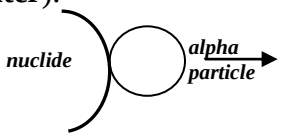
RADIOACTIVITY

- A nuclide is an *atom* with specific number of protons(Z) and neutrons(A-Z) in its nucleus. There is about **1500 different nuclide structures,** only **250 stable nuclides (**all with **$Z \leq 83$)** and not every nuclide with $Z < 83$ is a stable structure. What does it happen with unstable nuclides?
After existing for some time, an unstable nuclide (***parent***) *emits* ***spontaneously*** a "***radioactive emission"*** which is constituted by ***α or β - particles*** and transforms to another type of nuclide (**daughter**).
An **α-particle** is a **twice ionized He** atom $({}_2^4He)^{++}$ (i.e. a set of 2 protons & 2 neutrons, q = +2e)
A ***β - particle*** is an ***electron*** (m = m_e, q = -e) or a ***positron*** (m = m_e, q = +e).
Most of the ***radioactive nuclides*** emit ***either α*** or ***β*** particles; only ***a few emit both***.



The majority of "***daughter"*** nuclides " born " after the emissions of ***α*** or ***β*** particles, are initially in an ***excited energy*** level but they drop ***fast*** to the ***ground level***. This "*secondary process"* is accompanied by the **emission** of excitement energy in the form of a ***γ-photon*** ($m_0 = 0, q = 0$) which is a ***quantum particle*** with ***very high energy***. The corresponding electromagnetic wave for this type of photon has $\lambda_\gamma \leq 10^{-12}$ meter . Often, one considers that these ***γ-radiations*** are a type of ***radioactive emission***, too.

- *A radioactive event happens without "exterior action"*; so, it must obey to the **principle of energy conservation** " *the sum of energy of the products must be equal to the energy of the parent nuclide* ".
One ties the reference frame for calculations to the parent nuclide; so its energy is just the rest energy

$$E_{initial} = m_{0(parent)} * c^2 \qquad (1)$$

The ***products*** of radioactive emission ***move*** with respect to parent nuclide. So, they move versus the frame tied to parent nuclide. As they have a ***rest*** and a ***kinetic energy,*** the energy of decay products is

$$E_{final} = \sum_i m_0^i * c^2 + K \qquad (2)$$

where the second term contains the ***sum of kinetic energies of all the products***. From the principle of energy conservation one has $m_0(parent) * c^2 = \sum_i m_0^i c^2 + K \rightarrow m_0(parent) * c^2 > \sum_i m_0^i c^2$ and

$$m_0(parent) > \sum_i m_0^i \qquad (3)$$

The relation (3) is the ***basic condition*** for a ***radioactive process to happen***. It means that the total rest mass of the products must be smaller than the rest mass of parent nuclide. This requirement explains "*Why the natural radioactivity cannot produce any type of free particles"*; like.. a single proton or a single neutron or five protons (or neutrons); *the condition (3) cannot be met for these thought radioactive processes.*

The ***energy released*** at the end of ***a radioactive event*** is called ***disintegration energy*** Q.

$$Q = [m_0(parent) - \sum_i m_0^i] * c^2 \qquad (4)$$

Actually, this energy is the *sum of the kinetic energies* of emitted particles (daughter nuclide, α , β) plus the *energy of released gamma radiation* that appears ***after*** a radioactive event.

-A value ***Q < 0*** tells only that the disintegration process is ***impossible to happen spontaneously***. But, if one ***provides*** the necessary amount of ***energy*** to the ***parent nucleus, the energy restriction is overcome and*** the parent nuclide may be ***disintegrated***. If the *nuclide disintegration* happens via this **intermediary step** one calls the process a ***nuclear fission .*** *The* ***minimum amount of energy*** given to parent nuclide in a nuclear ***fission event*** *is called* the ***separation energy*** (*noted* ***S***) of ***fission*** and it is calculated as ***S = - Q*** .

