THE STRUCTURE OF NUCLEUS

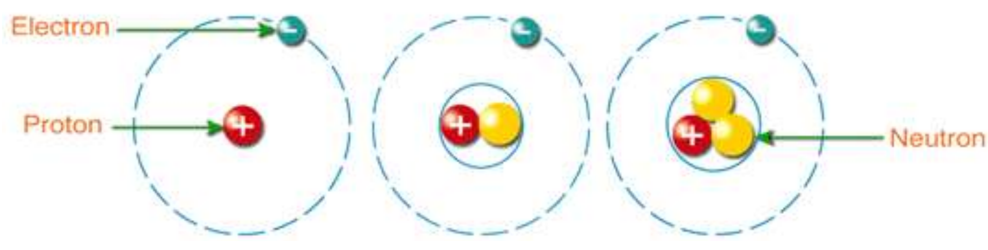
- ***Atomic model*** : Each atom contains a *nucleus* and a number of *electrons in its atomic orbits*. The atom is an *electrically neutral* structure. So, the nucleus contains an electric charge equal to the sum of electrons' charge but with positive sign. The forces between the nucleus and the electrons, i.e. *atomic forces are electromagnetic forces*. The quantum rules define the characteristics of electron orbits; the orbital electrons "***know only***" that there is a positive electric charge inside nucleus.

- The atomic nucleus contains a number of ***nucleons*** (***protons and neutrons***). A proton contains the amount "**+e**" of electric charge (electron charge is "**-e**"). Consequence: There is the **same number of protons** (*inside nucleus*) and electrons(*on atomic orbits*) inside an atom (*electrically neutral*).

- Each ***element of periodic table*** shown by ***symbols*** (*H*, *He*,..) is identified by its ***atomic number*** **Z,** i.e. the *number of protons in nucleus*. The *atomic number* of **natural** elements vary *from* ***Z = 1(H) to 92(U)***. The **artificial** elements have Z = ***93*** to ***118***. ***Atomic mass number*** ***A*** gives the total number of nucleons

$$A\text{(nucleon nb.)} = N\text{(neutron nb.)} + Z\text{(proton nb.)} \quad (1)$$

- The notation $^{A}_{Z}X$ ($^{16}_{8}O$, $^{14}_{7}N$ ..) indicates a ***nuclide*** , i.e. a *nucleus* with a ***precise number of*** *protons and neutrons*. This notation is very important for distinction of different ***isotopes*** (***A different***) of the ***same element*** (atoms with ***same Z***). Since the *chemical proprieties* depend on the electronic structure and this is the same for all *isotopes of the same element*, all ***isotopes of same element are chemically identical***.
The experimental records show the presence of three hydrogen ***isotopes*** in nature; protium (^{1}H **)**, deuterium (^{2}H) and tritium (^{3}H). The nuclei of all those three isotopes contain *one proton*; so, there is three nuclides that correspond to the same element; **hydrogen**(*Z=1).* Protium contains no neutrons, deuterium has one *neutron,* tritium has two *neutrons in its nucleus.* Protium constitutes 99.98% of the hydrogen isotopes in nature; deuterium is stable but rare; tritium is unstable (radioactive) and very rare.



Protium (stable) Deuterium(stable) Tritium(radioactive)

Fig 1 Three main isotopes of hydrogen

-The mass of a **nucleon (***proton or neutron)* is **~10^{-27} kg**, i.e. ~10^{4} times larger than **electron mass (m_e~10^{-31} kg)**. So, even for 100 electrons, the ***mass of an atom*** is practically defined by the **number** "**A**" **of nucleons** that it contains. One uses "*A number*" to express the *atoms' mass* by a proper mass unit at "atomic dimensions"; the ***unified mass unit-u defined*** as 1/12 part of the ***rest mass*** of the isotope $^{12}_{6}C$ of carbon atom; so $m_{C^{12}_{6}} = 12u$. One may find out that ***1u = 1.66056*10^{-27} kg = 931.5 MeV/c^2***. The atomic masses given in **u-units** in the periodic table are the ***weighted averages over all isotopes*** of the element. The ***rest mass*** for a proton, a neutron or electron may be expressed in units *kg, u* or *MeV/c^2*

m_p = 1.67264*10^{-27}kg = 1.007276 u = 938.28 MeV/ c^2
m_n = 1.67493*10^{-27}kg = 1.008665 u = 939.57 MeV/ c^2
m_e = 9.10938*10^{-31}kg = 0.000549 u = 0.511 MeV/ c^2

-The *radius* of the smallest ***atom*** (first orbit in Bohr's model for H) is ~0.0526nm = $5.26*10^{-11}$m. The experimental measurements show that, the *spherical model* of ***nuclei*** is a good first step approximation and one may estimate the ***radius of nuclei*** by expression

$$R \sim 1.2*A^{1/3}\ \text{fm} \qquad 1\text{fm(fermi)} = 10^{-15}\ \text{m} \qquad (2)$$

By using expression (2) at nucleus of Protium one gets $R \sim 1.2*1^{1/3}$ = 1.2fm i.e. 4 orders of magnitude smaller than radius of the smallest H-atom orbit. Note that practically all atom mass is concentrated inside the nucleus. This means very <u>big values of mass density</u> inside nuclei (ex. $\rho_M \sim 10^{17}\text{kg/m}^3$ at ${}^{16}_{8}O$). From the relation between mass and energy one may infer that <u>**energy density** has **very large values** inside the nuclei.</u> ($E_0 = m*c^2$; So, $\rho_E = E/V = (m/V)*c^2 = \rho_M*c^2 \sim 10^{17}*9*10^{16}[\text{kg/m}^3*\text{m}^2/\text{s}^2] \sim 10^{34}\,[\text{J/m}^3]$)

BINDING ENERGY AND NUCLEAR STABILITY

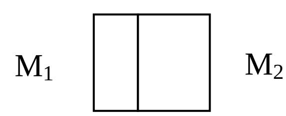
Introduction of binding energy concept

a) *Consider a **system of two objects** M_1 (magnet) and M_2 (metal) in free space in two configurations;*

a-1) Bound to each other.

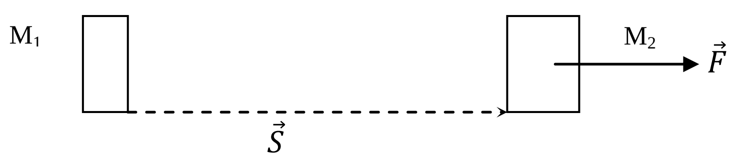
The binding energy of the system is

$E_{sys\text{-}initial} = E_{12}$



In this case, E_{12} is the magnetic binding energy

*a-2) Very far (infinite distance) from each other so that magnetic **attraction** is **zero**. Assume that one uses an **external force** $\vec{F}$, to detach and shift object M_2 far away from M_1(kept fixed).*



The new energy of interaction for the system is zero (no interaction M_1-M_2 means $E_{12}=0$)

$$E_{sys\text{-}final} = 0$$

<u>*The principle of energy conservation tells that the **positive work** ($dW_F = F*dS*\cos 0^0$) done **by** force $\vec{F}$ to shift the object M_2 away from M_1 is equal to the change of energy of the set of two objects.*</u>

$$W_{ext} = \Delta E = E_{sys\text{-}final} - E_{sys\text{-}inital} = 0 - E_{12} = -E_{12} \qquad \text{So,} \qquad E_{12} = -W_{ext}$$

<u>*The **magnitude** of **binding energy** is equal to the work, one must spent to detach the two (or all, if there are more) constitutive parts of the system and shift them far away from each other. As W_{ext} is positive, it comes out that the binding energy is **negative.** A **bound system** has **negative binding energy** and the **zero or positive energy of system** corresponds to an **unbound configuration**. This means that* **the energy of molecular, atomic and nuclear bound states should have negative values.**</u>