Remember: The *radioactivity is a natural* ***spontaneous*** *phenomenon. This means that a decay event may happen* ***with a certain probability*** ***at time "t"*** *and not " it happens at time t ".*

ALPHA DECAY

- Almost all **unstable** nuclides with **A > 150** do undergo **α -decay**. Only a few nuclides with **Z < 83** undergo **α-decay.** During a **α-decay** event, the **parent** nuclide ${}^{A}_{Z}X$ emits **one** α - particle (${}^{4}_{2}He^{++}$ i.e. a nucleus of the helium atom). This process decreases the charge from **+Ze** to **+(Z - 2)e** and the mass number from **A** to **A-4** at the new ***daughter*** nuclide "*Y* ". So the relation that describes an α-decay event is:

$$ {}^{A}_{Z}X \Rightarrow \alpha - decay \Rightarrow {}^{A-4}_{Z-2}Y + ({}^{4}_{2}He)^{++} \qquad (5) $$

The decay of Radium to Radon is an example of α-decay (Fig.1): ${}^{226}_{88}Ra \Rightarrow \alpha - decay \Rightarrow {}^{222}_{86}Rn + ({}^{4}_{2}He)^{++}$

*What is the **disintegration energy** of this radioactive event?* By applying the relation (4), one gets

$$Q = [m_0({}^{226}_{88}Ra) - m_0({}^{222}_{86}Rn) - m_0({}^{4}_{2}He^{++})] * c^2 = [m_0({}^{226}_{88}Ra) - m_0({}^{222}_{86}Rn) - m_0({}^{4}_{2}He^{++}) + 88m_e - 88m_e] * c^2$$

$$Q = [m_{atom}({}^{226}_{88}Ra) - m_{atom}({}^{222}_{86}Rn) - m_{atom}({}^{4}_{2}He)] * c^2 = (226.025408 - 222.01757 - 4.002603)u * c^2$$

$$Q = 0.00523(931.5MeV/c^2) * c^2 \approx 4.9MeV$$

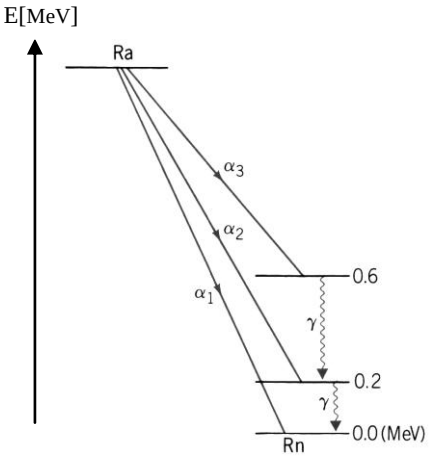


Figure 1

The "parent" **radium** nuclide in its ground energy level emits one **α-particle** and it is transformed into a **radon** nuclide "daughter". Due to the three energy levels of radon nuclide ${}^{222}_{86}Rn$ (ground & two excited levels), there exist three possibilities for the **kinetic energy** of the emitted α- particle. The ***maximum energy*** $K_{\alpha 1}$ corresponds to the transfer between the two respective energy ground levels. The experimental measurements give $K_{\alpha 1} \approx 4.8MeV$. This value fits exactly to Q calculations if one adds ~0.1MeV for recoiling kinetic energy of daughter nuclide ${}^{222}_{86}Rn$. Each of two other possible transitions versus the excited levels of radon nuclide is followed by additional de-excitement to lower energy levels through emission of a **γ particle**.
So, during a α-decay process produced in a sample containing ${}^{226}_{88}Ra$ isotopes, one might expect to record:

- The daughter nuclides Rn at different energy[1] levels;
- α particles with energy ~4.8 MeV, ~ 4.6MeV, ~ 4.2MeV
- γ particles with energy ~0.2MeV, ~0.4 MeV, ~0.6 MeV(rarely recorded)

Note: ***The γ "particles" that follow the α-decay have a discrete spectrum of energies*.**