***Example**: The molecular binding energy of molecule H_2O is equal to "-"the work one must do to shift to infinity all the three atoms H, H, O.*

***Note**: In nuclear physics, one refers to energy calculations based on $E=mc^2$ and prefers to avoid operations with negative numbers. So, one **assigns the 0 value to the energy of ground state** and this results to positive values for the energy of bound states. This does not change anything at the energy calculations for nuclear transitions because they are based on the difference between the energy values.*

-Having the same electric charge, the protons inside the nucleus, push each other. The fact that they remain bound means that another *attractive force* with *larger magnitude* acts between them.
This is the ***nuclear force*** which main characteristics are:

a) Its attractive action fades quickly for distances > 3 - 4fm. It's a *short distance force*.
b) It does not depend on nucleon electric charge. So, it is the same for protons and neutrons.

Nuclear forces are (1)<u>attractive</u>, (2)<u>short range</u> forces** and **(3)<u>do not depend</u> on electric charge.

-The **<u>nuclear binding energy</u>** (**BE**) is equal to the amount of work needed to be spent by external forces for separation of all particles located inside a nuclide structure.
One may calculate BE by the following step-by-step logic:

a) ***Z*** protons and (***A-Z***) neutrons constitute the nuclide which is a <u>stable bound system</u>.
b) This means that the ***system of protons and neutrons*** <u>*has smaller*</u> energy when they are together inside this nuclide compared to the <u>*sum of their energy*</u> when they are separated (fig.3).

When "bounded inside" the <u>*neutral atom "x" containing Z electrons*</u>, ***the mass*** and the corresponding ***energy*** *(using* $\boldsymbol{E = m*c^2}$*)* of ***the system*** *of nucleons* are

$m_{Nucl} = m_{nucleons\_in\_atom} = m_x - Z*m_e$ and $E_{Nucl} = m_{nucl} *c^2 = [m_x - Z*m_e]*c^2$ (3)

(m_x-the mass of neutral atom given in tables)

When separated (nucleons far away from each others) their total ***mass*** and ***energy are***

$m_{sep} = [Z*m_p+(A-Z)m_n]$ and $E_{sep} = m_{sep} *c^2 = [Z*m_p+(A-Z)m_n]*c^2$ (4)

By adding and subtracting the mass of "$\mathbf{Z*m_e}$" to expression (4), E_{sep} transforms to

$E_{sep} = [(Z*(m_p+ m_e) + (A-Z)m_n - Z* m_e]*c^2 = [Z*\mathbf{m_H} + (A-Z)\mathbf{m_n} - Z* \mathbf{m_e}]*c^2$ (5)

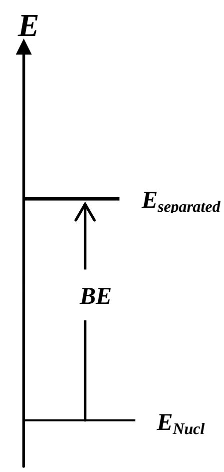


Fig.3

$\boldsymbol{m_H}$ is the ***mass*** of isotope ${}^1_1H$, i.e. a ***"protium structure"*** of hydrogen.

Then, the ***binding energy*** "BE" (see Fig.3) of nucleus can be calculated by subtracting (3) from (5)

$$BE = E_{sep} - E_{Nucl} = [Z*m_H + (A-Z)m_n - Z*m_e - m_x +Z*m_e]*c^2$$

$$\boldsymbol{BE = [Z*m_H + (A-Z)*m_n - m_x]*c^2 = \Delta m*c^2} \quad (6)$$

where $\boldsymbol{\Delta m = Z*m_H +(A-Z)*m_n - m_x}$ *known as* ***MASS EXCESS*** shows numerically by how much the total mass of nucleons inside a nuclide is ***<u>smaller</u>*** than the sum of their individual masses. <u>*This difference of mass is converted into binding energy.*</u>

<u>Example</u>: Calculate BE for ${}^{12}_{6}C$ and average BE per nucleon in ${}^{12}_{6}C$.

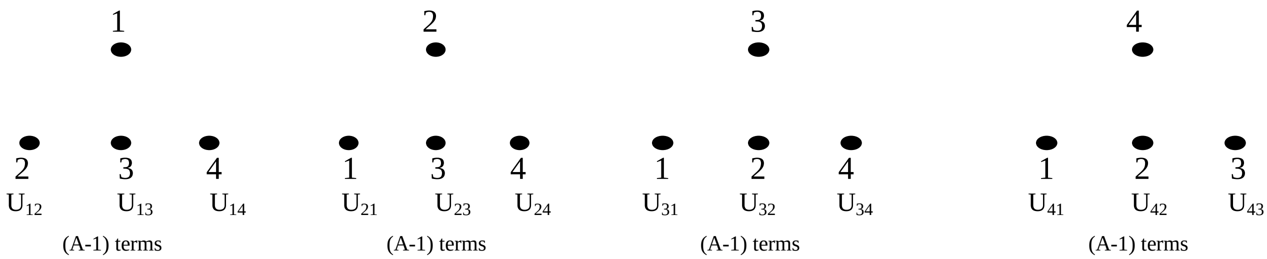
a) The mass of ${}^{12}_{6}C$ isotope is $\mathbf{m_x = m_C = 12u}$. Also **Z = 6** , **A-Z =12- 6 = 6** and
   $\mathbf{m_H = 1.007825u}$ **;** $\mathbf{m_n = 1.008665u}$ **.**

So, $\mathbf{\Delta m = [6*1.007825 + 6*1.008665 - 12]u = 0.09894u}$; As ***1u = 931.5 Mev/c²***
   $\boldsymbol{BE = \Delta m *c^2}$ = $(0.09894u)*c^2 = (0.09894*931.5Mev/c^2)*c^2 =$ **<u>92.16261Mev</u>**

b) The ***average energy binding per nucleon*** is ***BE / A*** = 92.16261/12 = **<u>7.680217Mev</u>**
c) Note that $\boldsymbol{\Delta m}$ and BE are always positive.

NUCLEAR STABILITY

- The <u>*average binding energy per nucleon*</u> (***BE/A***) is a key parameter for understanding the <u>*nuclear stability*</u>. If each nucleon would contribute equally to binding energy then *BE/A should increase linearly* with number of nucleons. The following scheme with four nucleons (A=4) helps to clarify this comment.



If one express BE as $BE = \sum_{i,j} U_{ij}$ (7) each term appears twice in this sum (Ex. $U_{12} = U_{21}$ or $U_{13} = U_{31}$)

As $U_{ij} = U_{\text{nucleon-nucleon}}$ is the same quantity, one get

$$BE = \frac{A(A-1)}{2} * U_{nucleon-nucleon} \qquad (8)$$

$$\frac{BE}{A} = (A/2 - 1/2) * U_{nucleon-nucleon} = \left(\frac{U_{nucleon-nucleon}}{2}\right) * A - \frac{U_{nucleon-nucleon}}{2} \equiv \alpha * A - \alpha \qquad (9)$$

Expression (9) shows that *BE/A* should be increased in a linear way with the increase of A value.
The graph in figure 4 presents the actual evolution of *BE/A* parameter when A increases following the different elements. The "*BE/A*" values in graph are calculated by using the formula (6) with tabulated data and are confirmed by experiments. This graph shows that *BE/A* increases <u>***almost linearly***</u> until A~20. Beyond A=20, it increases very slowly till a <u>***maximum value 8.79 MeV***</u> (nucleus $^{56}_{26}Fe$); next it decreases to 7.6MeV at the ***last natural element*** $^{238}_{92}U$. This curve informs about the <u>*spatial range of action for nuclear forces*</u>. As long as the ***average binding energy*** per *nucleon increases linearly with A (from 2 to ~20),* the ***nuclear attraction force*** is larger than the ***repulsive*** (p-p) ***electric force.***