BETA DECAY

-During a ***β-decay*** event, the **parent** nuclide ${}^{A}_{Z}X$ emits a **β** particle. Experiments show that while A-value remains unchanged, the value of "Z" can change by +/- 1. Actually, there are **two kinds of β particles**: " β^- ", which are simply **electrons** ($m = m_{el}$, $q = -e$) and " β^+ ", which are **positrons** ($m = m_{el}$, $q = +e$). The **positron** is an example of ***antimatter particles***. The antimatter particles appeared initially in theory, during the development of quantum mechanics. Later on, several of them are observed experimentally. Note that ***neither electrons nor positrons are particles that exist inside the nucleus***. So, it comes out th at both of them are ***created in nucleus only during a process of β-decay*** and emitted straight away.

[1] During an α-event the daughter nucleus gets some recoiling kinetic energy, too; Rn nucleus gets a kinetic energy ~ 0.1MeV

- The **two schemes** associated to a **β - decay** event are:

a) **β^- decay**. $Z \Rightarrow \beta^- \Rightarrow Z+1$ The ***daughter*** has ***one proton <u>more</u>*** *than parent* nuclide.

b) **β^+ decay**. $Z \Rightarrow \beta+ \Rightarrow Z-1$ The ***daughter*** has ***one proton <u>less</u>*** *than parent* nuclide.

-The experimental measurements showed that the ***kinetic energy*** *of the emitted* **β particles** *is* ***not discrete.*** As the ***parent*** and the ***daughter*** *nuclides have* ***only discrete*** *energy levels,* this experimental result was in contradiction to what is expected from the principle of energy conservation.
Enrico Fermi resolved this situation by ***predicting*** that, *during a β- event*, together with the **β** ***particle,*** *it* is emitted ***another "particle" that he called neutrino*** (***$m_{rest} \cong 0$, $q = 0$***).
This particle that <u>*interacts very weakly with the matter*</u> (weak interaction) was detected experimentally in 1956. The neutrino transports one part of released energy during a *β- event; this allows to explain the continuous spectrum of* **β** *-particles*.

Remember that: a) β-decay brought to the discovery of two "new" particles (***positron*** and ***neutrino***).
b) the ***energy spectrum of β-particles*** is ***continuous***.

-The complete scheme of a **β** decay event is:

$$ {}^{A}_{Z}X \Rightarrow \beta^- decay \Rightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu} \qquad \bar{\nu} \text{ is an } \textbf{\textit{antineutrino}} \tag{6}$$

or

$$ {}^{A}_{Z}X \Rightarrow \beta^+ decay \Rightarrow {}^{A}_{Z-1}Y + e^+ + \nu \qquad \nu \text{ is a } \textbf{\textit{neutrino}} \tag{7}$$

Exemples: ${}^{14}_{6}C \Rightarrow \beta^- decay \Rightarrow {}^{14}_{7}N + e^- + \bar{\nu}$ ${}^{13}_{7}N \Rightarrow \beta^+ decay \Rightarrow {}^{13}_{6}C + e^+ + \nu$

- Note that <u>*during "β" decay events, the* ***mass number A*** *of the* ***daughter*** *nucleus (**Y**) is the same as* ***mass number A*** *of parent nucleus(X)*</u>. One explains this experimental fact by considering that **β^- decay** is due to an ***initial transformation*** of one ***neutron to proton*** inside the parent nuclei following the scheme

$$n \Rightarrow p^+ + e^- + \bar{\nu} \tag{8}$$

Next, the electron and antineutrino leave the nucleus while the proton remains inside.

Similarly, the **β^+** decay is due to an ***initial transformation of one proton to neutron*** inside the parent nucleus following the scheme $p^+ \Rightarrow n + e^+ + \nu$ (9)

Notes: - **β** decay showed that *neutron* and *proton* are <u>*not true fundamental particles;*</u> they do <u>*disintegrate*</u>.
This opened the way for research of more fundamental elementary particle in physics.
- Since the rest mass of neutron is ***larger*** than rest mass of proton, the process (8) may happen even in **free space**. Meanwhile, the process (9) is **not allowed** to happen **in free space** by condition (4) but, it can happen ***<u>inside the nucleus</u>*** as a fission process. In this case the decaying proton collects the "*missing energy-mass*" from the binding energy of the system of all particles inside the nuclide.

<u>GAMMA RAYS</u>

-These are high-energy (***1KeV*** to several ***MeV***) ***photons*** that generally ***accompany α and β decay events***.
In general, after α and β decay events, ***majority of daughter nuclides are in excited levels of energy.*** Then, *daughter nuclei* fall in their ground energy level and emit γ radiation. Since the energy levels of nuclei are discrete, these ***photons have discrete spectrum of energy***.

THE DECAY LAW

- This is a ***statistical law*** of nature, i.e. knowing that a given nuclide is radioactive, it's ***impossible*** to predict *when precisely* it will decay but one may *predict the probability* that it decays during an *interval of time.* Actually, one uses the probability rules to describe the process over a set of radioactive nuclides[2].
If "***N"*** is the *total number of parent radioactive nuclides* in sample at the moment "***t"*** and "***dN"*** is the change of "*N"* due to those that decay during the interval of time (*t, t+dt),* the following relation holds on

$$\frac{dN}{dt} = -\lambda N \qquad (10)$$

The relation (10) is simply telling that the decay rate is proportional to number of parent nuclides.
The ***decay constant*** λ [1/sec] has a **characteristic** value for a given type of decay at a particular nuclide.
The ***decay rate*** $R = -\frac{dN}{dt}$ informs about the ***number of decays*** happening in an ***unit of time*** (say 1sec).

- One can get the expression that describes the evolution of number of *parent* nuclides in time as follows:

$$\frac{dN}{N} = -\lambda dt \rightarrow \int_{N_0}^{N} \frac{dN}{N} = \int_{0}^{t} -\lambda dt \Rightarrow\Rightarrow \ln N(t) - \ln N(0) = -\lambda t \rightarrow \ln \frac{N(t)}{N(0)} = -\lambda t \Rightarrow N(t) = N(0) * e^{-\lambda * t}$$

So, the number of ***remained radioactive parent nuclides*** in sample, ***N(t),*** decreases with time as

$$N(t) = N(0) * e^{-\lambda * t} \qquad (11)$$

where ***N(0***) is their number at the moment*(t=0)* when ***one starts counting*** .

- Two main ***characteristic parameters*** of a radioactive phenomena are:

a) ***Half time*** " $T_{½}$ " i.e. the time required to get to ***N(t) = N(0) / 2***. One may calculate it as follows

$$\frac{1}{2} N(0) = N(0) e^{-\lambda * T_{1/2}} \rightarrow -\ln 2 = -\lambda T_{1/2} \rightarrow \lambda T_{1/2} = 0.693 \qquad \text{and}$$

$$T_{1/2} = \frac{0.693}{\lambda} \qquad (12)$$

b) The ***evolution*** of ***decay rate*** in time $R(t) = -\frac{dN(t)}{dt} = \lambda * N(0) e^{-\lambda t} = R(0) e^{-\lambda t}$ (13)

where the ***initial decay rate*** is $R(0) = \lambda * N(0)$ (14)

Notes:

- The ***decay rate units*** are Becquerel [***1Bq = 1decay/sec***] and Curie [***1Ci = 3.7*10^{10} Bq***].
- If the *half – time is too large* , the *decay constant is very small* ($\lambda \approx 0$), $e^{-\lambda t} \cong 1$ and the laboratory measurements give a **constant value** for the ***decay rate*** $R(t) = R(0) = \lambda * N(0)$.

[2] Which is the situation in a real experiment.