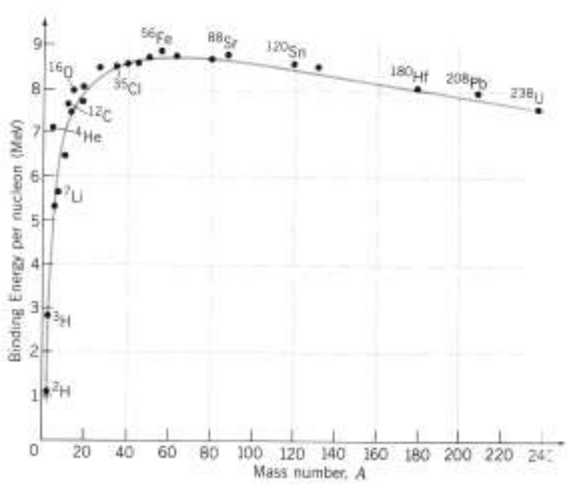


Fig. 4 Evolution of BE/A with A

For A= 20 to ~30 *the addition of a nucleon increases just in moderate way the average binding energy*. At this point, one should note that calculations of BE/A are based on the general formula E= $m*c^2$ which does not give any information about the specific contributions of nuclear attraction and proton-proton electric repulsion to the binding energy. Yet, the graph in fig.4 indicates that *for A~30,* "*in average*" the *attractive action of nuclear forces starts to become comparable to electric repulsion action between protons.* As the magnitude of *electric force does not change significantly in so short distances, it comes out that it is the nuclear force action that decreases fast with distance for A >30.*

So, one concludes that the nuclear forces are ***short-range forces*** acting significantly ***till a maximum distance*** (≈radius of nuclide with A=30)

$$R \leq 1.2*30^{1/3} = 3.73 \text{ fm} \qquad (10)$$

- The graph in fig 5 shows the variation of number of neutrons in **stable natural nuclides** (Z = 1- 83). One may see that the light atomic nuclei contain practically as many neutrons as protons. Above Z = 20 (Ca), the atomic nuclei contain more neutrons than protons "*in order to compensate*" the effect of electric protons-proton repulsion. As long as the number of protons is not too high, the (*attractive*) nuclear forces between *all nucleons* "*can manage to win*" over the *repulsive* Coulomb forces acting between the *protons*. So, the **atomic nuclei** remain **stable** if they contain an adequate number of neutrons, in order to "dilute" the concentration of positive charges brought about by the protons. The nucleus $^{209}_{83}Bi$ (**bismuth**) has 43 neutrons "*in excess"* (209-(83+83)=43) and it is the heaviest *natural element* for which at least one isotope is *stable*. All the nuclei with **Z > 83** are ***unstable*** and undergo the ***radioactive disintegration***. Even the elements with **Z < 83** do have radioactive isotopes but for them does exist, at least, one stable isotope.

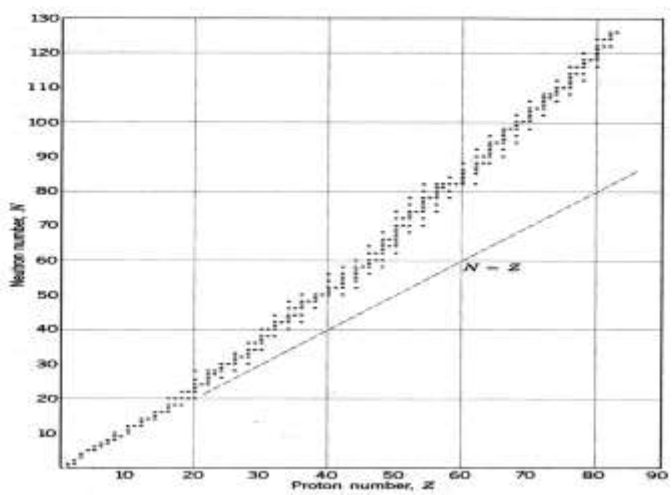


Fig.5 Number of neutrons in isotopes of different elements