RADIOACTIVE DATING

-***Basic idea is:*** *The* ***relative abundance*** *of two isotopes in a sample,* ***one stable*** *and* ***one radioactive****,* ***changes with time*** *in a determined way which depends on the* ***decay constant λ*** *of the radioactive nuclide.*

Critical requirement; One has to select and refer to a radioactive isotope with such $T_{½}$ value that the ratio of isotopes' ***abundances changes*** in a ***measurable way during the time interval*** one wants to estimate[3].

-The ***natural processes*** are often conveniently ***selective*** as to what ***nuclides*** they do incorporate during the ***formation of material structures***. ***In the simplest case, the material in its initial solidified form, i.e. at time t = 0 of its history as such, incorporates a parent nuclide and rejects the daughter nuclide***.
So, *no daughter nuclides* at all at t = 0. With this assumption, it comes out that *all the daughter nuclides contained in a sample are "born" inside material by radioactive decay of parent nuclides starting from the moment " t = 0 " when the solidified sample was formed.*
If the material incorporates both the parent and daughter nuclides " at t = 0 ", one has to make a correction for the *initial proportions* of the *parent radioactive nuclide* and its *daughter.*

The two main applications of radioactive dating concern the *measurement of the* ***time elapsed since***:
a) the rocks(***on Earth, Moon or...***) are formed b) a living organism has died

a] One uses *radioactive nuclides* ***with large*** $T_{1/2}$ like potassium $^{40}_{19}K$ or uranium $^{238}_{92}U$ for measuring the *age of rocks* by radioactive dating. The ***radioactive nuclide*** $^{40}_{19}K$ decays with $T_{1/2} = 1.25*10^9$ ***years*** to ***stable daughter nuclide*** of argon $^{40}_{18}Ar$ ($^{40}_{19}K \rightarrow \beta^+ + ^{40}_{18}Ar$) via a β+ event. The ***radioactive nuclide*** $^{238}_{92}U$ decays with $T_{1/2} = 4.5*10^9$ ***years*** (via a long chain of radioactive events) to ***the stable nuclide*** of lead $^{206}_{82}Pb$**.**

Considering that all $^{40}_{18}Ar$ inside the rocks[4] is due to radioactive ***β⁺ decay*** of potassium $^{40}_{19}K$, one estimates the rock age by use of the abundance ratio (N_{Ar} / N_K). Similarly, assuming that all of the lead $^{206}_{82}Pb$ in a rock is produced by the decay of original uranium one may estimate the rock age by use of ratio (N_{Pb} / N_U).

b] The *carbon atom in air* is met mostly inside the molecule of CO_2. The measurements have shown that there is just one ***radioactive nuclide*** $^{14}_{6}C$ ($T_{1/2} = 5730$ ***yrs***) in ~10^{13} **stable** ***nuclides*** $^{12}_{6}C$ *in air*. As the number of $^{14}_{6}C$ decreases with time (due to β⁻ decay to its ***daughter*** $^{14}_{7}N$; $^{14}_{6}C \rightarrow \beta^- + ^{14}_{7}N$), one would expect the ***ratio*** ($^{14}_{6}C/^{12}_{6}C$) in air to ***decrease with time***. But, the experimental measurements show that this ratio ***in air*** is practically ***the same*** *for a very long time. One* explains *this* by the *continuous production* of $^{14}_{6}C$ *isotopes* during the bombardment of ***nitrogen atoms*** by ***cosmic rays*** in the upper atmosphere.

-A ***living organism*** keeps a ***characteristic ratio*** ^{14}C / ^{12}C ***as long as it is living*** because of continuous exchange of CO_2 with air (***photosynthesis, breathing***). This ***ratio starts to decrease*** from the moment the organism dies (***^{14}C decreases due to its radioactivity while ^{12}C remains the same***). So, one can estimate the *time interval elapsed since* the ***organism*** died by comparison of its ratio ***^{14}C / ^{12}C*** to that of a similar living organism. This method is known as ^{14}C /^{12}C ***radioactive dating***.

[3] Most radioactive isotopes have short half-lives and lose their radioactivity within a few days or years.
[4] The daughter isotopes get trapped inside the rock material after solidification.